M01/540/S(1)

INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 1

Monday 14 May 2001 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.

- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-9750G*, Sharp EL-9400, Texas Instruments TI-85.

A correct answer with **no** *indication of the method used will usually receive* **no** *marks. You are therefore advised to show your working. In particular, where graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.*

1. (a) Explain when the Yates continuity correction needs to be used, giving a reason.

(b) In 200 tosses of a coin, 108 tails and 92 heads were observed. Test the hypothesis that it is a fair coin, at a significance level of 1%.

2. Let $(G, \circ)$ be a group with identity element e. Given that $x \circ x = e$ for all $x \in G$, prove that $(G, \circ)$ is an Abelian group.

3. The profit of an internet company at the end of a given year is 8000 dollars more than twice the profit for the previous year. If the profit at the end of the first year is \$30 000, find an expression for profit at the end of the nth year, for $n = 1, 2, \ldots$.

4. Let $(\mathbb{R}, +)$ be the group of real numbers under addition, and $(\mathbb{R}^+, \times)$ be the group of positive real numbers under multiplication. Prove that the two groups are isomorphic.

5. The points T, C and D lie on a circle with centre S. A tangent [OT] and a secant [OCD] are drawn from a point O to this circle. Prove that $\mathrm{OT}^2 = \mathrm{OC} \times \mathrm{OD}$.

6. (a) Prove that the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)^7}$ converges, for $n \in \mathbb{N}$.

(b) Approximate the sum of the series to an accuracy of six decimal places.

7. Let $I = \displaystyle\int_0^5 e^{-x^2}\,dx$. Find the number n, of intervals necessary to approximate correct to two decimal places, the value of I by the trapezium rule.

8. Prove that a line $y = mx + c$ is a tangent to an ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ if $c^2 = a^2 m^2 + b^2$.

9. A manager of two coal mines wants to test the heat-producing capacity of coal from each mine. The heat-producing capacity (in millions of calories per ton) of random samples of coal from each mine is given in the following table.

Mine 1	8260	8130	8350	8070	8340	
Mine 2	7950	7890	7900	8140	7920	7840

The manager knows that the two population variances are equal.

(a) Describe the test to be used with the choice of the test statistic, giving reasons for your answers.

(b) At the 5% level of significance, test if the average heat-producing capacity of the coal from the two mines is equal.

10. Let G be a simple graph. Prove that G has a spanning tree if and only if G is connected.