



**FURTHER MATHEMATICS  
STANDARD LEVEL  
PAPER 1**

Monday 15 May 2000 (afternoon)

1 hour

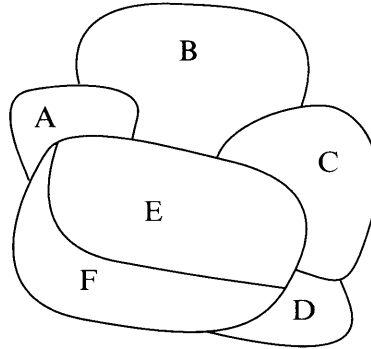
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**INSTRUCTIONS TO CANDIDATES**

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures, as appropriate.
- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-7400G*, Sharp *EL-9400*, Texas Instruments *TI-80*.

A correct answer with **no** indication of the method used will usually receive **no** marks. You are therefore advised to show your working. In particular, where graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.

1. Construct a graph for the following diagram of a map.



Use the graph to determine the number of colours required for a proper colouring of the map.

2. (a) Define and give an example of a well-ordered set.  
 (b) Prove that the intersection of two well-ordered, non-disjoint sets is well-ordered.
3. (a) The inside diameter of a cylindrical tube is a random variable with a mean of 3 inches and a standard deviation of 0.02 inches. The thickness of the tube is also a random variable with a mean of 0.3 inches and a standard deviation of 0.005 inches. Given that these two random variables are independent, calculate the mean and standard deviation of the outside diameter of the tube.  
 (b) If  $X$  is a discrete random variable, prove that  $\text{Var}(X) = E(X^2) - (E(X))^2$ .
4. (a) Explain what is meant by saying that the series  $\sum_{n=1}^{\infty} u_n$  is conditionally convergent.  
 (b) Show that the series  $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$  is conditionally convergent.
5. Let  $(G, \circ)$  be a group with four elements. Prove that  $(G, \circ)$  is Abelian.

6. Two groups of 100 students, from the graduating classes of two high schools, are randomly chosen to take a Mathematics Competency Test. The tests are given on the same day under identical conditions. It is found that 75 students from the first group and 65 from the second achieved a pass grade or better in that test. The test providers claim that 70% of graduating students would achieve a pass grade or better in the test. Test this claim at 5% level of significance.
7. An ellipse has centre O and major and minor axes of lengths  $2a$ ,  $2b$  respectively. The line segments (produced if necessary), joining any point P on the ellipse to the ends of the minor axis, meet the major axis at the points R and Q. Prove that

$$OQ \times OR = a^2.$$

8. Show that two positive integers  $a$  and  $b$  are congruent modulo  $m$  ( $m < a$  and  $m < b$ ) if and only if they have the same remainder when divided by  $m$ , where  $m$  is a positive integer.
9. Show that the series  $S_n = \sum_{k=1}^n \frac{(-1)^{k+1}}{\ln(k+1)}$  converges as  $n \rightarrow \infty$ . Hence find the approximate value of  $S_{10}$  to four decimal places, and find the maximum error in that estimation.
10. The sides [AB], [BC], [CD] and [AD] of a quadrilateral ABCD (produced if necessary) are cut by a transversal in the points P, Q, R and S, respectively. Prove that

$$\frac{AP}{PB} \times \frac{BQ}{QC} \times \frac{CR}{RD} \times \frac{DS}{SA} = 1.$$


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