IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 1

Wednesday 15 May 2002 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

You are advised to show all working, where possible. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Incorrect answers with no working will normally receive ***no*** *marks.*

1. Two independent random samples each of size of n observations are to be selected, one from each of two populations. If you wish to estimate the difference between the two population means correct to within 0.12 with probability equal to 90%, how large should n be? Assume that both variances are equal to 0.25.

2. The normal daily human potassium requirement is in the range of 2000 to 6000 milligrams. The amount of potassium in bananas is normally distributed with mean 630 mg. and standard deviation of 40 mg per banana. Anwar eats three bananas per day.

(a) Find the mean and standard deviation of Anwar's daily potassium intake.

(b) Find the probability that Anwar's daily intake exceeds the minimum requirement.

3. Find all possible remainders when $(2k + 1)^{151}$ is divided by 8, for $k \in \mathbb{Z}^+$.

4. A graph contains 22 vertices and 43 edges. Every vertex has a degree of 3 or 5. Find the number of vertices of degree 3.

5. Given that the order of a group is a prime number, prove that the group is cyclic.

6. Let $A = \left\{\sqrt{5}, -3, \frac{1}{5}, 2\pi, 6, \sqrt{20}\right\}$.

The relation R is defined on A by aRb if $\frac{a}{b} \in \mathbb{Q}$.

(a) Prove that R is an equivalence relation.

(b) Find the partition of the set A.

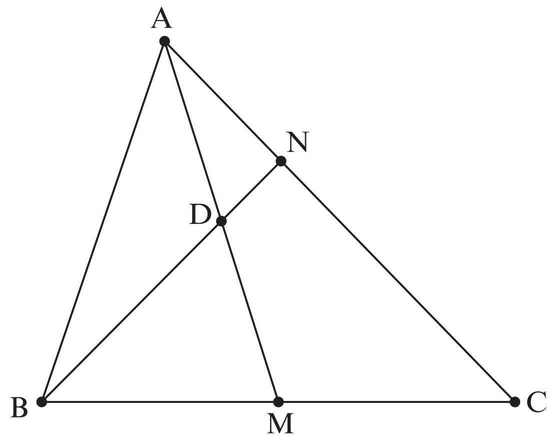
7. Determine whether the series $\sum_{n=0}^{\infty}\left(\frac{n}{n+4}\right)^n$ converges or diverges, giving clear reasons.

8. The convergent infinite sequence of positive real numbers u_n is defined recursively by

$$u_{n+1} = \sqrt{5 - 2u_n}\,, \quad n \in \mathbb{Z}^+ \,.$$

Find the **exact** value of the limit of the sequence.

9. The following diagram shows $\triangle$ABC . [AM] is a median. D is the midpoint of [AM] .



Prove that the line (BD) trisects [AC].

10. A hyperbola is defined by the parametric equations

$$x = t + \frac{1}{t}; \quad y = t - \frac{1}{t} \,.$$

Find its foci.