## Q. No. 1 – 25 Carry One Mark Each

1. If $x=\sqrt{-1}$, then the value of $x^x$ is:

(A) $e^{-\pi/2}$ (B) $e^{\pi/2}$ (C) x (D) 1

Answer:- (A)

Exp:- Given, $x=\sqrt{-1};\ x^x=\left(\sqrt{-1}\right)^{\sqrt{-1}}=i^i$

We know that $e^{i\theta}=\cos\theta+i\sin\theta\Rightarrow e^{i\frac{\pi}{2}}=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}=i$

$\therefore\ (i)^i=\left(e^{i\pi/2}\right)^i=e^{-\pi/2}$

2. With initial condition $x(1)=0.5$, the solution of the differential equation,

$t\frac{dx}{dt}+x=t$ is

(A) $x=t-\frac{1}{2}$ (B) $x=t^2-\frac{1}{2}$ (C) $x=\frac{t^2}{2}$ (D) $x=\frac{t}{2}$

Answer:- (D)

Exp:- Given DE is $t\frac{dx}{dt}+x=t\Rightarrow\frac{dx}{dt}+\frac{x}{t}=1$

$IF=e^{\int\frac{1}{t}dt}=e^{\log t}=t;$ solution is $x(IF)=\int(IF)\,tdt$

$xt=\int t\cdot tdt\Rightarrow xt=\frac{t^2}{2}+c;$ Given that $x(1)=0.5\Rightarrow 0.5=\frac{1}{2}+c\Rightarrow c=0$

$\therefore$ the required solution is $xt=\frac{t^2}{2}\Rightarrow x=\frac{t}{2}$

3. Two independent random variables X and Y are uniformly distributed in the interval $[-1,1]$. The probability that $\max[X,Y]$ is les than ½ is:

(A) 3/4 (B) 9/16 (C) 1/4 (D) 2/3

Answer:- (B)

Exp:-Uniform distribution X, Y on $[-1,1]; f(x)=f(y)=\frac{1}{2}$

$$P\left(\max(x,y)\le\frac{1}{2}\right)=P\left(X=\frac{1}{2},\ -1\le Y\le\frac{1}{2}\right).P\left(-1\le X\le\frac{1}{2},\ Y=\frac{1}{2}\right)$$
$$=\int_{-1}^{1/2}\frac{1}{2}dx\int_{-1}^{1/2}\frac{1}{2}dy=\frac{3}{4}\times\frac{3}{4}=\frac{9}{16}$$

4. The unilateral Laplace transform of $f(t)$ is $\frac{1}{s^2+s+1}$. The unilateral Laplace transform of $tf(t)$ is:

(A) $-\frac{s}{\left(s^2+s+1\right)^2}$ (B) $-\frac{2s+1}{\left(s^2+s+1\right)^2}$ (C) $\frac{s}{\left(s^2+s+1\right)^2}$ (D) $\frac{2s+1}{\left(s^2+s+1\right)^2}$

Answer:- (D)

Exp:- If $f(t) \leftrightarrow F(s)$, then $t f(t) \leftrightarrow -\frac{d}{ds}F(s)$

Thus if $F(s) = \frac{1}{s^2+s+1}$

$$t f(t) \to -\frac{d}{ds}\left(\frac{1}{s^2+s+1}\right) = \frac{2s+1}{s^2+s+1}$$

5. Given

$f(z) = \frac{1}{z+1} - \frac{2}{z+3}$. If C is a counter-clockwise path in the z-plane such that $|z+1| = 1$, the value of $\frac{1}{2\pi j}\oint_C f(z)\,dz$ is

(A) -2 (B) -1 (C) 1 (D) 2

Answer:- (C)

Exp:- $$\frac{1}{2\pi i}\oint_C f(z)\,dz = \frac{1}{2\pi i}\left[\underbrace{\oint_C \frac{1}{z+1}dz}_{I_1} - \underbrace{\oint_C \frac{z}{z+3}dz}_{I_2}\right]$$

$z = -1$ is singularity in c and z=-3 is not in c

By cauchy's integral formula $I_2 = \oint_C \frac{z}{z+3}dz = 0$

$\therefore I_1 = \oint_C \frac{1}{z+1}dz = 1;\ I_1 - I_2 = 1$

6. The average power delivered to an impedance $(4-j3)\Omega$ by a current $5\cos(100\pi t + 100)$ A is

(A) 44.2W (B) 50W (C) 62.5W (D) 125W

Answer:- (B)

Exp:- $Z = 4 - j3 = R_L - JX_C;\ R_L = 4;\ I = 5\cos(100\pi t + 100) = I_m\cos(\omega t + \alpha)$

$$P = \frac{1}{2}I_m^2 R_L = \frac{1}{2}\times 5^2 \times 4 = 50W$$

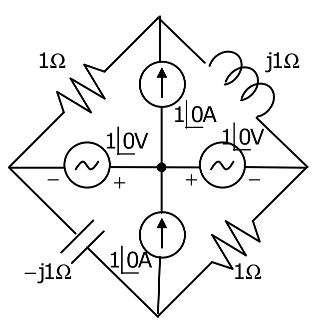
7. In the circuit shown below, the current through the inductor is:

(A) $\frac{2}{1+j}$ A

(B) $\frac{-1}{1+j}$ A

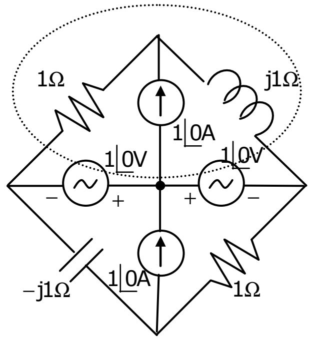
(C) $\frac{1}{1+j}$ A

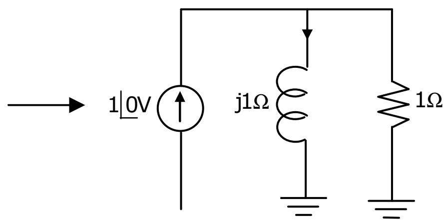
(D) 0A



Answer:- (C)

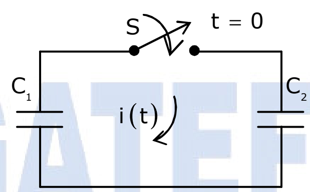
Exp:-




$$I_L = 1\angle 0 \times \frac{1}{1+j1} = \frac{1}{1+j1} A$$

8. In the following figure, $C_1$ and $C_2$ are ideal capacitors. $C_1$ has been charged to 12V before the ideal switch S is closed at t=0. The current i(t) for all t is:




(A) zero
(B) a step function
(C) an exponentially decaying function
(D) an impulse function

Answer:- (D)

Exp:- When the switch in closed at t = 0

Capacitor $C_1$ will discharge and $C_2$ will get charge since both $C_1$ and $C_2$ are ideal and there is no-resistance in the circuit charging and discharging time constant will be zero.

