MATHEMATICS

Time allowed : 3 hours ***Maximum Marks : 100***

General Instructions:

1. ***All*** *questions are compulsory.*
2. *The question paper consists of* ***29*** *questions divided into three sections, A, B and C. Section A comprises of* ***10*** *questions of one mark each, Section B comprises of* ***12*** *questions of* ***four*** *marks each and Section C comprises of* ***7*** *questions of* ***six*** *marks each.*
3. ***All*** *questions in Section A are to be answered in one word,* ***one*** *sentence or as per the exact requirement of the question.*
4. *There is no overall choice. However, internal choice has been provided in* ***4*** *questions of* ***four*** *marks each and* ***2*** *questions of* ***six*** *marks each. You have to attempt only* ***one*** *of the alternatives in all such questions.*
5. *Use of calculators is* ***not*** *permitted.*

QUESTION PAPER CODE 65/1/1
SECTION A

Question numbers 1 to 10 carry 1 mark each.

1. State the reason for the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} not to be transitive.

2. Write the value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

3. For a 2 × 2 matrix, A = $[a_{ij}]$, whose elements are given by $a_{ij} = \frac{i}{j}$, write the value of a_{12}.

4. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular ?

5. Write A^{-1} for $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$.

6. Write the value of $\int \sec x (\sec x + \tan x)\, dx$.

7. Write the value of $\int \frac{dx}{x^2 + 16}$

8. For what value of 'a' the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear?

9. Write the direction cosines of the vector $-2\hat{i} + \hat{j} - 5\hat{k}$.

10. Write the intercept cut off by the plane $2x + y - z = 5$ on x-axis.

SECTION - B

Question numbers 11 to 22 carry 4 marks each.

11. Consider the binary operation * on the set {1, 2, 3, 4, 5} defined by $a * b = \min. \{a, b\}$. Write the operation table of the operation *.

12. Prove the following:

$$\cot^{-1}\left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right] = \frac{x}{2},\ x \in \left(0, \frac{\pi}{4}\right)$$

OR

Find the value of $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$

13. Using properties of determinants, prove that

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$$

14. Find the value of 'a' for which the function f defined as

$$f(x)=\begin{cases} a\sin\frac{\pi}{2}(x+1), & x\le 0\\ \frac{\tan x-\sin x}{x^3}, & x>0\end{cases}$$

is continuous at x = 0.

15. Differentiate $x^{x\cos x}+\frac{x^2+1}{x^2-1}$ w.r.t. x

OR

If $x = a(\theta - \sin\theta)$, $y = a(1+\cos\theta)$, find $\frac{d^2y}{dx^2}$

16. Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the grou;nd in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4cm?

OR

Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to x-axis.

17. Evaluate: $\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$

OR

Evaluate: $\int \frac{2x}{(x^2+1)(x^2+3)}dx$

18. Solve the following differential equation:

$e^x \tan y\, dx + (1-e^x)\sec^2 y\, dy = 0$

19. Solve the following differential equation:

$\cos^2 x\frac{dy}{dx}+y=\tan x.$

20. Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$.

21. Find the angle between the following pair of lines:

$$\frac{-x+2}{-2}=\frac{y-1}{7}=\frac{z+3}{-3} \text{ and } \frac{x+2}{-1}=\frac{2y-8}{4}=\frac{z-5}{4}$$

and check whether the lines are parallel or perpendicular.

22. Probabilities of solving a specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that (i) the problem is solved (ii) exactly one of them solves the problem.

SECTION - C

Question numbers 23 to 29 carry 6 marks each.

23. Using matrix method, solve the following system of equations:

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2;\quad x, y, z \neq 0$$

OR

Using elementary transformations, find the inverse of the matrix

$$\begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}$$

24. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

25. Using integration find the area of the triangular region whose sides have equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

26. Evaluate: $\int_0^{\pi/2} 2 \sin x \cos x \tan^{-1}(\sin x)\, dx$

OR

Evaluate: $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\, dx$

27. Find the equation of the plane which contains the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$, $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$.

28. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time. If the profit on a racket and on a bat is Rs. 20 and Rs. 10 respectively, find the number of tennis rackets and crickets bats that the factory must manufacture to earn the maximum profit. Make it as an L.P.P. and solve graphically.

29. Suppose 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.

QUESTION PAPER CODE 65/1
SECTION A

Question numbers 1 to 10 carry 1 mark each.

1. Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. State whether f is one-one or not.

2. What is the principal value of $\cot^{-1}\left(\cos \frac{2\pi}{3}\right) + \sin^{-1}\left(\sin \frac{2\pi}{3}\right)$?

3. Evaluate :

$$\begin{vmatrix} \cos 15^o & \sin 15^o \\ \sin 75^o & \cos 75^o \end{vmatrix}$$

4. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, write A^{-1} in terms of A.

5. If a matrix has 5 elements, write all possible orders it can have.

6. Evaluate:

$$\int (ax + b)^3 \, dx$$

7. Evaluate:

$$\int \frac{dx}{\sqrt{1-x^2}}$$

8. Write the direction-cosines of the line joining the points (1, 0, 0) and (0, 1, 1).

9. Write the projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$.

10. Write the vector equation of a line given by $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$.

SECTION B

Question numbers 11 to 22 carry 4 marks each.

11. Let $f : R \to R$ be defined as $f(x) = 10x + 7$. Find the function $g : R \to R$ such that $gof = fog = I_R$.

OR

A binary operation * on the set {0, 1, 2, 3, 4, 5} is defined as :

$$a * b = \begin{cases} a+b, & \text{if } a+b < 6 \\ a+b-6, & \text{if } a+b \geq 6 \end{cases}$$

Show that zero is the identity for this operation and each element 'a' of the set is, invertible with 6 – a, being the inverse of 'a'.

12. Prove that:

$$\tan^{-1}\left|\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right| = \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x, -\frac{1}{\sqrt{2}} \le x \le 1$$

13. Using properties of determinants, solve the following for x :

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 2x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

14. Find the relationship between 'a' and 'b' so that the function 'f' defined by:

$$f(x) = \begin{cases} ax+1, & \text{if } x \le 3 \\ bx+3, & \text{if } x > 3 \end{cases} \text{ is continuous at } x = 3.$$

OR

If $x^y = e^{x-y}$, show that $\dfrac{dy}{dx} = \dfrac{\log x}{\{\log (x e)\}^2}$.

