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First Semester B.E. Degree Examination, Dec. 07 / Jan. 08

Engineering Mathematics I

Time: 3 hrs. Max. Marks:100

Note : ***Answer any FIVE full questions choosing at least ONE from each part.***

Part A

1 a. Find the angle between the pair of lines whose direction cosines satisfy the equations, $l+2m+3n=0$ and $mn-4nl+3lm=0$. **(06 Marks)**

b. Find the equation of the plane through the points (2, 1, -2) and (-1, 0, -4) parallel to the line joining the points (1, 2, -1) and (-4, -1, 0). **(07 Marks)**

c. Find the image of the point (1, 2, 3) in the line $\frac{x+1}{2}=\frac{y-3}{3}=\frac{z}{-1}$. **(07 Marks)**

2 a. Find the shortest distance and the equation of shortest distance between the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and the x-axis. **(06 Marks)**

b. Find the equation of the right circular cone whose axis is $\frac{x-1}{-1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the generator is $\frac{x-1}{-1}=\frac{y-1}{2}=\frac{z-2}{4}$. **(07 Marks)**

c. Find the equation of the right circular cylinder for which the radius is 4 units and the axis passes through the points (1, -2, 3) and (3, -1, 1). **(07 Marks)**

Part B

3 a. If $x=\tan(\ln y)$, show that $(1+x^2)y_{n+1}+(2nx-1)y_n+n(n-1)y_{n-1}=0$. **(06 Marks)**

b. Find the angle between the tangent and the radius vector on the cardioide $r=a(1+\cos\theta)$ at the points $\theta=\pi/3$ and $2\pi/3$. **(07 Marks)**

c. Show that the curves $r=a\sec^2\frac{\theta}{2}$ and $r=b\operatorname{cosec}^2\frac{\theta}{2}$ are orthogonal. **(07 Marks)**

4 a. State and prove Euler's theorem for a homogeneous function $u(x,y)$. If $u=\sin^{-1}\frac{x}{y}+\tan^{-1}\frac{y}{x}$, find the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$. **(06 Marks)**

b. If $z=f(u,v)$ and $u=x^2-y^2$, $v=2xy$; show that
$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=4(x^2+y^2)\left(\frac{\partial^2 z}{\partial u^2}+\frac{\partial^2 z}{\partial v^2}\right).$$ **(07 Marks)**

c. If $u=x^2-2y^2$, $v=2x^2-y^2$ and $x=r\cos\theta$, $y=r\sin\theta$; evaluate $\frac{\partial(u,v)}{\partial(r,\theta)}$. **(07 Marks)**

Part C

5 a. Obtain a reduction formula for $\int\tan^n x\,dx$. **(06 Marks)**

b. Evaluate $\int_0^{\pi}\sin^6 x\cos^4 x\,dx$. **(07 Marks)**

c. Trace the curve $y^2(2a-x)=x^3$. **(07 Marks)**

6 a. Find the area of a loop of the curve $r = a\sin 3\theta$. **(06 Marks)**

b. Find the perimeter of the curve $x^{2/3} + y^{2/3} = a^{2/3}$. **(07 Marks)**

c. Find the volume of the surface formed by the revolution of the curve, $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$ about the tangent at the vertex. **(07 Marks)**

Part D

7 a. Solve $(3y + 2x + 4)dx - (4x + 6y + 5)dy = 0$. **(06 Marks)**

b. Solve $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$. **(07 Marks)**

c. Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$. **(07 Marks)**

8 a. Find the orthogonal trajectories of the family of curves $r = a(1 - \cos\theta)$. **(06 Marks)**

b. Test the convergence of the series,

$\frac{4}{3} + \frac{4.7}{3.5} + \frac{4.7.10}{3.5.7} + \ldots\ldots\ldots$ **(07 Marks)**

c. Explain conditionally convergent and absolutely convergent series with examples. **(07 Marks)**

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