Thus current will exist like an impulse function.

$$i = \frac{V - 0.7}{500}$$

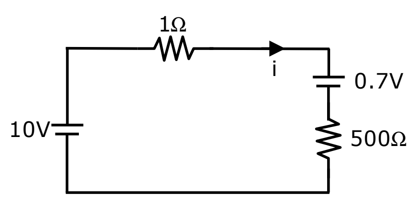
$$\frac{di}{dV} = \frac{1}{500}$$

$$\Rightarrow r_d = 500\Omega$$

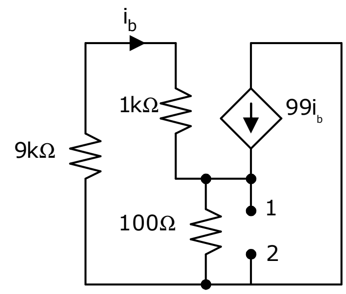
Since diode will be forward biased voltage across diode will be 0.7V

$$i = \frac{10 - 0.7}{1000 + 500}$$

$$= \frac{9.3}{1500} = 6.2\,mA$$



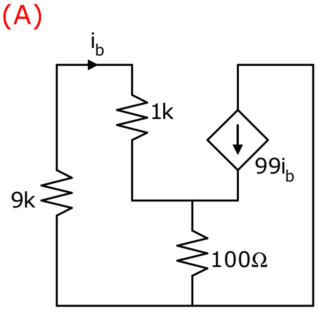
9. The impedance looking into nodes 1 and 2 in the given circuit is:

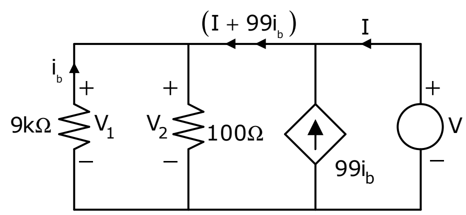



(A) 50Ω (B) 100Ω (C) 5kΩ (D) 10.1kΩ

Answer:- (A)

Exp:-




After connecting a voltage source of V

$V_1 = V_2 \Rightarrow \ (10k)(-i_b) = 100(I + 99i_b + i_b);$

$-10000i_b = 100I + 100 \times 100i_b = 100I + 10000i_b$

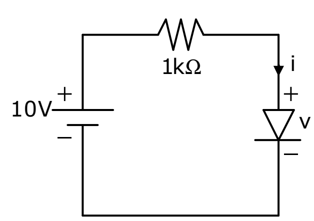
$-20000i_b = 100I \Rightarrow i_b = -\left(\frac{100}{20000}\right)I = \left[-\frac{I}{200}\right]$

$V = 100[I + 99i_b + i_b] = 100\left[I + 100\left(\frac{-I}{200}\right)\right] = 50I$

$R_{th} = \frac{V}{I} = \frac{50I}{I} = 50\Omega$

10. The i-v characteristics of the diode in the circuit given below are:

$$i = \begin{cases} \frac{v-0.7}{500} A, & v \geq 0.7V \\ 0A, & v < 0.7V \end{cases}$$




The current in the circuit is:

(A) 10mA (B) 9.3mA (C) 6.67mA (D) 6.2mA

Answer:- (D)

Exp:- $i = \frac{10 - 0.7}{1k + 500} = 6.2mA$

11. A system with transfer function

$$G(s)=\frac{(s^2+9)(s+2)}{(s+1)(s+3)(s+4)}$$ is excited by $\sin(\omega t)$. The steady state output of the system is zero at:

(A) $\omega = 1rad/s$ (B) $\omega = 2rad/s$ (C) $\omega = 3rad/s$ (D) $\omega = 4rad/s$

Answer:- (C)

Exp:- Steady state output of system is

$$y(t)=|G(j\omega)|\sin\left(\omega t+\angle G(j\omega)\right)$$

for $y(t)$ to be zero

$|G(j\omega)|$ can be zero

$$|G(j\omega)|=\frac{(-\omega^2+9)\sqrt{\omega^2+4}}{\sqrt{\omega^2+1}\sqrt{\omega^2+9}\sqrt{\omega^2+16}}$$

$\Rightarrow$ at $\omega = 3$ rad / sec

$|G(j\omega)|=0$

thus $y(t)=0$

12. The output Y of a 2-bit comparator is logic 1 whenever the 2-bit input A is greater than the 2-bit input B. The number of combinations for which the output is logic 1, is:

(A) 4 (B) 6 (C) 8 (D) 10

Answer:- (B)

Exp:-

| Input A | | Input B | | Y |
|---|---|---|---|---|
| $A_2$ | $A_1$ | $B_2$ | $B_1$ | |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 |

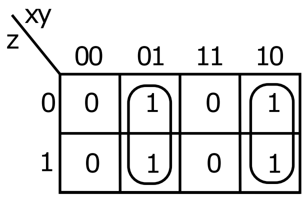
Thus for 6 combinations output in logic 1

13. In the sum of products function $f(X,Y,Z)=\sum(2,3,4,5)$, the prime implicants are:

(A) $\bar{X}Y, X\bar{Y}$
(B) $\bar{X}Y, X\bar{Y}\bar{Z}, X\bar{Y}Z$
(C) $\bar{X}Y\bar{Z},\ \bar{X}YZ,\ X\bar{Y}$
(D) $\bar{X}Y\bar{Z},\ \bar{X}YZ,\ X\bar{Y}\bar{Z},\ X\bar{Y}Z$

Answer:- (A)

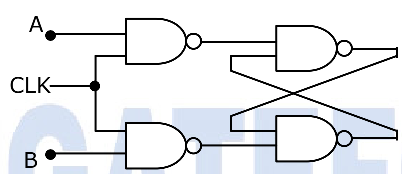
Exp:- .




Implicates are $\bar{x}y\bar{z}, \bar{x}yz, x\bar{y}\bar{z}, x\bar{y}z$

The prime implicants are $\bar{x}y$ and $x\bar{y}$

14. Consider the given circuit

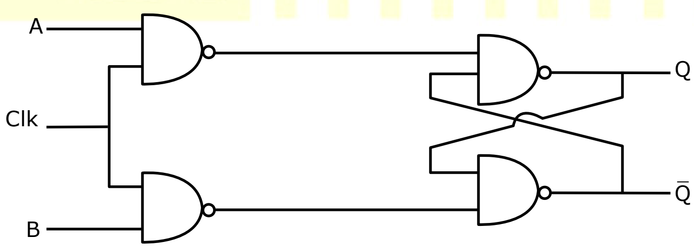



In this circuit, the race around

(A) does not occur
(B) occurs when CLK =0
(C) occurs when CLK=1 and A=B=1
(D) occurs when CLK=1 and A=B=0

Answer:- (A)

Exp:- .