15. Prove that $y = \dfrac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function in $\left[0, \dfrac{\pi}{2}\right]$.

OR

If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its surface area.

16. If $x = \tan\left(\dfrac{1}{a}\log y\right)$, show that

$$(1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$$

17. Evaluate:

$$\int_0^{\pi/2} \frac{x + \sin x}{1 + \cos x} \, dx$$

18. Solve the following differential equation:

$$x \, dy - y \, dx = \sqrt{x^2 + y^2} \, dx$$

19. Solve the following differential equation:

$$(y + 3x^2) \frac{dx}{dy} = x$$

20. Using vectors, find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).

21. Find the shortest distance between the following lines whose vector equations are:

$$\vec{r} = (1 - t)\,\hat{i} + (t - 2)\,\hat{j} + (3 - 2t)\,\hat{k} \quad \text{and}$$

$$\vec{r} = (s + 1)\,\hat{i} + (2s - 1)\,\hat{j} - (2s + 1)\,\hat{k}$$

22. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2 + K$

Determine:

(i) K

(ii) $P(X < 3)$

(iii) $P(X > 6)$

(iv) $P(0 < X < 3)$

OR

Find the probability of throwing at most 2 sixes in 6 throws of a single die.

SECTION C

Question numbers 23 to 29 carry 6 marks each.

23. Using matrices, solve the following system of equations:

$$4x + 3y + 2z = 60$$

$$x + 2y + 3z = 45$$

$$6x + 2y + 3z = 70$$

24. Show that the right-circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

OR

A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the rectangle that will produce the largest area of the window.

25. Evaluate:

$$\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$$

OR

Evaluate:

$$\int \frac{6x+7}{\sqrt{(x-5)(x-4}} dx$$

26. Sketch the graph of $y = |x + 3|$ and evaluate the area under the curve $y = |x + 3|$ above x-axis and between $x = -6$ to $x = 0$.

27. Find the distance of the point (- 1, - 5, - 10), from the point of intersection of the line $\vec{r} = \left(2\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ and the plane $\vec{r} \cdot \left(\hat{i} - \hat{j} + \hat{k}\right) = 5$.

28. Given three identical boxes I, II and III each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in box III, there is one gold

and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold?

29. A merchant plans to sell two types of personal computers - a desktop model and a portable, model that will cost Rs. 25,000 and Rs. 40,000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs. 70 lakhs and his profit on the desktop model is Rs. 4,500 and on the portable model is Rs. 5,000. Make an L.P.P. and solve it graphically.

Marking Scheme — Mathematics

General Instructions :

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question(s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

QUESTION PAPER CODE 65/1/1

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Q. No. Marks

1-10. 1. $(1,2)\in R,\ (2,1)\in R$ but $(1,1)\notin R$ 2. 1 3. $a_{12} = \frac{1}{2}$

4. $x = 3$ 5. $A^{-1} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$ 6. $\tan x + \sec x + c$ 7. $\frac{1}{4}\tan^{-1}\frac{x}{4} + c$

8. $a = -4$ 9. $-\frac{2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}$ 10. $5/2$ $1\times10 = 10$ m

SECTION - B

11.

a*b	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

1½ marks, for correct structuring of table.

½ mark for each correct row / column 4 m

12. $\text{LHS} = \cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$

$\therefore\ \text{LHS} = \tan^{-1}\left[\frac{\left(\cos x/2 + \sin x/2\right) + \left(\cos x/2 - \sin x/2\right)}{\left(\cos x/2 + \sin x/2\right) - \left(\cos x/2 - \sin x/2\right)}\right]$ 2 m

$= \cot^{-1}\left[\cot x/2\right]$ 1 m

$= \frac{x}{2}$ 1 m

OR

$$\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right) = \tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{\frac{x}{y}-1}{1+x/y}\right)$$ 1 m

$$= \tan^{-1}\left(\frac{x}{y}\right) - \left(\tan^{-1}\frac{x}{y} - \frac{\pi}{4}\right)$$ 2 m

$$= \tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x}{y}\right) + \frac{\pi}{4} = \frac{\pi}{4}$$ 1 m

13. Taking a, b, c respectivly common from R_1, R_2, R_3 toget

$$\text{LHS} = \text{Determinant} = abc \begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$$ 1 m

Taking a, b, c respectivly common from C_1, C_2, C_3 toget

$$\text{LHS} = a^2 b^2 c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$ 1 m

Applying $R_2 \to R_2 + R_1,\ R_3 \to R_3 + R_1$, toget

$$\text{LHS} = a^2 b^2 c^2 \begin{vmatrix} -1 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{vmatrix}$$ 1 m

$$= a^2 b^2 c^2 \cdot (-1)(-4) = 4a^2 b^2 c^2 = \text{RHS}$$ 1 m

14. $\text{L.H.L.} = \lim_{x\to 0^-} f(x) = a$ 1 m

$f(0) = a \cdot \sin \pi/2 = a$ ½ m

$$\text{R.H.L.} = \lim_{x\to 0^+} \frac{\tan x}{x} \cdot \frac{(1-\cos x)}{x^2}$$

$$= \lim_{x \to 0^+} \frac{\tan x}{x} \cdot 2\left(\frac{\sin x/2}{2\,x/2}\right)^2 = \frac{1}{2}$$ 2 m

$$\therefore \; a = \frac{1}{2}$$ ½ m

15. $y = x^{x\cos x} + \dfrac{x^2+1}{x^2-1} = u + v$ ½ m

$u = x^{x\cos x} \Rightarrow \log u = x\cos x \cdot \log x$ ½ m

$$\therefore \; \frac{1}{u} \cdot \frac{du}{dx} = \frac{x\cos x}{x} + \cos x \cdot \log x - x\sin x \log x$$

$$\Rightarrow \frac{du}{dx} = x^{x\cos x}\,(\cos x + \log x \cdot \cos x - x\sin x\log x)$$ 1 m

$$v = \frac{x^2+1}{x^2-1}, \quad \frac{dv}{dx} = \frac{(x^2-1)2x - (x^2+1)2x}{(x^2-1)^2} = -\frac{4x}{(x^2-1)^2}$$ 1 m

$$\therefore \; \frac{dy}{dx} = x^{x\cos x}\,(\cos x + \log x \cdot \cos x - x\sin x\log x) - \frac{4x}{(x^2-1)^2}$$ 1 m

