IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 1

Tuesday 12 November 2002 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all ten questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

*You are advised to show all working, where possible. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Incorrect answers with no working will normally receive **no** marks.*

1. Consider the group $(\mathbb{Z}_{12}, +)$.

(a) Find the order of the elements 4 , 5 and 9 .

(b) Show that this group is cyclic. Find all possible generators.

2. Consider κ_n a complete graph with n vertices.

(a) Draw κ_5 and find an Eulerian circuit in it.

(b) Find the value of n such that κ_n contains an Eulerian path but not an Eulerian circuit. Justify your answer.

3. Determine whether the following series converges or diverges.

$$\frac{1}{\sqrt{2}}+\frac{3}{2}+\frac{5}{2\sqrt{2}}+\frac{7}{4}+\frac{9}{4\sqrt{2}}+\cdots.$$

4. Find all the integers x that satisfy the equation $2x^3-3x+1\equiv 4 \pmod 6$.

5. Eggs are packed in boxes of four. During one day 200 boxes were selected and the number of broken eggs in each box was recorded.

Number of broken eggs	0	1	2	3	4
Number of boxes	73	80	31	14	2

Test at the 5 % level of significance whether this data follows a binomial distribution with $n = 4$ and $p = 0.24$.

6. The function $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 3^{\cos x} + \frac{1}{6}$.

(a) Determine whether the function is injective or surjective, giving your reasons.

(b) If the domain of f is restricted to $[0, \pi]$ find its inverse function.

7. Consider the triangle ABC. The points M, N and P are on the sides [BC], [CA] and [AB] respectively, such that the lines (AM), (BN) and (CP) are concurrent.

Given that $\frac{\text{AP}}{\text{AB}} = \lambda$, and $\frac{\text{CM}}{\text{CB}} = \mu$, where $\lambda, \mu, \in \mathbb{R}^+$, find $\frac{\text{NA}}{\text{CN}}$.

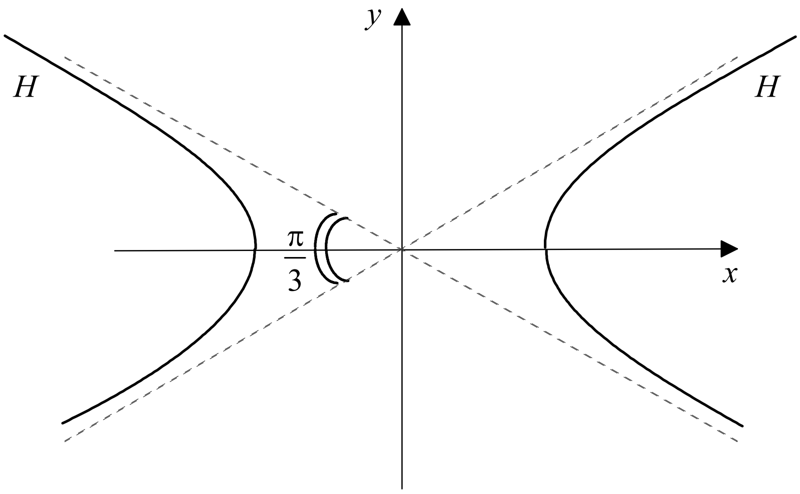
8. Find a cubic Taylor polynomial approximation for the function $f(x) = \tan x$, about $x = \frac{\pi}{4}$.

9. A school newspaper consists of three sections. The number of misprints in each section has a Poisson distribution with parameters 0.9 , 1.1 and 1.5 respectively. Misprints occur independently.

(a) Find the probability that there will be no misprints in the newspaper.

(b) The probability that there are more than n misprints in the newspaper is less than 0.5 . Find the smallest value of n.

10. Consider the hyperbola H with equation $b^2x^2 - a^2y^2 = a^2b^2$. The angle between the asymptotes of H is $\frac{\pi}{3}$, as shown in the diagram below.



(a) Calculate the eccentricity of H.

(b) Find the equations of the directrices of H, giving your answers in terms of a.