$$Q_{next}=\overline{\overline{A.CLK}.\bar{Q}}$$

$$=A.CLK+Q$$

$$\bar{Q}_{next}=A.CLK+\bar{Q}$$

If CLK = 1 and A and B = 1

then $Q_{next}=1$, $\bar{Q}_{next}=1$ } No race around

If CLK = 1 and A = B = 0

$Q_{next}=Q$, $\bar{Q}_{next}=\bar{Q}$ } No race around

Thus race around does not occur in the circuit

15. If $x[n] = (1/3)^{|n|} - (1/2)^n u[n]$, then the region of convergence (ROC) of its z-transform in the z-plane will be:

(A) $\frac{1}{3} < |z| < 3$ (B) $\frac{1}{3} < |z| < \frac{1}{2}$ (C) $\frac{1}{2} < |z| < 3$ (D) $\frac{1}{3} < |z|$

Answer:- (C)

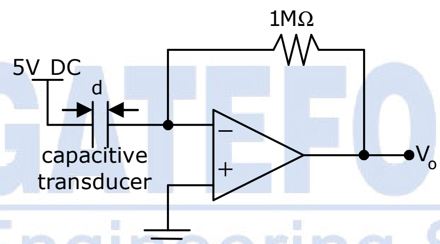
Exp:- $x[n] = \left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{2}\right)^n u[n]$

for $(1/3)^{|n|}$ ROC is $\frac{1}{3} < |z| < 3$

for $(1/2)^n u[n]$ ROC is $|z| > \frac{1}{2}$

Thus common ROC is $\frac{1}{2} < |z| < 3$

16. A capacitive motion transducer circuit is shown. The gap d between the parallel plates of the capacitor is varied as $d(t) = 10^{-3}\left[1 + 0.1\sin(1000\pi t)\right]m$. If the value of the capacitance is 2pF at t=0ms, the output voltage $V_o$ at t=2ms is:




(A) $\frac{\pi}{2}$mV (B) $\pi$mV (C) $2\pi$mV (D) $4\pi$mV

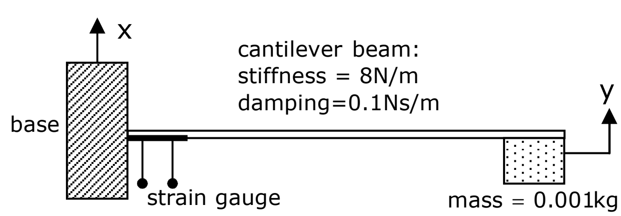
Answer:- (B)

17. A psychrometric chart is used to determine

(A) pH (B) Sound velocity in glasses
(C) $CO_2$ concentration (D) Relative humidity

Answer:- (D)

18. A strain gauge is attached on a cantilever beam as shown. If the base of the cantilever vibrates according to the equation $x(t) = \sin\omega_1 t + \sin\omega_2 t$, where 2rad/s<$\omega_1, \omega_2 < 3$rad/s then the output of the strain gauge is proportional to




(A) x (B) $\frac{dx}{dt}$ (C) $\frac{d^2x}{dt^2}$ (D) $\frac{d(x-y)}{dt}$

Answer:- (A)

19. The transfer function of a Zero-Order-Hold system with sampling interval T is:

(A) $\frac{1}{s}\left(1-e^{-Ts}\right)$ (B) $\frac{1}{s}\left(1-e^{-Ts}\right)^2$ (C) $\frac{1}{s}e^{-Ts}$ (D) $\frac{1}{s^2}e^{-Ts}$

Answer:- (A)

20. An LED emitting at 1μm with a spectral width of 50nm is used in a Michelson interferometer. To obtain a sustained interference, the maximum optical path difference between the two arms of the interferometer is:

(A) 200μm (B) 20 μm (C) 1 μm (D) 50nm

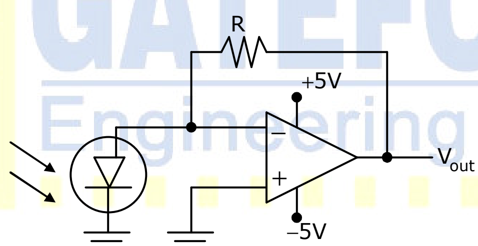
Answer:- (D)

21. Light of wavelength 630nm in vacuum, falling normally on a biological specimen of thickness 10μm, splits into two beams that are polarized at right angles. The refractive index of the tissue for the two polarizations are 1.32 and 1.333. When the two beams emerge, they are out of phase by:

(A) 0.13º (B) 74.3º (C) 90.0º (D) 128.6º

Answer:- (C)

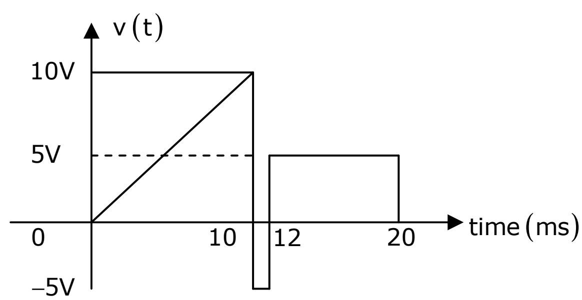
22. The responsivity of the PIN photodiode shown is 0.9A/W. To obtain $V_{out}$ of -1V for an incident optical power of 1mW, the value of R to be used is:




(A) 0.9Ω (B) 1.1Ω (C) 0.9kΩ (D) 1.1kΩ

Answer:- (C)

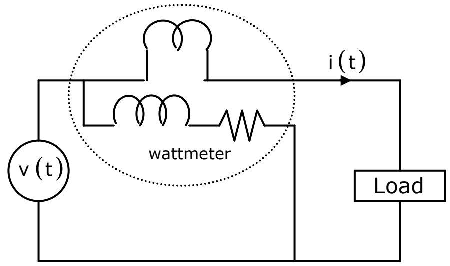
23. A periodic voltage waveform observed on an oscilloscope across a load is shown. A permanent magnet moving coil (PMMC) meter connected across the same load reads.




(A) 4V (B) 5V (C) 8V (D) 10V

Answer:- (A)

24. For the circuit shown in the figure, the voltage and current expressions are
$v(t) = E_1 \sin(\omega t) + E_3 \sin(3\omega t)$ and $i(t) = I_1 \sin(\omega t - \phi_1) + I_3 \sin(3\omega t - \phi_3) + I_5 \sin(5\omega t)$
The average power measured by the Wattmeter is:




(A) $\frac{1}{2} E_1 I_1 \cos\phi_1$ (B) $\frac{1}{2}\left[E_1 I_1 \cos\phi_1 + E_1 I_3 \cos\phi_3 + E_1 I_5\right]$

(C) $\frac{1}{2}\left[E_1 I_1 \cos\phi_1 + E_3 I_3 \cos\phi_3\right]$ (D) $\frac{1}{2}\left[E_1 I_1 \cos\phi_1 + E_3 I_1 \cos\phi_1\right]$

Answer:- (C)

25. The bridge method commonly used for finding mutual inductance is:
(A) Heaviside Campbell bridge (B) Schering bridge
(C) De Sauty bridge (D) Wien bridge