OR

$$\therefore \; \frac{dx}{d\theta} = a\,(1-\cos\theta) \text{ and } \frac{dy}{d\theta} = -a\sin\theta$$ 1 m

$$\Rightarrow \frac{dy}{dx} = -\frac{\sin\theta}{1-\cos\theta}$$ 1 m

$$\frac{d^2y}{dx^2} = \frac{(1-\cos\theta)\;(-\cos\theta) + \sin\theta\,(\sin\theta)}{(1-\cos\theta)^2} \cdot \frac{d\theta}{dx}$$ 1 m

$$= \frac{(1-\cos\theta)}{(1-\cos\theta)^2} \cdot \frac{1}{a\,(1-\cos\theta)}$$

$$= \frac{1}{a\,(1-\cos\theta)^2} \text{ or } \frac{1}{4a}\,\mathrm{cosec}^4\,\theta/2$$ 1 m

16. Here $\frac{dv}{dt} = 12\ cm^3/s$ and $h = \frac{1}{6} r$ 1 m

$v = \frac{1}{3} \pi r^2 h$ ½ m

$= \frac{1}{3} \pi (6h)^2 \cdot h = 12 \pi h^3$ ½ m

$\frac{dv}{dt} = 36\pi h^2 \cdot \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{36\pi h^2} \frac{dv}{dt}$ 1 m

when $h = 4$ cm, $\frac{dh}{dt} = \frac{1}{36\pi (16)} 12 = \frac{1}{48\pi}$ cm/s 1 m

OR

$x^2 + y^2 - 2x - 3 = 0 \Rightarrow 2x + 2y \frac{dy}{dx} - 2 = 0$ 1 m

$\therefore \frac{dy}{dx} = \frac{1-x}{y}$ ½ m

since tangents are parallel to x axis $\therefore \frac{1-x}{y} = 0 \Rightarrow x = 1$ 1 m

$\Rightarrow (1)^2 + y^2 - 2(1) - 3 = 0 \Rightarrow y^2 = 4 \therefore y = \pm 2$ 1 m

Hence the points are $(1, 2), (1, -2)$ ½ m

17. $I = \int \frac{5x+3}{\sqrt{x^2+4x+10}} dx = \int \frac{\frac{5}{2}(2x+4) - 7}{\sqrt{x^2+4x+10}} dx$ 1 m

$= \frac{5}{2} \int \frac{2x+4}{\sqrt{x^2+4x+10}} dx - 7 \int \frac{1}{\sqrt{(x+2)^2 + (\sqrt{6})^2}} dx$ 1 m

$= 5 \cdot \sqrt{x^2+4x+10} - 7 \log\left| (x+2) + \sqrt{x^2+4x+10} \right| + c$ 1+1 m

OR

$$I = \int \frac{2x}{(x^2+1)(x^2+3)}\,dx = \int \frac{dt}{(t+1)(t+3)} \text{ where } t = x^2$$ 1 m

$$= \frac{1}{2}\int \left[\frac{1}{(t+1)} - \frac{1}{(t+3)}\right] dt$$ 1 m

$$= \frac{1}{2}\left[\log|t+1| - \log|t+3|\right] + c$$ 1 m

$$= \frac{1}{2}\left[\log(x^2+1) - \log(x^2+3)\right] + c$$ 1 m

18. Given differential equation can be written as

$$\frac{e^x}{1-e^x}\,dx + \frac{\sec^2 y}{\tan y}\,dy = 0$$ 1 m

Integrating to get $-\log|1-e^x| + \log|\tan y| = \log|c|$ 2 m

$$\log|\tan y| = \log|c(1-e^x)|$$ ½ m

$$\therefore \quad \tan y = c(1-e^x)$$ ½ m

19. Given differential equation can be written as

$$\frac{dy}{dx} + \sec^2 x \cdot y = \tan x \cdot \sec^2 x$$ ½ m

$$\text{I.F.} = e^{\int \sec^2 x\,dx} = e^{\tan x}$$ 1 m

$\therefore$ The solution is $y \cdot e^{\tan x} = \int \tan x \cdot e^{\tan x} \sec^2 x\,dx$ 1 m

$$= \int t\,e^t\,dt, \text{ where } \tan x = t$$

$$\Rightarrow y \cdot e^{\tan x} = (t-1)\,e^t + c$$ 1 m

$$\Rightarrow y \cdot e^{\tan x} = (\tan x - 1)\,e^{\tan x} + c$$ ½ m

or $y = (\tan x - 1) + c \cdot e^{-\tan x}$

20. $\vec{a} + \vec{b} = 4\hat{i} + 4\hat{j}, \quad \vec{a} - \vec{b} = 2\hat{i} + 4\hat{k}$ 1 m

A vector perpendicular to $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ is $\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} - \vec{b}\right)$ ½ m

$$\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} - \vec{b}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix} = 16\hat{i} - 16\hat{j} - 8\hat{k}$$ 1½ m

$\therefore$ Unit vector in the direction of $\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} - \vec{b}\right)$ is

$$\frac{16}{24}\hat{i} - \frac{16}{24}\hat{j} - \frac{8}{24}\hat{k} = \frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$$ 1 m

21. The direction ratios of given lines are

$2, 7, -3$ and $-1, 2, 4$ 1 m

Let θ be the angle between these lines, then

$$\cos\theta = \frac{2(-1) + 7(2) + (-3)4}{\sqrt{4+49+9} \cdot \sqrt{1+4+16}} = 0$$ 1 m

$$\Rightarrow \theta = \pi/2$$ 1 m

Hence the lines are perpendicular to each other. 1 m

22. Here $P(A) = \frac{1}{2} \quad \therefore \quad P(\text{Not } A) = \frac{1}{2}$

and $P(B) = \frac{1}{3} \quad \therefore \quad P(\text{Not } B) = \frac{2}{3}$ 1 m

