INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

M01/540/S(2)

FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 2

Tuesday 15 May 2001 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.

- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-9750G*, Sharp EL-9400, Texas Instruments TI-85.

You are advised to start each new question on a new page. A correct answer with **no** *indication of the method used will usually receive* **no** *marks. You are therefore advised to show your working. In particular, where graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.*

1. *[Maximum mark: 20]*

(i) (a) Find the derivative of e^{x^n}, and hence, show that $2xe^{-x^2} - 3x^2e^{-x^3}$ is the derivative of the function $f(x) = e^{-x^3} - e^{-x^2}$. *[2 marks]*

(b) Use **the fixed point** iterative process of the form $x_{n+1} = g(x_n)$ to find the maximum point of the function f, giving your answer correct to three significant figures. *[5 marks]*

(c) Use the trapezium rule with five intervals to find the area of the region enclosed by the function f and the x-axis. *[4 marks]*

(ii) Find an interval over which the series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{2^n \sqrt{n}}$ will converge. Show your work completely and test the endpoints of this interval. *[9 marks]*

2. *[Maximum mark: 20]*

(a) Show that the set of matrices $\left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} \mid x \in \mathbb{R},\ x \neq 0 \right\}$ is an Abelian group under matrix multiplication. *[10 marks]*

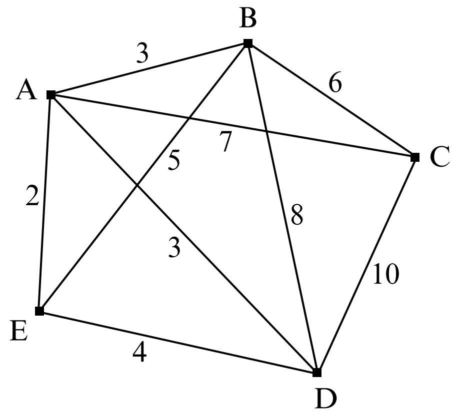
(b) Prove that for all $a, b \in G$, $(a^{-1}ba)^n = a^{-1}b^n a$, where G is a multiplicative group and $n \in \mathbb{Z}^+$. *[5 marks]*

(c) If $f: A \to B$ and $g: B \to A$ are such that $g \circ f(x) = x$ for all $x \in A$, show that f is an injection. *[5 marks]*

3. *[Maximum mark: 21]*

(i) (a) The degree sequence of a graph G is 1 , 1 , 1 , 2 , 2 , 2 , 3 , 3 , 3 , 4 , 4 , 4 , 5 , 5 . What is the degree sequence of the complement of G? *[4 marks]*

(b) The diagram below shows four wells in an offshore oilfield (vertices A, B, C, and D) and an onshore terminal (vertex E).



The four wells in this field must be connected together via a pipeline network to the onshore terminal. The various pipelines that can be constructed are shown as edges in the diagram and the cost of each pipeline (in thousands of dollars) is given next to it. Use an appropriate algorithm to determine which pipelines should be built to minimise the cost. Describe the algorithm you used and explain how it works, perform the steps, and draw your final tree. *[5 marks]*

(ii) (a) Show that if 3 divides $(a^2 + b^2)$ then 3 divides a and 3 divides b, where $a , b \in \mathbb{Z}^+$. *[5 marks]*

(b) Show that if p is a prime number and p divides a and p divides $(a^2 + b^2)$ then p divides b, where $a , b , p \in \mathbb{Z}^+$. *[3 marks]*

(c) The greatest common divisor of x and y is denoted by (x , y). Show that if a and b are relatively prime, then $(a , bc) = (a , c)$, where $a , b , c \in \mathbb{Z}^+$. *[4 marks]*

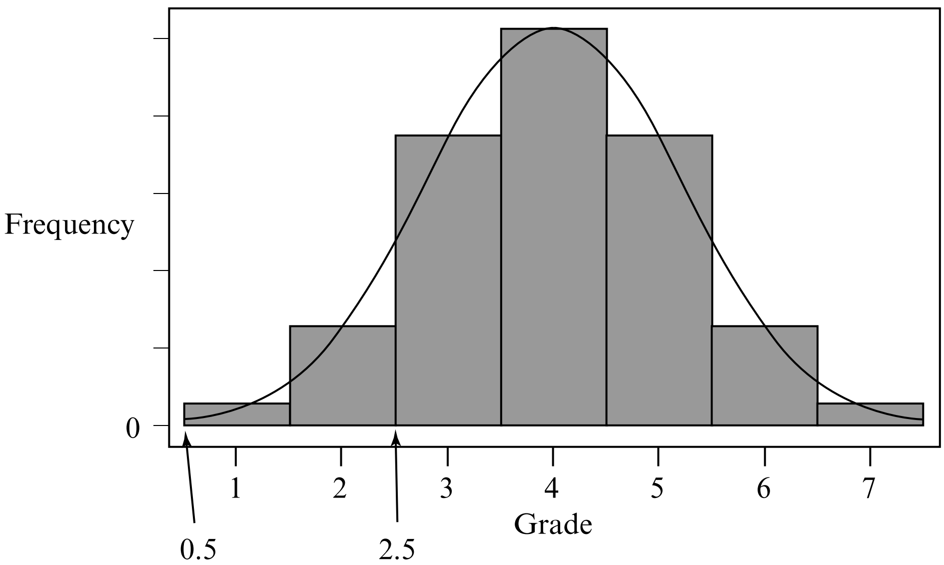
4. *[Maximum mark: 20]*

The table below shows the frequency distribution for Mathematics grades for a certain country, *Utopia*, in May 1999.