Answer:- (A)

**Q. No. 26 – 51 carry Two Marks Each**

26. A fair coin is tossed till a head appears for the first time. The probability that the number of required tosses is odd, is:
(A) 1/3 (B) 1/2 (C) 2/3 (D) ¾

Answer:- (C)

Exp:- $P(\text{odd tosses}) = P(H) + P(TTH) + P(TTTTH) + .....$

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + ...... = \frac{1}{2}\left(1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^4 + .....\right)$$

$$= \frac{1}{2}\left[1 + \left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 + ......\right] = \frac{1}{2}\left(\frac{1}{1 - \frac{1}{4}}\right) = \frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$$

27. Given that

$A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the value of $A^3$ is

(A) 15A+12I (B) 19A+30I (C) 17A+15I (D) 17A+21I

Answer:- (B)

Exp :- Given : $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$;

Characteristic equation of A is $|A\text{-}I\lambda| = 0 \Rightarrow \begin{vmatrix} -5-\lambda & -3 \\ 2 & 0-\lambda \end{vmatrix} = 0$

$\Rightarrow (-5-\lambda)(-\lambda) + 6 = 0 \Rightarrow 5\lambda + \lambda^2 + 6 = 0$

$\Rightarrow \lambda^2 = -5\lambda - 6$ and $\lambda^3 = -5\lambda^2 - 6\lambda = -5(-5\lambda - 6) - 6\lambda$

$\lambda^3 = 25\lambda - 6\lambda + 30 = 19\lambda + 30$

Every satisfies its characteristic equation; $\therefore\ A^3 = 19A + 30I$

28. The direction of vector A is radially outward from the origin, with $|A| = kr^n$ where $r^2 = x^2 + y^2 + z^2$ and k is constant. The value of n for which $\nabla \cdot A = 0$ is:

(A) -2 (B) 2 (C) 1 (D) 0

Answer:- (A)

Exp:- We know that, $\nabla.\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right)$

Now, $\nabla.\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right)$

$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^{rH2}\right) = \frac{k}{r^2}\ (n+2)r^{n+1}$

$= k\ (n+2)r^{n+1}$

$\therefore$ For, $\nabla\ .\ \vec{A} = 0, \Rightarrow (n+2) = 0 \Rightarrow n = -2$

29. The maximum value of $f(x) = x^3 - 9x^2 + 24x + 5$ in the interval $[1, 6]$ is:

(A) 21 (B) 25 (C) 41 (D) 46

Answer:- (C)

EXP:- Given, $f(x) = x^3 - 9x^2 + 24x + 5$

$f'(x) = 0$ for stationary values $\Rightarrow\ 3x^2 - 18x + 24 = 0 \Rightarrow x=2,4$

$f''(x) = 6x - 18;\ f''(2) = 12 - 18 < 0;\ f''(4) = 24 - 18 > 0$

Hence $f(x)$ has maximum value at x=2

$\therefore$ The maximum value is $2^3 - 9\times 2^2 + 24\times 2 + 5 = 25$

But we have to find the maximum value in the interval $[1, 6]$

$\therefore f(6) = 6^3 - 9\times 6^2 + 24\times 6 + 5 = 41$

30. Consider the Differential equation

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \delta(t) \text{ with } y(t)\Big|_{t=0^-} = -2 \text{ and } \frac{dy}{dt}\Big|_{t=0^-} = 0$$

The numerical value of $\frac{dy}{dt}\Big|_{t=0^+}$ is:

(A) -2 (B) -1 (C) 0 (D) 1

Answer:- (D)

Exp:- $\frac{d^2y(t)}{dt^2} + \frac{2\,dy(t)}{dt} + y(t) = \delta(t)$

Converting to s – domain,

$s^2y(s) - sy(0) - y'(0) + 2[sy(s) - y(0)] + y(s) = 1$

$[s^2 + 2s + 1]y(s) + 2s + 4 = 1$

$y(s) = \frac{-3-2s}{(s^2+2s+1)}$

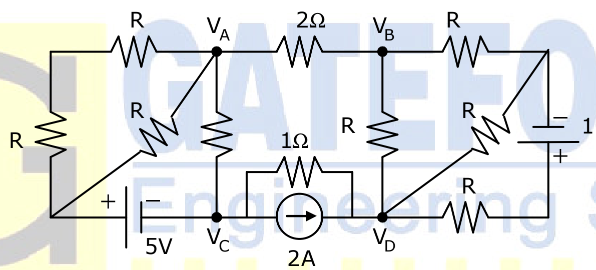
Find inverse lapalce transform

$y(t) = [-2e^{-t} - te^{-t}]u(t)$

$\frac{dy(t)}{dt} = 2e^{-t} + te^{-t} - e^{-t}$

$\left.\frac{dy(t)}{dt}\right|_{t=0^+} = 2 - 1 = 1$

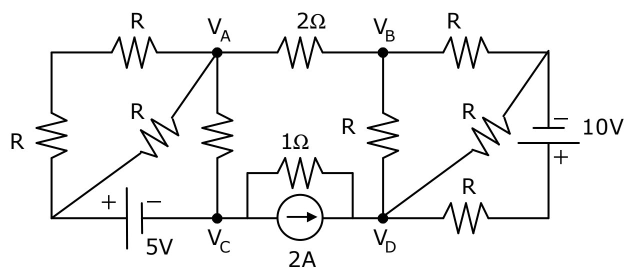
31. If $V_A - V_B = 6V$, then $V_C - V_D$ is

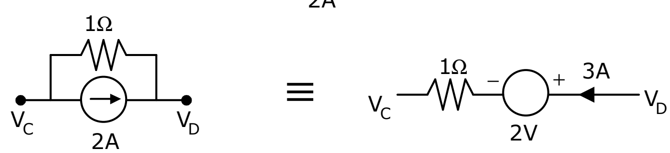



(A) -5V (B) 2V (C) 3V (D) 6V

Answer:- (A)

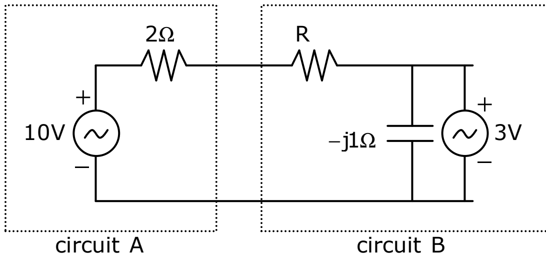
Exp:- $I = \frac{V_A - V_B}{2} = \frac{6}{2} = 3A$; Since current entering any network is same as leaving in $V_C - V_D$ branch also it is I = 3A




$V_D = 2 + 3 + V_C = 5 + V_C$; $V_C - V_D = -5V$

32. Assuming both the voltage sources are in phase, the value of R for which maximum power is transferred from circuit A to circuit B is:



