

1261/A21

MAY 2008

DISCRETE MATHEMATICS

Time : Three hours Maximum : 100 marks

PART A — (6 × 5 = 30 marks)

Answer any SIX questions.

1. Given an example of a relation which is neither reflexive nor irreflexive.
2. Prove that the equality of numbers on a set of real numbers is an equivalence.
3. Draw truth table for the five basic connectives.
4. Differentiate tautology and contradiction.
5. Obtain the recurrence relation whose solution is $D(k) = 5.2^k$.
6. What are the properties of a generating functions?
7. Define the terms simple, complete and regular graph with example.
8. Define isomorphism of two graphs with suitable graphs.

9. Write the properties of the operations meet and join.

10. Find atoms in the Boolean algebra of all positive divisors of 70.

PART B — (4 × 10 = 40 marks)

Answer any FOUR questions.

11. Let $R = \{(1, 2), (3, 4), (2, 2)\}$ and $S = \{(4, 2), (2, 5), (3, 1), (3, 1)\}$ find $R \circ S, S \circ R, R \circ R$ and $S \circ S$.

12. If $f: A \rightarrow B$ and $g: B \rightarrow C$ be bijective prove that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

13. Solve the recurrence relation $a(n) = a(n-1) + 2(n-1)$ with boundary condition $a(1) = 2$.

14. Prove that a tree with n vertices has $n - 1$ edges.

15. Prove that, in a simple graph, the number of odd degree vertices is always even.

16. State and prove the De Morgan's law for a complemented distributive Lattice.

PART C — (2 × 15 = 30 marks)

Answer any TWO questions.

17. Prove

$$(a) \quad P \rightarrow (Q \rightarrow R) \Leftrightarrow P \rightarrow (\sim Q \vee R) \\ \Leftrightarrow (P \wedge Q) \rightarrow R$$

(b) Object PDNF of

$$(P \wedge Q) \vee (\sim P \wedge R) \vee (Q \wedge R).$$

18. Prove that a graph is Eulerian iff all the vertices are of even degree.

19. (a) Using Karnaugh map method find the minimal sum-of-product of $f(a, b, c, d) = \sum 0, 5, 7, 8, 12, 14$.

(b) Implement Boolean function $f(x, y, z) = (x \wedge y) \vee (x' \wedge y') \vee (y' \wedge z')$ using AND, NOT gates only.