(i) P (problem is solved) = 1 – P (problem is not solved) ½ m

$= 1 - P(\text{Not } A) \cdot P(\text{Not } B)$ ½ m

$$= 1 - \frac{1}{2} \cdot \frac{2}{3} = \frac{2}{3}$$ ½ m

(ii) P (exactly one of them solves) = $P(A) \cdot P(\overline{B}) + P(B) \cdot P(\overline{A})$ ½ m

$$= \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{1}{2}$$ ½ m

$$= \frac{1}{2}$$ ½ m

SECTION - C

23. Writing the given system of equations as

$$\begin{pmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{pmatrix} \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} \text{ or } A \cdot X = B$$ 1 m

$|A| = 2(120 - 45) - 3(-80 - 30) + 10(36 + 36) = 1200, \therefore X = A^{-1}B$ 1 m

Cofactors are
$C_{11} = 75, \quad C_{12} = 110, \quad C_{13} = 72$
$C_{21} = 150, \quad C_{22} = -100, \quad C_{23} = 0$
$C_{31} = 75, \quad C_{32} = 30, \quad C_{33} = -24$
1 mark for any 4 correct cofactors 2 m

$$A^{-1} = \frac{1}{1200} \begin{pmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{pmatrix}$$ ½ m

$$\therefore \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{pmatrix} = \frac{1}{1200} \begin{pmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{pmatrix}$$

$\therefore \quad x = 2,\ y = 3,\ z = 5$ 1½ m

OR

Given matrix A can be written as

$$\begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$$ 1 m

Applying $R_2 \to R_2 + 3R_1 \;\; R_3 \to R_3 - 2R_1$ $\begin{pmatrix} 1 & 3 & -2 \\ 0 & 9 & -7 \\ 0 & -5 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} A$ 1 m

$$R_2 \to R_2 + 2R_3 \Rightarrow \begin{pmatrix} 1 & 3 & -2 \\ 0 & -1 & 1 \\ 0 & -5 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 2 \\ -2 & 0 & 1 \end{pmatrix} A$$ ½ m

$$R_3 \to R_3 + 5R_2 \Rightarrow \begin{pmatrix} 1 & 3 & -2 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 2 \\ 3 & -5 & -9 \end{pmatrix} A$$ ½ m

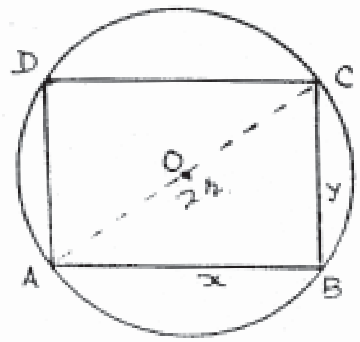
$$R_2 \to R_2 + R_3,\ R_3 \to -R_3 \begin{pmatrix} 1 & 3 & -2 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -4 & -7 \\ -3 & 5 & 9 \end{pmatrix} A$$ 1 m

$$R_1 \to R_1 + 2R_3,\ R_2 \to -R_2 \begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -5 & 10 & 18 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{pmatrix} A$$ 1 m

$$R_1 \to R_1 - 3R_2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{pmatrix} A$$ ½ m

$$\therefore \quad A^{-1} = \begin{pmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{pmatrix}$$ ½ m

24.



Let ABCD be the rectangle of maximum area inscribed in a circle of radius r. For Figure — 1 m

If AB = x and BC = y, then

$x^2 + y^2 = 4r^2 \Rightarrow y^2 = 4r^2 - x^2$ — 1 m

and Area (A) = xy

let $S = x^2 y^2 = x^2 (4r^2 - x^2) = 4r^2 x^2 - x^4$ — 1 m

$$\frac{ds}{dx} = 0 \Rightarrow 8r^2x - 4x^3 = 0$$

$$\Rightarrow x^2 = 2r^2 \Rightarrow x = \sqrt{2}\, r$$ — 1 m

$$\frac{d^2s}{dx^2} = 16r^2 - 12x^2 = 16r^2 - 12(2r^2) = -8r^2 < 0$$ — 1 m

$\therefore$ For maximum area $x = \sqrt{2}$ r and $y = \sqrt{4r^2 - 2r^2} = \sqrt{2}$ r

$\therefore$ square has the maximum area. — 1 m

25.

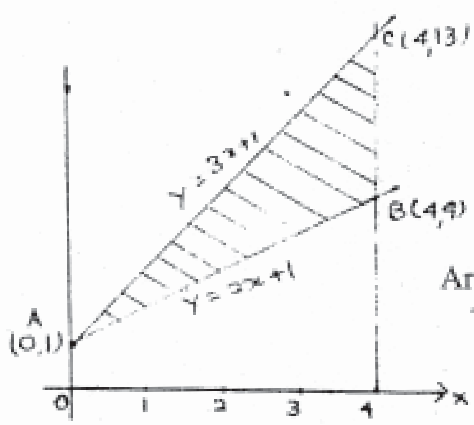


Figure — 1 m

Getting the points of intersection as

A (0, 1), B (4, 9) and C (4, 13) — 1½ m

$$\text{Area } \Delta ABC = \int_0^4 (3x+1)\,dx - \int_0^4 (2x+1)\,dx$$ — 2 m

$$= \int_0^4 x\,dx = \left[\frac{x^2}{2}\right]_0^4 = 8 \text{ sq.units}$$ — 1½ m

26. $I = \int_0^{\pi/2} 2 \sin x \cos x \tan^{-1}(\sin x)\,dx = \int_0^1 \tan^{-1} t \cdot (2t)\,dt$ where $\sin x = t$ — 1 m

$$= \left[\tan^{-1} t \times t^2\right]_0^1 - \int_0^1 \frac{t^2}{t^2+1}\,dt$$ — 1 m

$$= \frac{\pi}{4} - \int_0^1 \left(1 - \frac{1}{t^2+1}\right) dt$$ ½+1 m

$$= \frac{\pi}{4} - \left[t - \tan^{-1} t\right]_0^1$$ 1 m

$$= \frac{\pi}{4} - \left[1 - \frac{\pi}{4}\right] = \left(\frac{\pi}{2} - 1\right)$$ ½+1 m