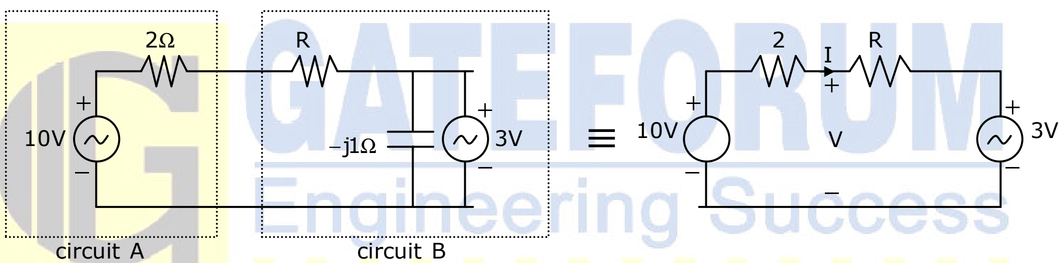

(A) 0.8Ω (B) 1.4Ω (C) 2Ω (D) 2.8Ω

Answer:- (A)

Exp:- Power transferred from circuit A to circuit A = $VI=\left(\frac{7}{R+2}\right)\left(\frac{6+10R}{R+2}\right)=\frac{42+70R}{(R+2)^2}$

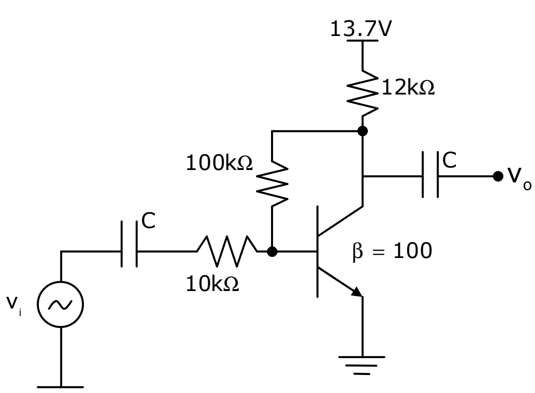
$$I=\frac{10-3}{2+R}=\frac{7}{2+R};\ V=3+IR=3+\frac{7R}{2+R}=\left(\frac{6+10R}{2+R}\right)$$

$$\frac{dP}{dR}=\frac{(R+2)^2(70)-(42+70R)2(R+2)}{(R+2)^4}=0$$




$$70(R+2)^2=(42+70R)2(R+2);\ 5(R+2)=2(3+5R)$$
$$5R+10=6+10R;\ 4=5R;\ R=0.8\Omega$$

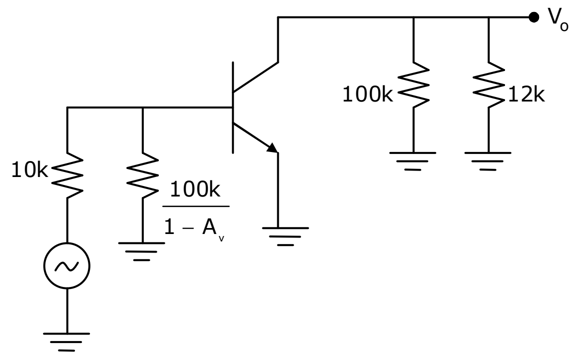
33. The voltage gain $A_v$ of the circuit shown below is:




(A) $|A_v|\approx 200$ (B) $|A_v|\approx 100$ (C) $|A_v|\approx 20$ (D) $|A_v|\approx 10$

Answer:- (D)

Exp:-




KVL in input loop, $13.7-(I_C + I_B)12k - 100k(I_B) - 0.7 = 0$

$\Rightarrow I_B = 9.9\mu A;\ I_C = \beta I_B = 0.99mA;\ I_E = 1mA$

$\therefore\ r_e = \frac{26mA}{I_E} = 26\Omega;\ z_i = \beta r_e = 2.6k\Omega;\ \therefore\ A_v = \frac{(100k\,||\,12k)}{26} = 412$

$z_i' = z_i\,||\left(\frac{100k}{1+412}\right) = 221\Omega;\ A_{vs} = A_v \frac{z_i'}{z_i' + R_s} = (412)\left(\frac{221}{221+10k}\right)$

$|A_{vs}| \approx 10$

34. The state variable description of an LTI system is given by

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{pmatrix} = \begin{pmatrix}0 & a_1 & 0\\ 0 & 0 & a_2\\ a_3 & 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} + \begin{pmatrix}0\\0\\1\end{pmatrix}u;\ y = (1\ \ 0\ \ 0)\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$$

where y is the output and u is the input. The system is controllable for:

(A) $a_1 \neq 0; a_2 = 0; a_3 \neq 0$ (B) $a_1 = 0; a_2 \neq 0; a_3 \neq 0$

(C) $a_1 = 0; a_2 \neq 0; a_3 = 0$ (D) $a_1 \neq 0; a_2 = 0; a_3 = 0$

Answer:- (D)

Exp:- The controllability matrix

$$= \begin{bmatrix} B & AB & A^2B \end{bmatrix}$$

$$A = \begin{bmatrix}0 & a_1 & 0\\ 0 & 0 & a_2\\ a_3 & 0 & 0\end{bmatrix}$$

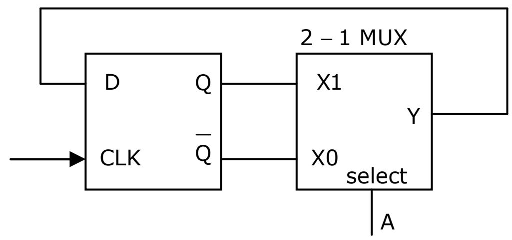
$$B = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

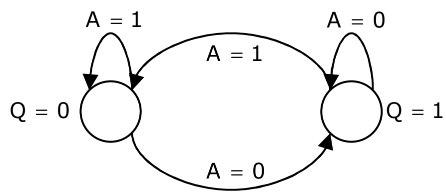
$$\Rightarrow \text{controllability matrix} = \begin{bmatrix}0 & 0 & a_1 a_2\\ 0 & a_2 & 0\\ 1 & 0 & 0\end{bmatrix}$$

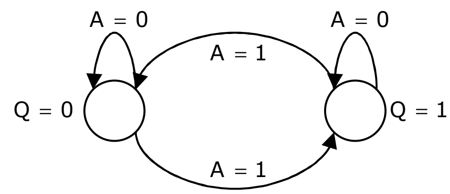
$\Rightarrow a_1 \neq 0$
$a_2 \neq 0$
$a_3$ can be zero

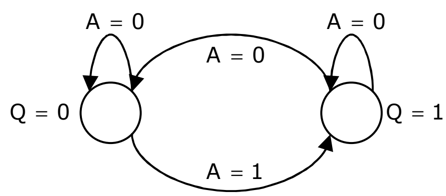
for system to be controllable, determinant of controlability matrix should not be zero

