M05/5/FURMA/SP2/ENG/TZ0/XX+

22057102

FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 2

Tuesday 24 May 2005 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. *[Maximum mark: 20]*

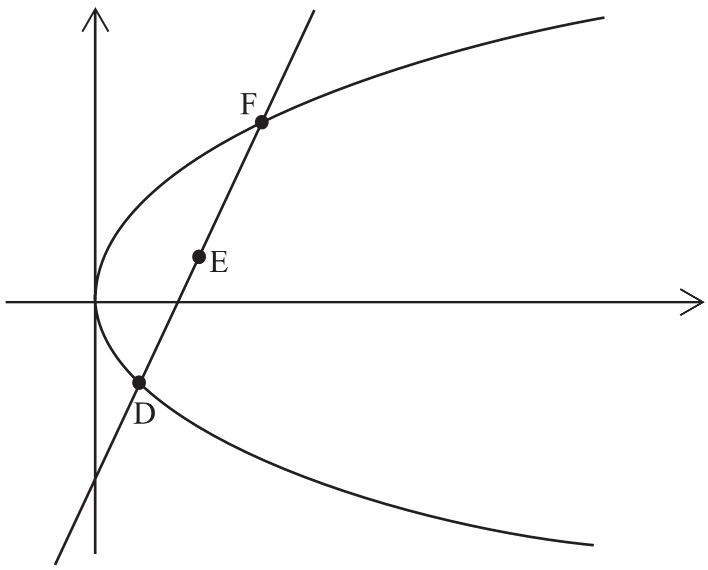
(i) In triangle ABC the angles at the vertices A, B and C are $\frac{5\pi}{8}, \frac{\pi}{8}$ and $\frac{\pi}{4}$ respectively. Point M is the foot of perpendicular from A to the side [BC], N is the midpoint of the side [AC], and P is the intersection of the angle bisector of $\hat{C}$ and the side [AB].

(a) Show that $\frac{\text{AP}}{\text{BP}} = \tan\frac{\pi}{8}$. *[3 marks]*

(b) Show that $\frac{\text{BM}}{\text{CM}} = \tan\frac{3\pi}{8}$. *[3 marks]*

(c) Use the results in parts (a) and (b) to show that the lines (AM), (BN) and (CP) are concurrent. *[4 marks]*

(ii) The following diagram shows the parabola $y^2 = 4x$. The point E(2, 1) is the midpoint of the chord [DF].



(a) Find the equation of (DF). *[4 marks]*

(b) Find the coordinates of the points D and F. *[4 marks]*

(c) Find DF. *[2 marks]*

2. *[Maximum mark: 21]*

Let $f(x) = x^5 + 2x^3 + 3x - 5$.

(a) Calculate the values of $f(x)$ at $x = 0$ and $x = 1$.
Explain why there is at least one zero in the interval]0, 1[. *[3 marks]*

(b) (i) State Rolle's theorem.

(ii) Show that there is only one zero of f in the interval]0, 1[. *[5 marks]*

(c) Use the Newton-Raphson method to find the zero of f correct to six significant figures. Write down all your successive approximations. *[4 marks]*

Fixed-point iteration with the initial value of $x_0 = 1$ is used to solve the equation $x^5 + 2x^3 + 3x - 5 = 0$.

(d) (i) Show that one possible rearrangement of the equation is $x = \sqrt[3]{\dfrac{5 - 3x - x^5}{2}}$.

(ii) Explain why the fixed-point iteration based on this rearrangement diverges. *[4 marks]*

