

**FURTHER MATHEMATICS  
STANDARD LEVEL  
PAPER 1**

Thursday 22 May 2003 (afternoon)

1 hour

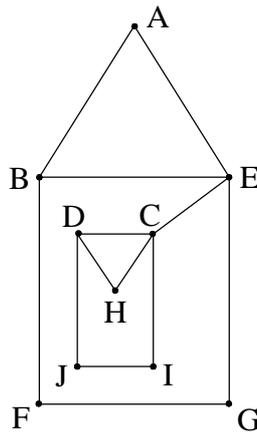
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**INSTRUCTIONS TO CANDIDATES**

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator in the appropriate box on your cover sheet  
*e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. Consider two sets  $S$  and  $T$  with a mapping  $\alpha : S \rightarrow T$ . If  $A$  and  $B$  are subsets of  $S$ , show that  $\alpha(A \cup B) = \alpha(A) \cup \alpha(B)$ .
  
2. Use the Euclidean algorithm to show that 7 is the greatest common divisor of 1001 and 357. Hence find two integers  $m$  and  $n$  such that  $7 = 1001n + 357m$ .
  
3. Let  $S_3$  be the set of permutations of  $\{1, 2, 3\}$ , under composition of permutations (i.e.  $S_3$  is the symmetric group of degree 3).
  - (a) Find a proper subgroup containing  $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ .
  - (b) Show that this subgroup is cyclic.
  
4. Use the graph below to answer the following.
  - (a) Use the depth-first search to obtain a “depth-first” numbering of the vertices.
  - (b) Use the numbering obtained to form a spanning tree of the graph.



5. A random variable  $X$  has a Poisson distribution with mean 9.

(a) If  $X_1$  and  $X_2$  are two such independent variables, find  $P(X_1 + X_2 = 10)$ .

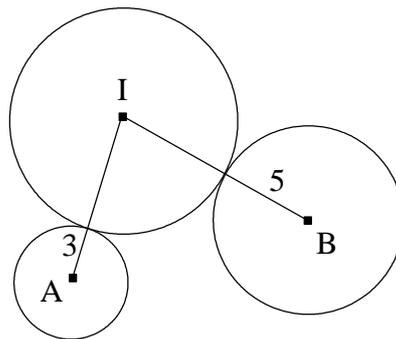
(b) The random variable  $\bar{X}$  is the mean of a random sample of 64 values of  $X$ . Find  $P(\bar{X} < 8.5)$ .

6. A series expansion of  $e^x$  is  $\sum_{k=0}^{\infty} \frac{x^k}{k!}$ . A random variable  $X$  has a Poisson distribution.

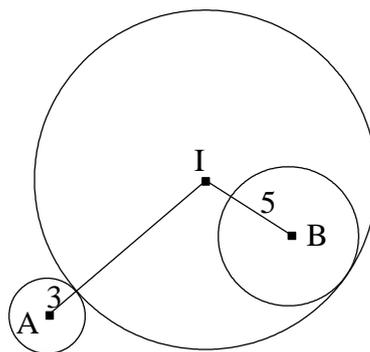
Show that  $\sum_{x=0}^{\infty} P(X = x) = 1$ .

7. A circle with centre A and radius 3 cm, and a circle with centre B and radius 5 cm are given with their centres 10 cm apart. A third circle is tangent to the two given circles simultaneously. The following diagrams show two of the four possible cases.

(i)

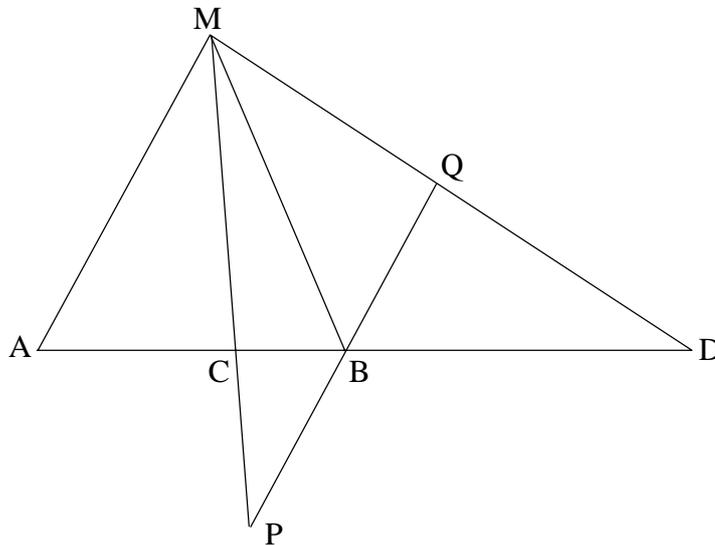


(ii)



Find and describe the locus of the centres of the third circle in the two cases shown.

8. The diagram below shows the line (AD), where the points C and D are in harmonic ratio to points A and B. The point M is outside line (AB). Line (PQ) is drawn through B parallel to (AM), where Q lies on (MD) and P on (MC).



Show that B is the midpoint of [PQ].

9. The probability density function of a random variable  $X$  is given in the table below. Calculate each of the following:

$x$	4	6	8
$P(X = x)$	0.35	0.51	0.14

- $E(X)$
  - $E(X^2)$
  - $\text{Var}(X)$
  - $\text{Var}(3X - 2)$
10. Determine whether  $\sum_{n=2}^{\infty} \frac{1}{\ln n}$  is convergent or divergent, giving reasons for your answer.