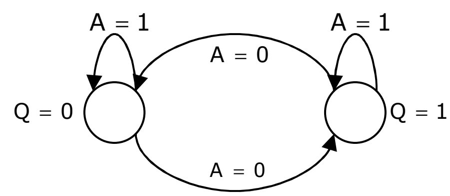
35. The state transition diagram for the logic circuit shown is:




(A)


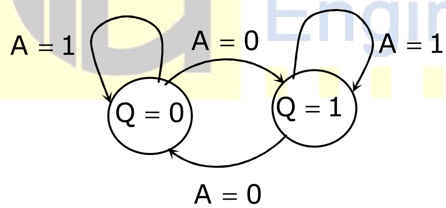

(B)



(C)



(D)



Answer:- (D)

Exp:- $\left.\begin{array}{l} A = 0, \quad y = Q \\ A = 1, \quad y = \overline{Q} \end{array}\right\}$ when ever $A = 1$, output gets into same state




whenever $A = 0$, output gets toggled

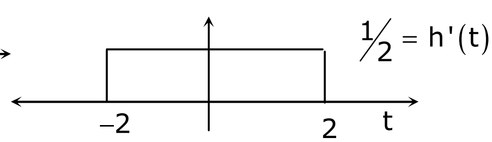
36. The Fourier transform of a signal h(t) is H(j□) $= (2\cos\omega)(\sin 2\omega)/\omega$. The value of h(0) is:

(A) 1/4 (B) 1/2 (C) 1 (D) 2

Answer:- (C)

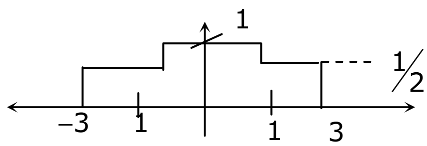
Exp:-

$\frac{\sin 2\omega}{\omega} \longleftrightarrow$




$2\cos\omega\left(\frac{\sin 2\omega}{\omega}\right) \longleftrightarrow h(t) = h'(t-1) + h'(t+1)$

$\left[e^{j\omega} + e^{-j\omega}\right]\left(\frac{\sin 2\omega}{\omega}\right)$



37. Let $y[n]$ denote the convolution of $h[n]$ and $g[n]$, where $h[n] = (1/2)^n u[n]$ and $g[n]$ is a causal sequence. If $y[0] = 1$ and $y[1] = 1/2$, then $g[1]$ equals:

(A) 0 (B) 1/2 (C) 1 (D) 3/2

Answer:- (A)

Exp:- $y[n] = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k g(n-k)$

$y[0] = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k g(-k) = 1$

$\Rightarrow \left(\frac{1}{2}\right)^O g(0) = 1$

$\Rightarrow g(0) = 1$

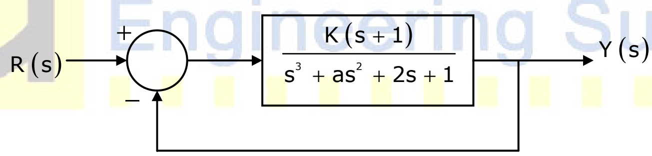
Since g(n) is Causal sequence $g(-1), g(-2), \ldots\ldots\ldots\ldots\ldots = 0$

$y[1] = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k g[1-k]$

$\Rightarrow \left(\frac{1}{2}\right)^0 g[1] + \left(\frac{1}{2}\right)^1 g(0) = 1/2$

$g[1] = 0$

38. The feedback system shown below oscillates at 2rad/s when




(A) K=2 and a=0.75 (B) K=3 and a=0.75

(C) K=4 and a=0.5 (D) K=2 and a=0.5

Answer:- (A)

Exp:- $1 + G(S)H(S) = S^3 + as^2 + (2+k)s + 1 + k$

$s^3 \quad 1 \quad (2+k)$

$s^2 \quad a \quad (2+k)$

$s \quad a(2+k) - (2+k)\,0$

$s^o \quad (1+k)^a$

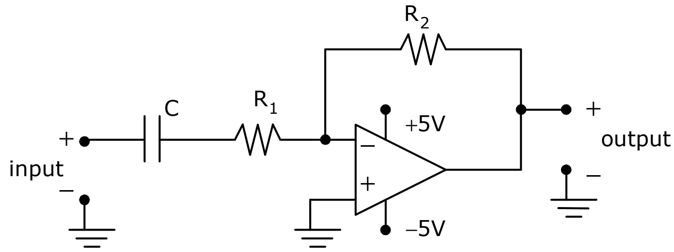
For system to oscillate $a(2+k) - (1+k) = 0 \Rightarrow a = \left(\frac{1+k}{2+k}\right)$

$A.E \Rightarrow as^2 + (1+k) = 0 \Rightarrow s = \sqrt{\frac{1+k}{a}} = 2$

$\Rightarrow 2 + k = 4 \Rightarrow k = 2$

Thus $a = 0.75$

39. The circuit shown is a




(A) low pass filter with $f_{3dB} = \frac{1}{(R_1 + R_2)C}$ rad / s

(B) high pass filter with $f_{3dB} = \frac{1}{R_1C}$ rad / s

(C) low pass filter with $f_{3dB} = \frac{1}{R_1C}$ rad / s

(D) high pass filter with $f_{3dB} = \frac{1}{(R_1 + R_2)C}$ rad / s

Answer:- (B)

Exp:- $V_0(S) = -\left(\frac{R_2}{R_1 + \frac{1}{CS}}\right)\upsilon_1(s)$

$V_0(S) = -\frac{R_2CS}{(R_1CS+1)}V_i(S)$

Thus cutoff frequency is $\frac{1}{R_1C}$

and the filter in high pass filter

40. The input $x(t)$ and output $y(t) = \int_{-\infty}^{t} x(\tau)\cos(3\tau)d\tau$ . The system is

(A) time-invariant and stable
(B) stable and not time invariant
(C) time-invariant and not stable
(D) not time-invariant and not stable

Answer:- (B)

Exp:- $y(t) = \int_{-\infty}^{t} x(\tau)\cos(3\tau)d\tau$

Since y(t) and x(t) are related with some function of time, so they are not time-invariant.

Let x(t) be bounded to some finite value k.

$y(t) = \int_{-\infty}^{t} K\cos(3\tau)d\tau < \infty$

y(t) is also bounded. Thus System is stable.