OR

$$I = \int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx \text{ ......(i)}$$

$$x \to \left(\frac{\pi}{2} - x\right)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\left(\pi/2 - x\right) \sin\left(\pi/2 - x\right) \cos\left(\pi/2 - x\right)}{\sin^4\left(\pi/2 - x\right) + \cos^4\left(\pi/2 - x\right)} dx = \int_0^{\frac{\pi}{2}} \frac{\left(\pi/2 - x\right) \cos x \sin x \, dx}{\sin x^4 + \cos x^4} \text{ ....(ii)}$$ 1+1 m

$$2I = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{\tan x \cdot \sec^2 x \; dx}{1 + \tan^4 x}$$ 1+1 m

$$I = \frac{\pi}{4} \frac{1}{2} \int_0^{\infty} \frac{dt}{1+t^2} \text{ where } t = \tan^2 x$$ 1 m

$$= \frac{\pi}{8} \cdot \left[\tan^{-1} t\right]_0^{\infty} = \frac{\pi}{8} \cdot \frac{\pi}{2} = \frac{\pi^2}{16}$$ 1 m

27. Given equation of planes can be written as 1 m

$$x + 2y + 3z - 4 = 0, \text{ ......(i)}$$

and $2x + y - z + 5 = 0$, ...(ii) 1 m

Equation of plane through the intersection of (i) and (ii) is

$x + 2y + 3z - 4 + \lambda\ (2x + y - z + 5) = 0$ 1 m

$\Rightarrow\ (1 + 2\lambda)x + (2 + \lambda)y + (3 - \lambda)z - 4 + 5\lambda = 0$ ....(iii) 1½ m

Plane (iii) is perpendicular to the plane $5x + 3y - 6z + 8 = 0$

$\therefore\ 5(1 + 2\lambda) + 3(2 + \lambda) - 6(3 - \lambda) = 0 \Rightarrow \lambda = \frac{7}{19}$ 1½ m

$\therefore$ Equation of plane is $33x + 45y + 50z - 41 = 0$ 1 m

or $\vec{r} \cdot (33\hat{i} + 45\hat{j} + 50\hat{k}) - 41 = 0$

28. Let number of tennis rackets be 'x' and cricket bats be 'y'

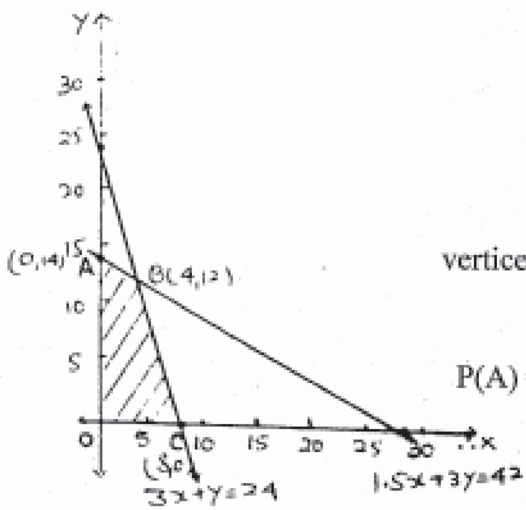
$\therefore$ LPP is

Maximise $z = 20x + 10y$ 1 m

s.t. $1.5x + 3y \le 42$

$3x + y \le 24$

$x \ge 0,\ y \ge 0$ 2 m



For correct graph 2 m

vertices of feasible region are

A (0, 14), B(4, 12), C(8, 0)

P(A) = Rs. 140, P(B) = Rs. 200, P(C) = Rs. 160

For Maximum profit

Number of Rackets = 4 1 m

Number of bats = 12

29. Let event E_1 : A male is selected

E_2 : A female is selected

A : selected person is grey haired 1 m

$P(E_1) = \frac{1}{2},\ P(E_2) = \frac{1}{2}$ 1 m

$P(A/E_1) = \frac{5}{100} = \frac{1}{20},\ P(A/E_2) = \frac{25}{10000} = \frac{1}{400}$ 1 m

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$ 1 m

$$= \frac{\frac{1}{2} \cdot \frac{1}{20}}{\frac{1}{2} \cdot \frac{1}{20} + \frac{1}{2} \cdot \frac{1}{400}} = \frac{1}{1+\frac{1}{20}} = \frac{20}{21}$$ 2 m

QUESTION PAPER CODE 65/1

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1-10. 1. f is a one-one function 2. π 3. Zero 4. $A^{-1} = \frac{1}{19} A$

5. $1\times5,\ 5\times1$ 6. $\frac{1}{4a}(ax+b)^4 + c$ 7. $\sin^{-1}x + c$ 8. $-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

9. zero 10. $\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + (3\hat{i} + 7\hat{j} + 2\hat{k})$ $1\times10 = 10$ m

SECTION - B

11. Let $y = 10x + 7 \quad \therefore \quad x = \frac{1}{10}(y-7)$

let $g(y) = \frac{1}{10}(y-7)$ 1 m

$\therefore \quad gof(x) = g(10x+7) = \frac{1}{10}(10x+7-7) = x \Rightarrow I_R = gof$ 1½ m

and $\therefore \quad fog(y) = f\left(\frac{1}{10}(y-7)\right) = 10\left(\frac{1}{10}(y-7)\right) + 7 = y \Rightarrow I_R = fog$ 1½ m

Hence $g(y) = \frac{1}{10}(y-7)$

OR

$$\left.\begin{array}{ll} \text{since} & a * 0 = a + 0 = a \\ \text{and} & 0 * a = 0 + a = a \end{array}\right\} \quad \forall\, a \in \{0, 1, 2, 3, 4, 5\}$$ 2 m

$\therefore$ 0 is the identity for *.

Also, $\forall\, a \in \{0, 1, 2, 3, 4, 5\}$, $a * (6 - a) = a + (6 - a) - 6$

$= 0$ (which is identity) 2 m

$\therefore$ Each element 'a' of the set is invertible with (6 – a), being the inverse of 'a'.

