IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 2

Wednesday 13 November 2002 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

Please start each question on a new page. You are advised to show all working, where possible. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Incorrect answers with no working will normally receive ***no*** *marks.*

1. *[Maximum mark: 22]*

(i) The marks for a class are given in the table below.

Marks	0	1	2	3	4	5	6	7
Frequency	0	5	7	7	2	4	3	2

(a) Calculate the mean and the standard deviation for the marks. *[2 marks]*

(b) Test at the 5 % level of significance whether a Poisson distribution is a suitable model for the data. *[9 marks]*

(ii) The heights in cm of players in two basketball teams A and B are recorded.

The heights for team A are 203, 214, 187, 188, 196, 199, 205, 203, 199 and 208.

For team B, the sum of the heights is 2388, and the sum of the squares of the heights is 475 770. The mean height is 199 cm.

i.e. $\sum h = 2388$, $\sum h^2 = 475\,770$, $\bar{h} = 199$.

(a) Calculate

(i) the standard deviation of the heights for team A;

(ii) the standard deviation of the heights for team B;

(iii) the mean height of all the players, correct to the nearest millimetre. *[5 marks]*

(b) (i) Calculate the pooled two-sample estimate of the population variance for the combined sample.

(ii) Hence test at the 5 % level of significance whether these two teams come from the same population. *[6 marks]*

2. *[Maximum mark: 20]*

(i) The relation $\circ$ on the set $\mathbb{N} \times \mathbb{N}$ is defined by $a \circ b \Leftrightarrow 3^a \equiv 3^b \pmod{10}$.

(a) Show that $\circ$ is an equivalence relation. *[7 marks]*

(b) Find the equivalence classes of $\circ$. *[3 marks]*

(c) What is the last digit of the number 3^{2002}? *[2 marks]*

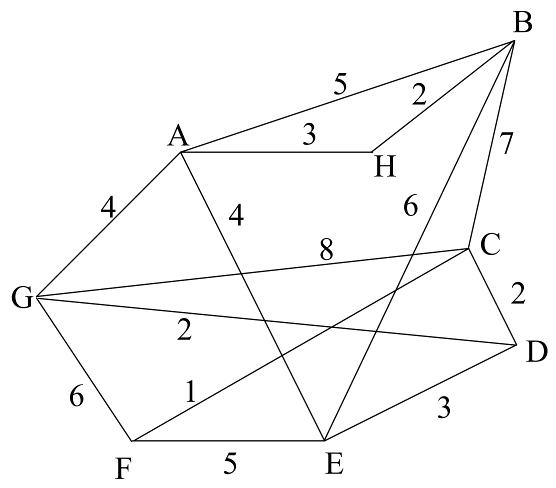
(ii) Let $\min\{a, b\}$ be the minimum value of two numbers a and b. The operation $\otimes$ is defined on the set of negative integers by $a \otimes b = \min\{a, b\}$.

(a) Show that $\otimes$ is commutative. *[1 mark]*

(b) Determine which of the group axioms are satisfied. *[7 marks]*

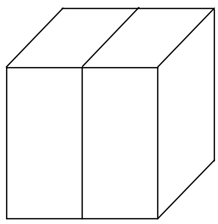
3. *[Maximum mark: 20]*

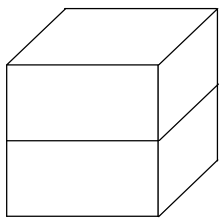
(i) The following diagram shows a weighted graph.



Use an appropriate algorithm to find a minimum spanning tree for the graph, and write down the weight of this tree. *[6 marks]*

(ii) A box has dimensions $2 \times 1 \times n$, where n is a positive integer. It must be filled with identical bricks of dimensions $2 \times 1 \times 1$.
When $n = 1$ there is only one way to fill the box.
When $n = 2$ there are two ways to fill the box as shown in the diagram below.





(a) Draw similar diagrams to show the number of ways the box can be filled for $n = 3$. *[3 marks]*

(b) Let a_n denote the number of ways of filling a box of dimensions $2 \times 1 \times n$.