41. A double convex lens is used to couple a laser beam of diameter 5mm into an optical fiber with a numerical aperture of 0.5. The minimum focal length of the lens that should be used in order to focus the entire beam into the fiber is:

(A) 1.44mm (B) 2.50mm (C) 4.33mm (D) 5.00mm

Answer:- (B)

42. An analog voltmeter uses external multiplier settings. With a multiplier setting of $20k\Omega$, it reads 440V and with a multiplier setting of $80k\Omega$, it reads 352V. For a multiplier setting of $40k\Omega$, the voltmeter reads:

(A) 371V (B) 383V (C) 394V (D) 406V

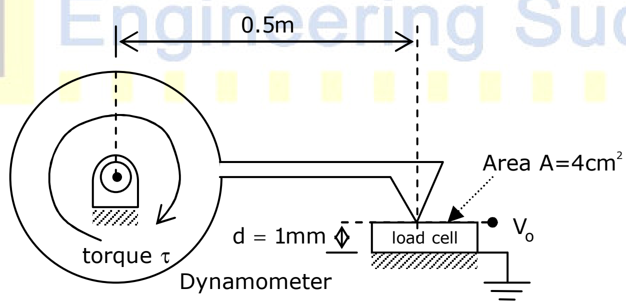
Answer:- (D)

43. The open loop transfer function of a unity negative feedback control system is given by $G(s) = \frac{150}{s(s+9)(s+25)}$. The gain margin of the system is:

(A) 10.8dB (B) 22.3dB (C) 34.1dB (D) 45.6dB

Answer:- (C)

44. A dynamometer arm makes contact with the piezoelectric load cell as shown. The g-constant of the piezoelectric material is $50\times10^{-3}\,Vm/N$ and the surface area of the load cell is $4cm^2$. If a torque $\tau = 20Nm$ is applied to the dynamometer, the output voltage $V_o$ of the load cell is:




(A) 4V (B) 5V (C) 10V (D) 16V

Answer:- (B)

45. Water (density: $1000kgm^{-3}$) stored in cylindrical drum of diameter 1m is emptied through a horizontal pipe of diameter 0.05m. A pitot static tube is placed inside the pipe facing the flow. At the time when the difference between the stagnation and static pressures measured by the pitot-static tube is 10kPa, the rate of reduction in water level in the drum is:

(A) $\frac{1}{200\sqrt{5}}ms^{-1}$ (B) $\frac{1}{75\sqrt{10}}ms^{-1}$ (C) $\frac{1}{50\sqrt{10}}ms^{-1}$ (D) $\frac{1}{40\sqrt{5}}ms^{-1}$

Answer:- (D)

46. A U-tube manometer of tube diameter D is filled with a liquid of zero viscosity. If the volume of the liquid filled is V, the natural frequency of oscillations in the liquid level about its mean position, due to small perturbations is:

(A) $\frac{D}{2\sqrt{2\pi}}\sqrt{\frac{g}{V}}$ (B) $\frac{2\sqrt{2}}{\sqrt{\pi}}\frac{\sqrt{gV}}{D^2}$ (C) $\frac{1}{2\sqrt{\pi}}\frac{\sqrt{gD}}{V^{1/3}}$ (D) $\frac{1}{\sqrt{\pi}}\sqrt{\frac{g}{D}}$

Answer:- (A)

47. The open loop transfer function of a unity gain negative feedback control system is given by $G(s)=\frac{s^2+4s+8}{s(s+2)(s+8)}$. The angle $\theta$, at which the root locus approaches the zeroes of the system, satisfies:

(A) $|\theta| = \pi - \tan^{-1}\left(\frac{1}{4}\right)$ (B) $|\theta| = \frac{3\pi}{4} - \tan^{-1}\left(\frac{1}{3}\right)$

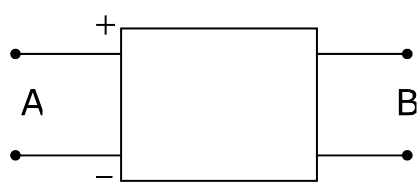
(C) $|\theta| = \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$ (D) $|\theta| = \frac{\pi}{4} - \tan^{-1}\left(\frac{1}{3}\right)$

Answer:- (D)

**Common Data for Questions: 48 & 49**

With 10V d.c. connected at port A in the linear nonreciprocal two-port network shown below, the following were observed:

1. 1ohm connected at port B draws a current of 3A
2. 2.5ohm connected at port B draws a current of 2A




48. With 10V dc connected at port A, the current drawn by 7ohms connected at port B is:

(A) 3/7A (B) 5/7A (C) 1A (D) 9/7A

Answer:- (C)

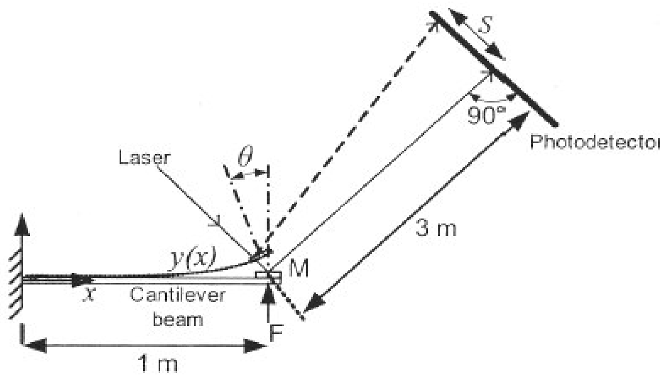
49. For the same network, with 6V dc connected at port A, 1ohm connected at port B draws 7/3A. If 8V dc is connected to port A, the open circuit voltage at port B is:

(A) 6V (B) 7V (C) 8V (D) 9V

Answer:- (B)

**Common Data for Questions: 50 & 51**

The deflection profile y(x) of a cantilever beam due to application of a point force F(in Newton) as a function of distance x from its base, is given by $y(x) = 0.001Fx^2\left(1-\frac{x}{3}\right)m$. The angular deformation $\theta$ at the end of the cantilever is measured by reflecting a laser beam off a mirror M as shown in the figure:




50. The translation S of the spot of laser on the photo-detector when a force of F=1N is applied to the cantilever is:
(A) 1mm (B) 3mm (C) 6mm (D) 12mm

Answer:- (B)

51. If linear variable differential transformers (LVDTs) are mounted at $x=\frac{1}{2}m$ and $x=\frac{1}{4}m$ on the cantilever to measure the effect of time varying forces, the ratio of their outputs is:
(A) 12/7 (B) 40/11 (C) 176/23 (D) 112/15

Answer:- (B)

**Linked Answer Questions: Q.52 to Q.55 Carry Two Marks Each**

**Statement for Linked Answer Questions: 52 & 53**

The transfer function of a compensator is given as:

$$G_c(s) = \frac{s+a}{s+b}$$

52. $G_c(s)$ is a lead compensator if:

(A) a=1, b=2 (B) a=3, b=2 (C) a=-3,b=-1 (D) a=3,b=1

Answer:- (A)

Exp:- $\phi = \tan^{-1}\frac{\omega}{a} - \tan^{-1}\frac{\omega}{\beta}$

for phase lead $\phi$ should be $+$ve $\Rightarrow \tan^{-1}\frac{\omega}{a} > \tan^{-1}\frac{\omega}{\beta}$ $\Rightarrow a < b$

both option (A) and (C) satisfier
but option (C) will pot polar and zero as
RHS of s-plane thus not possible
Option (A) is right

