



**FURTHER MATHEMATICS  
STANDARD LEVEL  
PAPER 2**

Thursday 16 May 2002 (morning)

2 hours

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**INSTRUCTIONS TO CANDIDATES**

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-9750G*, Sharp *EL-9600*, Texas Instruments *TI-85*.

Please start each question on a new page. You are advised to show all working, where possible. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Incorrect answers with no working will normally receive **no** marks.

1. [Maximum mark: 18]

- (i) A farmer produces duck eggs at his farm. He estimates that he will make a profit of 30 cents per egg on 20% of the eggs, 20 cents per egg on 50%, 10 cents per egg on 20%, no profit on 6% and a loss of 10 cents per egg on 4% of the eggs.
- (a) Construct the probability distribution table and calculate the expected profit per egg. [3 marks]
- (b) This year he has estimated that he will produce 700 000 eggs. What is the probability, correct to four decimal places, that the farmer will earn more than \$123 000 this year? [6 marks]
- (ii) An international company owns two factories, one in Europe and one in Australia. The accidents in the factories in one month follow the Poisson distribution with parameters  $\lambda_1$  and  $\lambda_2$  respectively, and happen independently.
- (a) Show that the probability that the company will have only one accident in the factories in a given month is given by the formula  $p = (\lambda_1 + \lambda_2)e^{-(\lambda_1 + \lambda_2)}$ . [3 marks]
- (b) Derive the formula for  $k$  accidents in the factories (where  $k \geq 0$ ). What can you deduce from your formula about the distribution of the number of accidents? [6 marks]

2. [Maximum mark: 22]

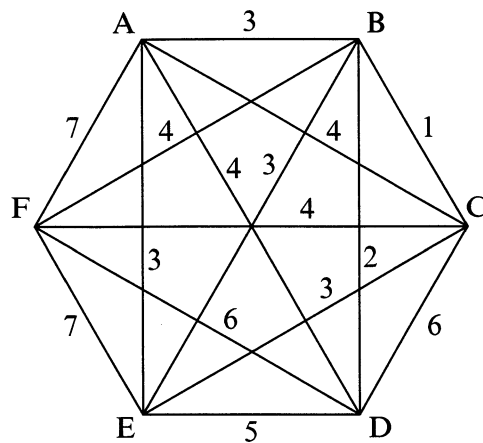
- (i) The operation  $\circ$  is defined on the set  $S = \{x \mid -1 < x < 1, x \in \mathbb{R}\}$  by  $x \circ y = \frac{x + y}{1 + xy}$ ,  $x, y \in S$ .

Determine whether  $S$  forms an Abelian group under  $\circ$ , giving reasons. [14 marks]

- (ii) The function  $f: A \rightarrow B$  is defined by  $f(x, y) = (x \cos y, \sin y)$ , where  $A = [0, \infty[ \times \left[0, \frac{\pi}{2}\right[$  and  $B = [0, \infty[ \times [0, 1[$ . Investigate whether  $f$  is injective, surjective or both. If possible, define the inverse function. If not, explain why not. [8 marks]

3. [Maximum mark: 20]

(i) The following diagram represents a graph  $G$ .



Graph  $G$

- (a) Use an appropriate algorithm to find the minimum spanning tree of  $G$  and state its weight. [4 marks]
- (b) Adapt Prim's algorithm to find the maximum weighted tree and state its weight. Explain how the adapted algorithm works. [5 marks]
- (ii) (a) The sum of the digits of a three-digit number of the form  $abb$  is divisible by 7. Show that the number itself is divisible by 7. [4 marks]
- (b) Use Euclid's algorithm to find the smallest positive integers  $x$  and  $y$  that satisfy the equation  $57x - 13y = 7$ . [7 marks]

4. [Maximum mark: 20]

- (i) (a) Find the error in approximating the integral  $\int_0^2 xe^{-2x} dx$  by using Simpson's rule with 10 strips. [3 marks]
- (b) How many strips are necessary so that the error term is less than  $5 \times 10^{-6}$ ? [3 marks]
- (ii) (a) Find the integral  $\int \frac{dx}{x(x-1)(x-2)}$ . [7 marks]

(b) **Hence** determine whether the following series converges.

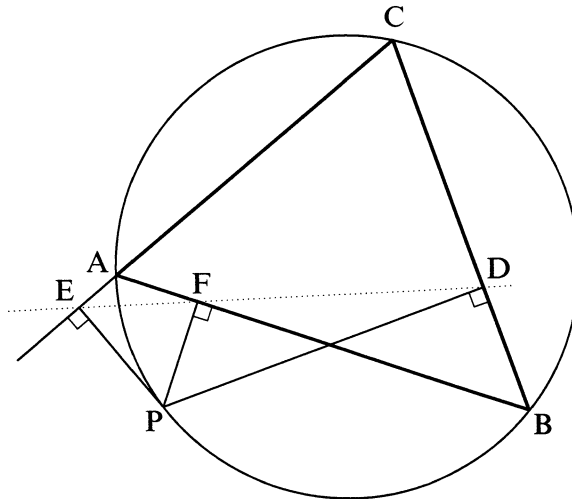
$$\frac{1}{2 \times 3 \times 4} + \frac{1}{3 \times 4 \times 5} + \frac{1}{4 \times 5 \times 6} + \dots + \frac{1}{(n-2) \times (n-1) \times n} + \dots, \quad n \geq 4. \quad [7 \text{ marks}]$$

5. [Maximum mark: 20]

(i) (a) State Menelaus' theorem and its converse.

[3 marks]

(b) The following diagram shows a triangle ABC and its circumscribed circle. The point P is on the circle such that its perpendicular projections to the lines (BC), (CA) and (AB) are D, E and F respectively.



Show that the points D, E and F are collinear.

[7 marks]

(ii) The parabola  $y^2 = 4px$  is given.

(a) Prove that the tangent to the parabola at the point  $(x_0, y_0)$  has equation  $yy_0 = 2p(x + x_0)$ .

[4 marks]

(b) The point D is on the parabola, and is not the vertex. The point L is the foot of the perpendicular from the point D to the directrix. Prove that the tangent to the parabola at D is the angle bisector of  $\widehat{FDL}$ , where F is the focus of the parabola.

[6 marks]