IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 1

Monday 24 May 2004 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator in the appropriate box on your cover sheet *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

http://www.xtremepapers.net

You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. The table below defines the operation $\otimes$ on the set $\{1, a, b, c, d\}$.

$\otimes$	1	a	b	c	d
1	1	a	b	c	d
a	a	c	d	b	1
b	b	1	c	d	a
c	c	d	a	1	b
d	d	b	1	a	c

Give two reasons, based on different group properties, why the table does not define a group.

2. Let X and Y be two points on the sides [AB] and [AC] of a triangle ABC such that (XY) is parallel to (BC). Let Z be the point of intersection of (BY) and (CX). Show that Z lies on the median from A.

3. Define the operation Δ on $\mathbb{Z} \times \mathbb{Z}$ by

$$(a, b) \Delta (c, d) = (ac + bd, ad + bc), \text{ where } a, b, c, d \in \mathbb{Z}.$$

Find the identity element for this operation.

4. Show that the series $\sum_{1}^{\infty} \frac{1+r}{1+r^2}$ diverges.

5. Industrial accidents in a factory occur at random at a rate of three per month.

(a) Calculate the probability that more than three accidents happen in a particular month.

(b) Let X denote the number of accidents occurring in a particular month. Find the minimum value of k such that $\mathrm{P}(X > k) < 0.05$.

6. Points A and B are chosen on the x-axis and y-axis respectively of a coordinate system such that the length AB = 18 units. P is a point on the line segment [AB] such that $\frac{\mathrm{AP}}{\mathrm{PB}} = \frac{4}{5}$. Give a full geometric description of the locus of P as A and B move along the axes.

7. The amount of liquid dispensed into a bottle by an automatic filling machine is normally distributed with preset mean μ cl and standard deviation 3 cl. The output from the machine is monitored every hour and regulations require that the mean volume of liquid contained in a randomly chosen sample of bottles from the output of the machine should lie within 0.9 cl of the preset mean μ cl.

(a) Find the probability that the regulations are satisfied when the sample contains nine bottles.

(b) Let n be the number of bottles in a sample. Find the minimum value of n so that the regulations are satisfied at least 95% of the time.

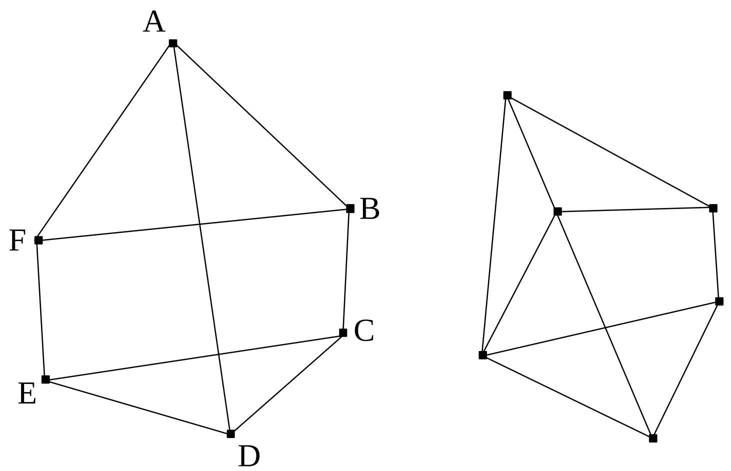
8. Let p be a positive integer and n an integer larger than 1. Consider the numbers $a = np$ and $b = (n-1)p$.

(a) Find the greatest common divisor (gcd) of a and b.

(b) Hence find the gcd of $15x(8y+5)$ and $24x(5y+3)$ where x and y are positive integers.

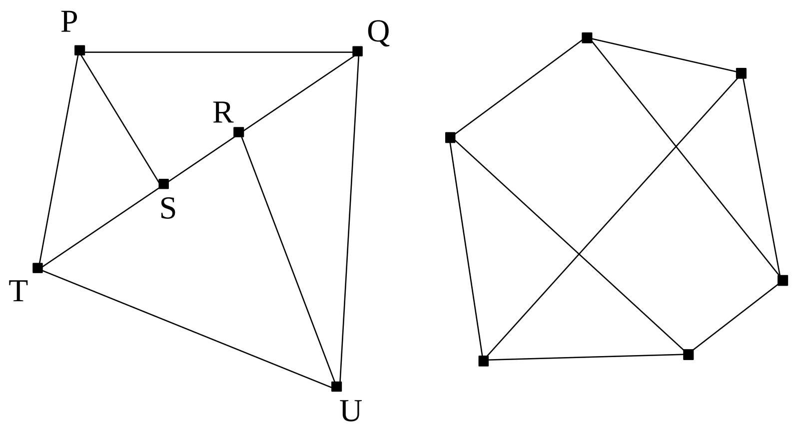
9. Consider the following two pairs of graphs, G_1, G_2 and H_1, H_2 shown in Figures 1 and 2. Determine whether or not the graphs in each pair are isomorphic. If they are isomorphic, copy and label the second one to show the isomorphism. If the two are not isomorphic, justify your answer.

Figure 1



G_1 G_2

Figure 2



H_1 H_2

10. The mean value theorem states that if a function f is continuous over $[a, a+h]$ and differentiable over $]a, a+h[$, then there exists a number θ with $0 < \theta < 1$ such that

$$f(a+h) - f(a) = hf'(a + \theta h).$$

Let $f(x) = e^x$. Find θ in terms of h.