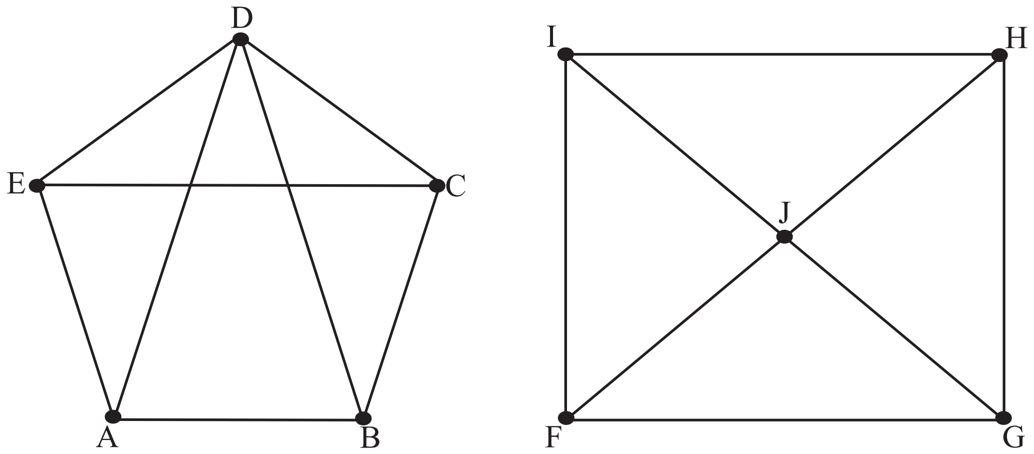
(e) (i) Calculate an estimate for the area of the region bounded by $y = f(x)$, $x = 1$, $x = 2$ and the x-axis using Simpson's rule with three ordinates. Give your answer correct to six significant figures.

(ii) What is the percentage error made in this approximation? *[5 marks]*

3. *[Maximum Mark: 19]*

(i) A maximum graph with n vertices is a graph with the maximum number of edges not containing κ_4.

Consider two maximum graphs with 5 vertices.



(a) Are these two graphs isomorphic? If yes, construct an isomorphism, otherwise explain why not. *[3 marks]*

The vertices P, Q, R, S, T and U of a regular hexagon are the vertices of κ_6.

(b) (i) How many edges does this κ_6 have?

(ii) The graph G is a subgraph of κ_6 omitting the edges PQ, RS and TU. Prove that G cannot contain κ_4.

(iii) Show that any subgraph of κ_6 from which only two edges are removed must contain κ_4, and hence that G defined in the above part is a maximum graph with 6 vertices. Deduce the number of edges in a maximum graph with 6 vertices. *[6 marks]*

(ii) (a) Prove that two positive integers x and y have the same remainder when divided by m if and only if $x \equiv y \pmod{m}$. *[5 marks]*

(b) Find the smallest positive integer that satisfies **both** the following congruencies.

$$2x \equiv 3 \pmod{5} \text{ and } 3x \equiv 2 \pmod{7}.$$ *[5 marks]*

4. *[Maximum mark: 20]*

(i) Prove that $(A \cap B') \cup B = B$ if and only if $A \subset B$. *[5 marks]*

(ii) Let $(G, \times)$ be the cyclic group generated by $\omega = \cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}$.

(a) (i) Find all the elements of the group $(G, \times)$.

(ii) Plot and describe geometrically all the elements in the complex plane.

(iii) Write down all possible generators of the group $(G, \times)$. *[7 marks]*

(b) List all possible proper subgroups of the group. *[4 marks]*

(c) Consider the group $(S, *)$, where $S = \{1, 2, 3, 4\}$ and $*$ is multiplication modulo 5. Draw the group table for $(S, *)$. Is there a proper subgroup in part (b) that is isomorphic to the group $(S, *)$? If yes construct an isomorphism, otherwise explain why not. *[4 marks]*

5. *[Maximum mark: 20]*

Analysis of the fouls committed by a basketball player during a season gave the following results:

Number of fouls per game	0	1	2	3	4	5
Number of games	21	11	24	18	17	9

(a) Calculate the mean and the standard deviation of the number of fouls per game. *[2 marks]*

(b) Perform a χ^2 goodness of fit test at the 5 % level to determine whether or not the distribution can be modelled by a Poisson distribution. *[10 marks]*

Another player played 80 games, having a mean of 2 fouls, and standard deviation of 0.9.

(c) Test, at the 5 % level of significance, whether there is a significant difference in the mean values of fouls per game of the two players. *[5 marks]*

(d) Test, at the 10 % level of significance, whether the first player makes more fouls per game than the second player. *[3 marks]*