Further Mathematics SL P1

2007 May

School Level 12th IB Diploma Programme

Board Exam

International Baccalaureate (IB Board)

IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

M07/5/FURMA/SP1/ENG/TZ0/XX

22077101

FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 1

Wednesday 16 May 2007 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. All students should therefore be advised to show their working.

1. *[Maximum mark: 8]*

The point $P(x, y)$ moves in such a way that its distance from the point $(1, 0)$ is three times its distance from the point $(-1, 0)$.

(a) Find the equation of the locus of P. *[4 marks]*

(b) Show that this equation represents a circle and state its radius and the coordinates of its centre. *[4 marks]*

2. *[Maximum mark: 8]*

Calculate the following limits

(a) $\lim_{x \to 0} \frac{2^x - 1}{x}$; *[3 marks]*

(b) $\lim_{x \to 0} \frac{(1+x^2)^{\frac{3}{2}} - 1}{\ln(1+x) - x}$. *[5 marks]*

3. *[Maximum mark: 12]*

(a) Show that the set S of numbers of the form $2^m \times 3^n$, where $m, n \in \mathbb{Z}$, forms a group $\{S, \times\}$ under multiplication. *[6 marks]*

(b) Show that $\{S, \times\}$ is isomorphic to the group of complex numbers $m + ni$ under addition, where $m, n \in \mathbb{Z}$. *[6 marks]*

4. *[Maximum mark: 12]*

(a) Use the Euclidean Algorithm to show that 275 and 378 are relatively prime. *[5 marks]*

(b) Find the general solution to the diophantine equation $275x + 378y = 1$. *[7 marks]*

5. *[Maximum mark: 9]*

Solve the differential equation $x\frac{dy}{dx}+2y=\sqrt{1+x^2}$

given that $y=1$ when $x=\sqrt{3}$. *[9 marks]*

6. *[Maximum mark: 11]*

The weights, X kg, of male birds of a certain species are normally distributed with mean 4.5 kg and standard deviation 0.2 kg. The weights, Y kg, of female birds of this species are normally distributed with mean 2.5 kg and standard deviation 0.15 kg.

(a) (i) Find the mean and variance of $2Y-X$.

(ii) Find the probability that the weight of a randomly chosen male bird is more than twice the weight of a randomly chosen female bird. *[6 marks]*

(b) Two randomly chosen male birds and three randomly chosen female birds are placed together on a weighing machine for which the recommended maximum weight is 16 kg. Find the probability that this maximum weight is exceeded. *[5 marks]*