INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL



**FURTHER MATHEMATICS**
**STANDARD LEVEL**
**PAPER 1**

Monday 12 November 2001 (afternoon)

1 hour

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INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.
- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

*A correct answer with* **no** *indication of the method used will usually receive* **no** *marks. You are therefore advised to show your working. In particular, where graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.*

**1.** Let $S = \{1\ ,\ 2\ ,\ 3\ ,\ 4\}$ and let $A = S \times S$. Define the relation $R$ on $A$ by:

$$(a\ ,\ b)\ R\ (x\ ,\ y) \text{ if and only if } a + b = x + y\ .$$

Show that $R$ is an equivalence relation and find the partition it creates on $A$.

**2.** Determine whether the series $\displaystyle\sum_{k=1}^{\infty} \frac{k}{\mathrm{e}^k}$ converges or diverges. Note the test you use.

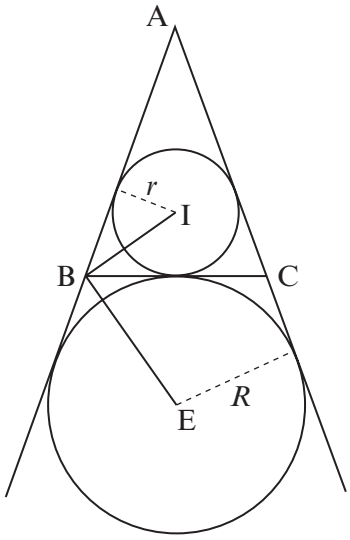
**3.** Find the order of a group $G$ generated by two elements $x$ and $y$, subject only to the following relations $x^3 = y^2 = (xy)^2 = 1$. List all subgroups of $G$.

**4.** Draw a graph given by the following adjacency matrix.

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

Determine how many graphs with the same number of edges are possible on this set of vertices.

**5.** The following diagram shows an isosceles triangle ABC, and 2 circles. The circle whose centre is I and radius is $r$ is inscribed in $\triangle ABC$. The circle whose centre is E and radius is $R$ is the escribed circle, *ie* it is outside $\triangle ABC$, and the lines (BC), (AB) and (AC) are tangents to this circle.




(a) Show that angle IBE is a right angle.

(b) Find BC in terms of $r$ and $R$.

**6.** Find the solution to the recurrence relation

$$a_n = 7a_{n-1} - 6a_{n-2}, \text{ with } a_0 = -1 \text{ and } a_1 = 4.$$

**7.** Use a binary search tree to find 43 on the following list

10 , 15 , 20 , 28 , 37 , 39 , 43 , 58 , 67 , 77 , 81 , 99 .

Show all steps.

**8.** A computer repair shop replaces corrupt hard disks at a rate of 4 per week. Assuming that such repairs occur at random, find the probability that

(a) exactly 7 hard disks are replaced in one week;

(b) in a 3-week period, at least 7 disks are replaced in two of these weeks.

**9.** In a triangle $ABC$, $AB = 8$, $AC = 10$, and the median to the side [BC] has length $8$. Find the area of the triangle.

**10.** Estimate $e^{0.2}$ correct to 3 decimal places, using the Taylor approximation

$$f(a+x) = f(a) + x f'(a) + \ldots + \frac{x^n}{n!} f^{(n)}(a) + \frac{x^{n+1}}{(n+1)!} f^{(n+1)}(c)$$