(i) Explain why $a_{n+2} = a_{n+1} + a_n$.

(ii) Solve the difference equation.

(iii) Find the number of ways of filling the box when $n = 15$. *[11 marks]*

4. *[Maximum mark: 18]*

In this question give your answers to the accuracy displayed by your calculator.

Consider the function $f(x) = \log_{10} x - \dfrac{1}{x}$.

(a) Find the area of the region bounded by the graph of the function, the x-axis and the two vertical lines $x = 1$ and $x = 2$. *[2 marks]*

(b) Use Simpson's rule with four strips to estimate the area in part (a). Find the percentage error of the estimate, giving your answer to **three** significant figures. *[6 marks]*

(c) Solve the equation $f(x) = 0$. *[2 marks]*

(d) Given that $x = \dfrac{1}{\log_{10} x}$ is a possible fixed-point iteration, determine whether this iteration converges or diverges with the initial value $x_0 = 2.5$. Give a reason for your answer. *[4 marks]*

(e) Use the Newton-Raphson method to solve the equation $f(x) = 0$. *[4 marks]*

5. *[Maximum mark: 20]*

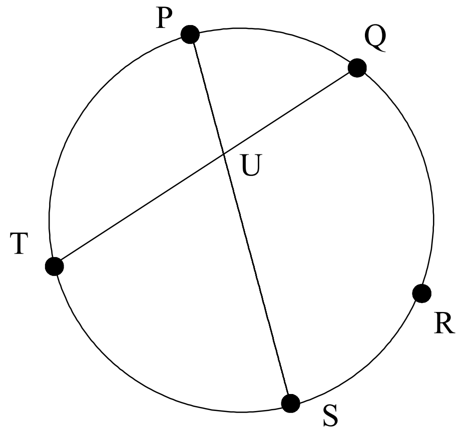
(i) A family of curves is given by the equation

$$(\lambda^2 - 9)x^2 + \lambda^2 y^2 = (\lambda^2 - 9)\lambda^2, \quad \lambda \in \mathbb{R}.$$

(a) Classify the curves with respect to λ (*i.e.* identify the curves for different values of λ). *[5 marks]*

(b) Show that the curves have the same foci if $\lambda \neq 0, \pm 3$. *[4 marks]*

(ii) On a circle, the points P, Q, R, S and T are given as shown below.



Arc PQ is equal to arc RS. The point U is the intersection of the line segments [PS] and [QT].

(a) Show that the triangles PTU and RTS are similar. *[2 marks]*

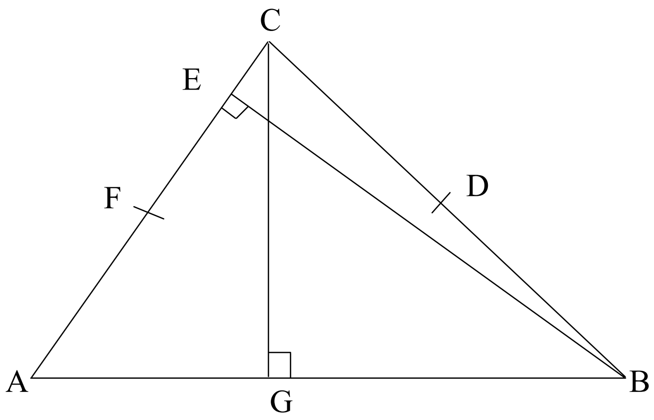
(b) Show that the triangles SUT and RPT are similar. *[2 marks]*

(c) Hence prove Ptolemy's theorem for the cyclic quadrilateral PRST. *[4 marks]*

(This question continues on the following page)

(Question 5 continued)

(iii) Triangle ABC is shown below.



D is the mid-point of the side [BC];
F is the mid-point of the side [CA];
E is the foot of the perpendicular from the vertex B to [AC];
G is the foot of the perpendicular from the vertex C to [AB].

Show that $\text{ED} \times \text{FG} + \text{EF} \times \text{GD} = \text{DF} \times \text{EG}$. *[3 marks]*