12. Putting $x = \cos\theta$ to get LHS $= \tan^{-1}\left[\dfrac{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}\right]$ 1 m

$\therefore$ LHS $= \tan^{-1}\left[\dfrac{\cos\theta/2 - \sin\theta/2}{\cos\theta/2 + \sin\theta/2}\right] = \tan^{-1}\left[\tan\left(\dfrac{\pi}{4}-\theta/2\right)\right]$ 1+1 m

$= \dfrac{\pi}{4} - \dfrac{1}{2}\theta = \dfrac{\pi}{4} - \dfrac{1}{2}\cos^{-1} x$ 1 m

13. Applying $C_2 \to C_2 - 2C_1$ and $C_3 \to C_3 - 3C_1$, we get

$$\begin{vmatrix} x-2 & 1 & 2 \\ x-4 & -1 & -4 \\ x-8 & -11 & -40 \end{vmatrix} = 0$$ 2 m

Applying $R_1 \to R_1 + R_2$ and $R_3 \to R_3 - 11R_2$, We get

$$\begin{vmatrix} 2x-6 & 0 & -2 \\ x-4 & -1 & -4 \\ -10x+36 & 0 & 4 \end{vmatrix} = 0$$ 1 m

Expanding along C_2, we get $-1[8x - 24 - 20x + 72] = 0$

or $12x = 48$ i.e. $x = 4$ 1 m

14. L.H.L. $= 3a + 1$

$f(3) = 3a + 1$ 2 m

RHL $= 3b + 3$

since f(x) is continuous at $x = 3$, $\therefore$ $3a + 1 = 3b + 3$ 1 m

or $3a - 3b = 2$, which is the required relation. 1 m

OR

$x^y = e^{x-y} \Rightarrow y \cdot \log x = (x - y)\log e = x - y$ 1 m

$\Rightarrow y = \dfrac{x}{1+\log x}$ ½ m

$$\frac{dy}{dx} = \frac{(1+\log x)\cdot 1 - x\cdot\left(\frac{1}{x}\right)}{(1+\log x)^2} = \frac{\log x}{(1+\log x)^2}$$ 1+1 m

$$= \frac{\log x}{(\log e + \log x)^2} = \frac{\log x}{[\log(xe)]^2}$$ ½ m

15. $$\frac{dy}{d\theta} = \frac{(2+\cos\theta)\,4\cos\theta - 4\sin\theta\,(-\sin\theta)}{(2+\cos\theta)^2} - 1$$ 1 m

$$= \frac{8\cos\theta + 4\left(\cos^2\theta + \sin^2\theta\right) - \left(4+\cos^2\theta + 4\cos\theta\right)}{(2+\cos\theta)^2}$$ ½ m

$$= \frac{4\cos\theta - \cos^2\theta}{(2+\cos\theta)^2} = \frac{4-\cos\theta}{(2+\cos\theta)^2}\cdot\cos\theta$$ 1 m

since, $\frac{4-\cos\theta}{(2+\cos\theta)^2} > 0$ for $\forall\,\theta$ and $\cos\theta \geq 0$ in $\left[0, \frac{\pi}{2}\right]$ 1 m

$\therefore\ \frac{dy}{d\theta} \geq 0$ in $\left[0, \frac{\pi}{2}\right]$, Hence the function is increasing in $\left[0, \frac{\pi}{2}\right]$ ½ m

OR

Here $r = 9$ cm and $\Delta r = 0.03$ cm. ½ m

Error in surface area $\Delta A \approx \frac{dA}{dr}\cdot\Delta r$ where $A = 4\pi r^2$ 1 m

$$\approx 8\pi r\cdot\Delta r$$ 1 m

$$\approx 8\pi\,(9)\,(0.03) = 2.16\pi\ \text{cm}^2$$ 1½ m

16. $x = \tan\left(\frac{1}{a}\log y\right) \Rightarrow \log y = a\tan^{-1}x\ \therefore\ y = e^{a\cdot\tan^{-1}x}$ 1 m

$$\frac{dy}{dx} = e^{a\cdot\tan^{-1}x}\cdot\frac{a}{1+x^2}$$ 1 m

$$\Rightarrow (1+x^2)\frac{dy}{dx} = a \cdot y$$ ½ m

differentiating again w.r.t., x we get

$$(1+x^2)\frac{d^2y}{dx^2} + 2x \cdot \frac{dy}{dx} = a \cdot \frac{dy}{dx}$$ 1 m

$$(1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$$ ½ m

17. $I = \int_0^{\pi/2} \frac{x}{1+\cos x} dx + \int_0^{\pi/2} \frac{\sin x}{1+\cos x} dx$ ½ m

$$= \int_0^{\pi/2} \cdot x \cdot \frac{1}{2}\sec^2\frac{x}{2} dx + \int_0^{\pi/2} \tan\frac{x}{2} dx$$ 1 m

$$= \left[x \tan\frac{x}{2}\right]_0^{\pi/2} - \int_0^{\pi/2} \tan\frac{x}{2} dx + \int_0^{\pi/2} \tan\frac{x}{2} dx$$ 1½ m

$$= \frac{\pi}{2}\ 1 - 0 = \frac{\pi}{2}$$ 1 m

18. Given equation can be written as $\frac{dy}{dx} = \frac{y}{x} + \sqrt{1+\left(\frac{y}{x}\right)^2}$ ½ m

$$\Rightarrow v + x\frac{dv}{dx} = v + \sqrt{1+v^2} \quad \text{where } \frac{y}{x} = v$$ 1 m

$$\Rightarrow \int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x}$$ ½ m

$$\Rightarrow \log\left|v + \sqrt{1+v^2}\right| = \log cx$$ 1 m

$$\Rightarrow v + \sqrt{1+v^2} = cx \quad \therefore \quad y + \sqrt{x^2+y^2} = cx^2$$ 1 m

19. Given equation can be written as

$$x\frac{dy}{dx} - y = 3x^2 \quad \text{or} \quad \frac{dy}{dx} - \frac{1}{x}\cdot y = 3x$$ 1 m

$$\text{I.F.} = e^{-\int \frac{1}{x}dx} = e^{-\log x} = e^{\log\frac{1}{x}} = \frac{1}{x}$$ 1 m

$$\therefore \text{ solution is, } y\cdot\frac{1}{x} = \int 3x\cdot\frac{1}{x}\,dx = 3x + c$$ 1½ m

$$\Rightarrow \quad y = 3x^2 + cx$$ ½ m

20. Area Δ ABC $= \frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{BC}\right|$ 1 m

Here, $\overrightarrow{AB} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\overrightarrow{BC} = -\hat{i} + 2\hat{j}$ 1 m

$$\overrightarrow{AB}\times\overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 2 & 0 \end{vmatrix} = -6\hat{i} - 3\hat{j} + 4\hat{k}$$ 1 m

$$\Rightarrow \text{Area} = \frac{1}{2}\sqrt{36+9+16} = \frac{1}{2}\sqrt{61}\text{ sq. units}$$ 1 m