Grade	1	2	3	4	5	6	7
Number of candidates	10	25	38	42	25	12	6

(a) Calculate the mean grade for these candidates. *[2 marks]*

(b) The grades for all Mathematics candidates in May 1999 are distributed as shown in the histogram below. This distribution may be approximated by a normal distribution with a mean of 4 and standard deviation of 1.17.



It is believed that the results of the *Utopia* candidates come from this normal distribution (mean 4, standard deviation 1.17).

(i) Copy and complete the following expected frequency table for all candidates.

Grade	1	2	3	4	5	6	7
Expected number of candidates	2.6		37.1	52.3			2.6

(ii) Test the hypothesis at the 5% level of significance. *[8 marks]*

(This question continues on the following page)

(Question 4 continued)

(c) Given that the population mean is 4, is there any evidence, at the 5% level of significance, that the performance of the candidates from *Utopia* is below that of the population? *[4 marks]*

(d) It is believed that for those candidates who take both Mathematics and Physics, there is a strong relationship between the grades they receive in each of these two subjects. The table below gives the distribution of these grades.

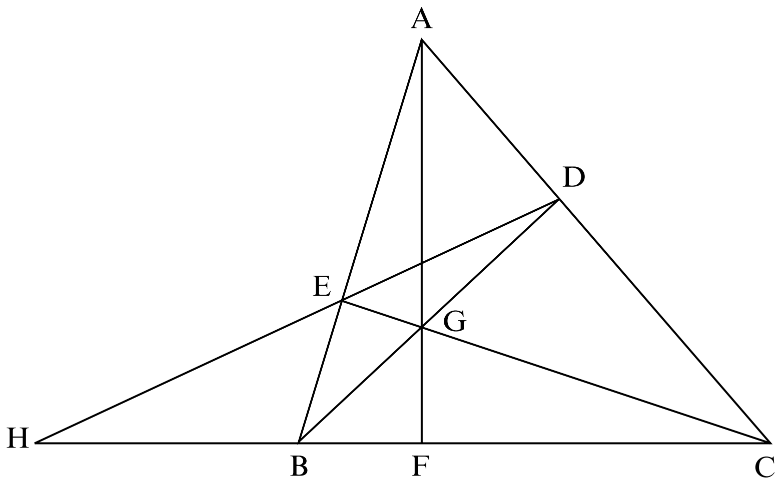
		Physics			
		7	6	5	4 & below
Mathematics	7	40	50	56	20
	6	42	65	90	28
	5	32	48	52	24
	4 & below	60	80	100	45

Is there any evidence at the 5% level of significance that this relationship holds? *[6 marks]*

5. *[Maximum mark: 19]*

(i) In the diagram below, ABC is a triangle with points D and E chosen on [AC] and [AB] respectively, such that $\dfrac{\text{CD}}{\text{DA}} = \dfrac{\text{AE}}{\text{EB}} = \dfrac{3}{2}$.

The lines (AF) and (CE) meet at the point G. The line (AG) meets the side [BC] at the point F. The line (DE) meets the line (CB) at the point H.



(a) Calculate $\dfrac{\text{BF}}{\text{FC}}$ and $\dfrac{\text{BH}}{\text{HC}}$, providing reasons for your steps. *[5 marks]*

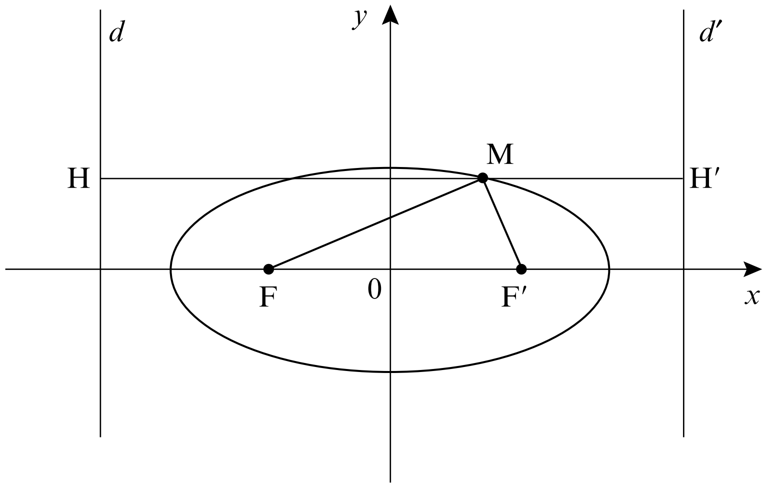
(b) Calculate $\dfrac{\text{AG}}{\text{GF}}$. *[4 marks]*

(ii) In triangle PQR , the base [QR] is fixed and the point P moves such that the ratio of PQ to QR is $3 : 2$. What is the locus of the point P ? *[5 marks]*

(*This question continues on the following page*)

(*Question 5 continued*)

(iii) M is a point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, with eccentricity e, as shown in the diagram. F and F′ are the foci, with $\mathrm{FF}' = 2c$. The directrices are d and d', and (MH) and (MH′) are perpendicular to d and d' respectively.



Prove that $\mathrm{MF} + \mathrm{MF}' = 2a$. *[5 marks]*