53. The phase of the above lead compensator is maximum at:

(A) $\sqrt{2}$rad / s (B) $\sqrt{3}$rad / s

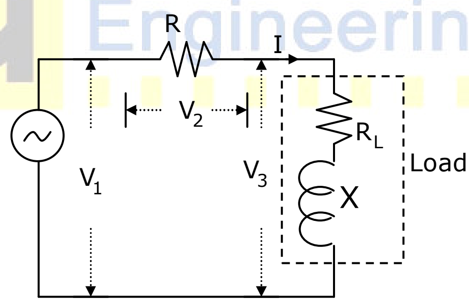
(C) $\sqrt{6}$rad / s (D) $\frac{1}{\sqrt{3}}$rad / s

Answer:- (A)

Exp:- $\omega$ = geometric mass of two carrier frequencies $= \sqrt{2 \times 1} = \sqrt{2}$ rad / sec

**Statement for Linked Answer Questions: 54 & 55**

In the circuit shown, the three voltmeter readings are $V_1 = 220V,\ V_2 = 122V,\ V_3 = 136V$




54. The power factor of the load is

(A) 0.45 (B) 0.50 (C) 0.55 (D) 0.60

Answer:- (A)

55. If $R_L = 5\Omega$, the approximate power consumption in the load is:

(A) 700W (B) 750W (C) 800W (D) 850W

Answer:- (B)

### Q. No. 56 –60 Carry One Mark Each

56. Choose the most appropriate alternative from the options given below to complete the following sentence:

**If the tried soldier wanted to lie down, he __________ the mattress out on the balcony**

(A) should take (B) shall take

(C) should have taken (D) will have taken

Answer:- (C)

57. If $(1.001)^{1259} = 3.52$ and $(1.001)^{2062} = 7.85$, then $(1.001)^{3321} =$

(A) 2.23 (B) 4.23 (C) 11.37 (D) 27.64

Answer:- (D)

Exp:- let 1.001 = x

$x^{1259} = 3.52 \text{ and } x^{2062} = 7.85$

$x^{3321} = x^{1259}.x^{2062} = 3.52 \text{ x } 7.85 = 27.64$

58. One of the parts (A, B, C, D) in the sentence given below contains an ERROR. Which one the following is **INCORRECT**?

**I requested that he should be given the driving test today instead of tomorrow.**

(A) requested that (B) should be given

(C) the driving test (D) instead of tomorrow

Answer:- (B)

59. Which one of the following options is the closest in meaning to the word given below?

**Latitude**

(A) Eligibility (B) Freedom

(C) Coercion (D) Meticulousness

Answer:- (B)

60. Choose the most appropriate word from the options given below to complete the following sentence:

**Given the seriousness of the situation that he had to face, his __ was impressive.**

(A) beggary (B) nomenclature (C) jealousy (D) nonchalance

Answer:- (D)

**Q. No. 61 –65 Carry Two Marks Each**

61. Raju has 14 currency notes in his pocket consisting of only Rs.20 notes and Rs. 10 notes. The total money value of the notes is Rs.230. The number of Rs. 10 notes that Raju has is

(A) 5 (B) 6 (C) 9 (D) 10

Answer:- (A)

Exp:- Let the number of Rs. 20 notes be x and Rs. 10 notes be y

$20x + 10y = 230$

$x + y = 14$

$x=9$ and $y=5$

Hence the numbers of 10 rupee notes are 5

62. **One of the legacies of the Roman legions was discipline. In the legions, military law prevailed and discipline was brustal. Discipline on the battlefield kept units obedient, intact and fighting, even when the odds and conditions were against them.**

Which one of the following statements best sums up the meaning of the above passage?

(A) Through regimentation was the main reason for the efficiency of the Roman legions even in adverse circumstances.

(B) The legions were treated inhumanly as if the men were animals.

(C) Discipline was the armies' inheritance from their seniors.

(D) The harsh discipline to which the legions were subjected to led to the odds and conditions being against them.



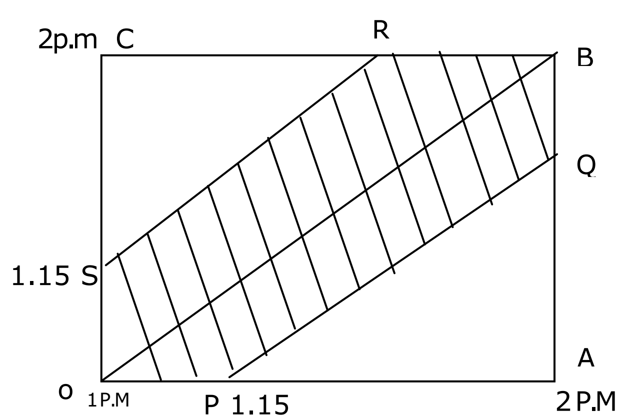
Answer:- (A)

63. A and B are friends. They decide to meet between 1 PM and 2 PM on a given day. There is a condition that whoever arrives first will not wait for the other for more than 15 minutes. The probability that they will meet on that day is

(A) 1/4 (B) 1/16 (C) 7/16 (D) 9/16

Answer:- (C)

Exp:-



OB is the line when both A and B arrive at same time.

Total sample space = 60 x 60 = 3600

Favourable cases = Area of OABC – Area of PQRS

$$= 3600 - 2 \times \left( \frac{1}{2} \times 45 \times 45 \right) = 1575$$

$\therefore$ The required probability = $\frac{1575}{3600} = \frac{7}{16}$

64. The data given in the following table summarizes the monthly budget of an average household.

| **Category** | **Amount (Rs)** |
|---|---|
| Food | 4000 |
| Clothing | 1200 |
| Rent | 2000 |
| Savings | 1500 |
| Other expenses | 1800 |

The approximate percentage of the monthly budget **NOT** spent on saving is

(A) 10% (B) 14% (C) 81% (D) 86%

Answer:- (D)

Exp:- Total budget = 10,500

Expenditure other than savings = 9000

Hence, $\frac{9000}{10500} = 86\%$



65. There are eight bags of rice looking alike, seven of which have equal weight and one is slightly heavier. The weighting balance is of unimited capacity. Using this balance, the minimum number of weighings required to identify the heavier bag is

(A) 2 (B) 3 (C) 4 (4) 8

Answer:- (A)

Let us categorize the bags in three groups as

$A_1$ $A_2$ $A_3$ $B_1$ $B_2$ $B_3$ $C_1$ $C_2$

$1^{st}$ weighing A vs B

**Case -1**

$A_1 A_2 A_3 = B_1 B_2 B_3$

Then either $C_1$ or $C_2$ is heavier

**Case -2**

$A_1 A_2 A_3 \neq B_1 B_2 B_3$

Either A or B would be heavier(Say A >B)

**2nd weighing**

$C_1$ vs $C_2$

If $C_1 > C_2$, then $C_1$

If $C_1 < C_2$, then $C_2$

If $A_1 < A_2$, then $A_2$

$A_1$ vs $A_2$

If $A_1 = A_2$, then $A_3$

If $A_1 > A_2$, then $A_1$