21. Equations of the lines are,

$$\vec{r} = \left(\hat{i} - 2\hat{j} - 3\hat{k}\right) + t\left(-\hat{i} + \hat{j} - 2\hat{k}\right) \text{ and}$$ ½ m

$$\vec{r} = \left(\hat{i} - \hat{j} - \hat{k}\right) + s\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$$

shortest distance $= \dfrac{\left|\left(\vec{a}_2 - \vec{a}_1\right)\cdot\left(\vec{b}_1\times\vec{b}_2\right)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}$ where ½ m

$\vec{a}_1 = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{a}_2 = \hat{i} - \hat{j} - \hat{k}$, $\vec{b}_1 = -\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b}_2 = \hat{i} + 2\hat{j} - 2\hat{k}$ ½ m

$\vec{a_2} - \vec{a_1} = \hat{j} - 4\hat{k}$, $\vec{b_1} \times \vec{b_2} = 2\hat{i} - 4\hat{j} - 3\hat{k}$ ½+1 m

$\therefore$ S.D. $= \dfrac{0-4+12}{\sqrt{29}} = \dfrac{8}{\sqrt{29}}$ 1 m

22. Here $k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1 \Rightarrow 10k^2 + 9k - 1 = 0$ 1 m

$\Rightarrow (10k-1)(k+1) = 0 \Rightarrow k = \dfrac{1}{10}$

$\therefore$ (i) $k = \dfrac{1}{10}$ ½ m

(ii) $P(x < 3) = 0 + k + 2k = 3k = \dfrac{3}{10}$ 1 m

(iii) $P(x > 6) = 7k^2 + k = \dfrac{7}{100} + \dfrac{1}{10} = \dfrac{17}{100}$ 1 m

(iv) $P(0 < x < 3) = k + 2k = 3k = \dfrac{3}{10}$ ½ m

OR

Here n = 6, probability of success (p) $= \dfrac{1}{6}$

probability of failure (q) $= \dfrac{5}{6}$ ½ m

P (at most 2 sixes) = P(0) + P(1) + P(2) 1 m

$= 6_{C_0}\left(\dfrac{1}{6}\right)^0 \cdot \left(5/6\right)^6 + 6_{C_1}\left(\dfrac{1}{6}\right)^1 \cdot \left(5/6\right)^5 + 6_{C_2}\left(\dfrac{1}{6}\right)^2 \cdot \left(5/6\right)^4$ 1½ m

$= \left(5/6\right)^6 + \left(5/6\right)^5 + \dfrac{1}{2}\left(5/6\right)^5 = \dfrac{7}{3}\left(5/6\right)^5$ 1 m

SECTION - C

23. Given system of equations can be written as

$$\begin{pmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 60 \\ 45 \\ 70 \end{pmatrix} \text{ or } A \cdot X = B$$ 1 m

$|A| = 4(0) - 3(-15) + 2(-10) = 45 - 20 = 25 \neq 0 \therefore X = A^{-1}B$ 1 m

Cofactors are $\begin{pmatrix} C_{11} = 0 & C_{12} = +15 & C_{13} = -10 \\ C_{21} = -5 & C_{22} = 0 & C_{23} = 10 \\ C_{31} = 5 & C_{32} = -10 & C_{33} = 5 \end{pmatrix}$ 1 mark for any 4 correct cofactors 2 m

$$\therefore A^{-1} = \frac{1}{25}\begin{pmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{pmatrix}$$ ½ m

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{25}\begin{pmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{pmatrix}\begin{pmatrix} 60 \\ 45 \\ 70 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \\ 8 \end{pmatrix}$$

$\therefore \quad x = 5, y = 8, z = 8$ 1½ m

24. Let radius of cone be r and height h

$$\therefore \; v = \frac{1}{3}\pi r^2 h \text{ (given)} \Rightarrow h = \frac{3v}{\pi r^2}$$ 1 m

$$\text{C.S.A.} = A = \pi r l = \pi r\sqrt{r^2 + h^2} = \pi r\sqrt{r^2 + \frac{9v^2}{\pi^2 r^4}}$$ 1 m

$$\text{Let } S = \pi^2 r^2\left(r^2 + \frac{9v^2}{\pi^2 r^4}\right) = \pi^2 r^4 + \frac{9v^2}{r^2}$$ ½ m

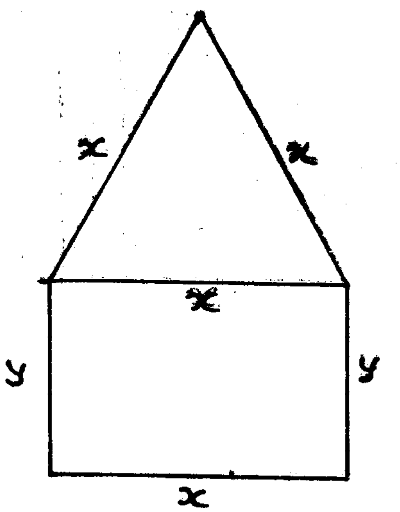
$$\frac{ds}{dr} = 0 \Rightarrow 4\pi^2 r^3 - \frac{18v^2}{r^3} = 0 \quad \textit{or} \quad 18v^2 = 4\pi^2 r^6$$ 1½ m

$$\Rightarrow 18\left(\frac{1}{9}\pi^2 r^4 h^2\right) = 4\pi^2 r^6 \Rightarrow h = \sqrt{2}\, r$$ 1 m

$$\frac{d^2s}{dr^2} = 12\pi^2 r^2 + \frac{54v^2}{r^4} > 0 \Rightarrow$$ curved surface area is least when $h = \sqrt{2}\, r$ 1 m

OR

Correct fiqure 1 m



let sides of rectangle be x and y and the sides of equilateral triangle be x

$$\therefore \quad 3x + 2y = 12 \Rightarrow y = \frac{12-3x}{2}$$ 1 m

$$\text{Area} = xy + \sqrt{3}\frac{x^2}{4}$$ 1 m

$$= x\frac{(12-3x)}{2} + \sqrt{3}\frac{x^2}{4}$$ ½ m

$$A = \frac{1}{4}\left[24x - 6x^2 + \sqrt{3}x^2\right]$$

$$\frac{dA}{dx} = 0 \Rightarrow 24 - 12x + 2\sqrt{3}x = 0$$

$$\Rightarrow x = \frac{24}{12-2\sqrt{3}} \text{ or } \frac{4\left(6+\sqrt{3}\right)}{11} \text{ m}$$ 1 m

$$\therefore \quad y = \frac{30-6\sqrt{3}}{11} \text{ m}$$ ½ m

$$\frac{d^2A}{dx^2} = (-12 + 2\sqrt{3}) < 0$$ $\therefore$ Area is maximum for 1 m

$$x = \frac{4\left(6+\sqrt{3}\right)}{11} \text{ m and } y = \frac{30-6\sqrt{3}}{11} \text{ m}$$

25. $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\, dx$ .... (i) 1 m

$x \to \left(\pi/3 + \pi/6 - x\right)$ 1 m

$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\pi/2 - x\right)}}{\sqrt{\cos\left(\pi/2 - x\right)}+\sqrt{\sin\left(\pi/2 - x\right)}}\, dx = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx$ ....(ii) 1 m

Adding (i) and (ii) to get

$2I = \int_{\pi/6}^{\pi/3} 1.\, dx = [x]_{\pi/6}^{\pi/3} = \pi/3 - \pi/6 = \pi/6$ 1+1 m

$\Rightarrow I = \pi/12$ 1 m

OR

$I = \int \frac{6x+7}{\sqrt{(x-5)(x-4)}}\, dx = \int \frac{6x+7}{\sqrt{x^2-9x+20}}\, dx$ 1 m

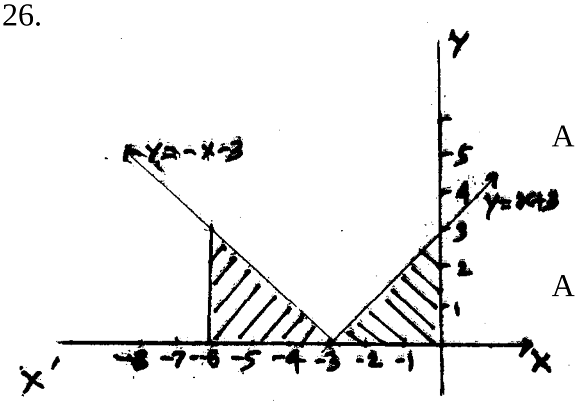
$= \int \frac{3(2x-9)+34}{\sqrt{x^2-9x+20}}\, dx$ 1 m

$= 3\int \frac{2x-9}{\sqrt{x^2-9x+20}}\, dx + 34\int \frac{dx}{\sqrt{\left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2}}\, dx$ 1½

$= 3.2\sqrt{x^2-9x+20} + 34.\log\left|\left(x-\frac{9}{2}\right) + \sqrt{x^2-9x+20}\right| + c$ 1+1

$= 6\cdot\sqrt{x^2-9x+20} + 34.\log\left|\left(\frac{2x-9}{2}\right) + \sqrt{x^2-9x+20}\right| + c$ ½

26. For correct graph — 1 m



$$A = \int_{-6}^{-3} -(x+3)\,dx + \int_{-3}^{0} (x+3)\,dx$$ 2 m

$$A = \left[-\frac{(x+3)^2}{2}\right]_{-6}^{-3} + \left[\frac{(x+3)^2}{2}\right]_{-3}^{0}$$ 2 m

$$= -0 + \frac{9}{2} + \frac{9}{2} - 0 = 9 \text{ sq. U.}$$ 1 m

27. Any point on the given line is $(2+3\lambda)\hat{i} + (-1+4\lambda)\hat{j} + (2+2\lambda)\hat{k}$ — 1 m

If this point lies on plane, it must satisfy its equation

$$\therefore \left[(2+3\lambda)\hat{i} + (-1+4\lambda)\hat{j} + (2+2\lambda)\hat{k}\right]\cdot\left(\hat{i} - \hat{j} + \hat{k}\right) = 5$$ 1 m

$$\Rightarrow 2+3\lambda+1-4\lambda+2+2\lambda-5 = 0 \Rightarrow \lambda = 0$$ 1 m

$\therefore$ Point of intersection is (2, –1, 2) — 1 m

$$\text{Distance} = \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} = 13$$ 2 m

28. Let E_1 : selecting box I, E_2 : selecting box II and E_3 : selecting box III — ½ m

$$\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$ 1 m

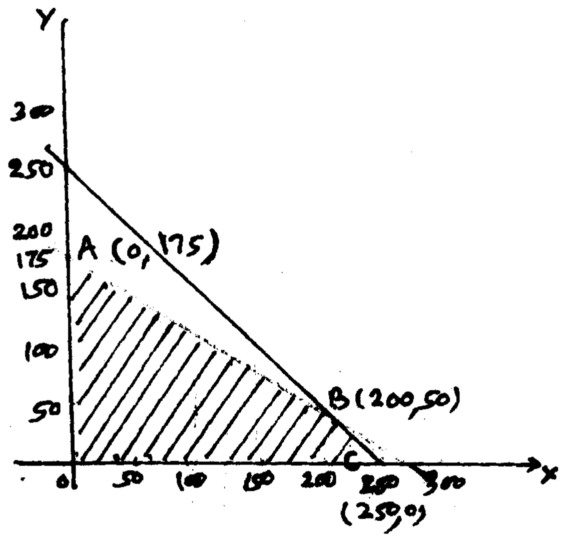
let event A : Getting a gold coin

$$\therefore P(A/E_1) = 1 \quad P(A/E_2) = 0 \quad P(A/E_3) = \frac{1}{2}$$ 1½ m

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)}$$ 1 m

$$= \frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot 1 + 0 + \frac{1}{3}\cdot\frac{1}{2}} = \frac{2}{3}$$ 1+1 m

29. Let the number of desktop models, he stock be x and the number of portable model be y



L.P.P. is, Maximise P = 4500 x + 5000y 1 m

subject to $x + y \leq 250$

$25000\,x + 40000\,y \leq 7000000$

(or $5x + 8y \leq 1400$) 2 m

$x \geq 0, \quad y \geq 0$

For correct graph 2 m

Vertices of feasible region are

A (0, 175), B (200, 50), C (250, 0)

P(A) = Rs. 875000

P(B) = Rs. 900000 + 250000 = Rs. 1150000

P(C) = Rs. 1125000

$\therefore$ For max. Profit destop model = 200

portable model = 50 1 m