Mathematics

2008 November

Technology BCA

Semester 1

University Exam

Mangalore University

BCACAC 103   Reg. No. [ ][ ][.]1.[ ][ ][ ][ ] 19

Credit Based First Semester B.C.A. Degree Examination
October / November 2008
MATHEMATICS

Time : 3 Hours   Max. Marks : 80

Note: Answer any TEN questions from PART A and ONE full question from
        PART B.

PART A

1.   (10x2=20)

   a)   Show that the improper fraction $\dfrac{3x^2 - 4x + 5}{x - 1}$ can be written as a sum of
        polynomial $(3x - 1)$ and $\dfrac{4}{x - 1}$.

   b)   Find the value of x if $\log_{10} (x-9) = 2$

   c)   Find the value of n if $^nP_2 = 30$

   d)   In which quadrant will θ terminate if (i) $\cos \theta$ and $\sin \theta$ are both negative.
                                              (ii) $\sin \theta$ is positive and $\cos \theta$ is negative.

   e)   Prove that $2 \cos^2 \dfrac{\pi}{4} - 1 = \cos \dfrac{\pi}{2}$

   f)   Simplify $(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2$

   g)   Prove that the equation of the locus of a point which moves such that the triangle
        ABP of area 8 sq. units, where A (1, 2) and B (-2, 5) and triangle ABP is traced
        in the clockwise direction is $3x + 3y - 25 = 0$.

   h)   Find the equation of the line passing through (3, 2) and perpendicular to the
        y-axis.

   i)   Show that the line AC is perpendicular to BD where A is (2, 1), B is (-1, 8),
        C is (0, 1) and D is (4, 3).

   j)   Find the midpoint of the line joining the points, (-2, 3) and (3, 7).

   k)   Define parabola. Write the standard form.

   l)   If the major axis of an ellipse is double the minor axis, find its eccentricity?

PART B

UNIT-I

2.   a)   Define proper and improper fraction ; illustrate with example.

b) Prove that $\dfrac{\log\sqrt{27} + \log\sqrt{8} - \log\sqrt{125}}{\log 6 - \log 5} = \dfrac{3}{2}$

c) Show that $(2 + \sqrt{3})^5 - (2 - \sqrt{3})^5 = 724$ (3+6+6)

OR

3. a) Resolve $\dfrac{x + 3}{x^3 - x}$ into partial fractions.

b) Using logarithms, find the value of (i) $(1.035)^{-12}$ (ii) $\sqrt{88.5 \times 129.7}$

c) Find the term independent of x in $\left(x - \dfrac{3}{x^2}\right)^9$ (4+6+5)

UNIT-II

4. a) Find the acute angle in radians between the minute and hour hands of a clock when time is 4 hours 20 minutes.

b) If $\cos\theta = \dfrac{24}{25}$ and θ is acute, find the values of other trigonometric functions of θ.

c) Solve $\triangle ABC$, given that $a = 2$, $c = \sqrt{3} + 1$, and $B = 60°$.   (5+5+5)

OR

5. a) Express both in degrees and radians the angles of a triangle whose angles are in the ratio 2:3:5.

b) Prove that $\dfrac{\sin\left(\frac{5\pi}{3}\right) \tan\left(\frac{4\pi}{3}\right) \sec\left(\frac{-7\pi}{3}\right)}{\cot\left(\frac{-3\pi}{4}\right) \cos\left(\frac{7\pi}{6}\right) \csc\left(\frac{-7\pi}{4}\right)} = \sqrt{6}$

c) i) Prove that $\dfrac{\tan A + \tan B}{\cot A + \cot B} = \tan A \tan B$

ii) simplify $\dfrac{\sin^2\theta}{1 - \cos\theta} - \dfrac{\cos^2\theta}{1 - \sin\theta}$ (4+6+5)

UNIT-III

6. a) Prove that the points, (7, 9), (3, -7) and (-3, 3) are the vertices of an isosceles right angled triangle and also find its area.

b) If A (1, -3), B(5, 2) and C(3, 4) are the vertices of $\triangle ABC$, find the area of $\triangle ABC$ and hence find the length of altitude from A.

c) Find the value of k if the line joining P(2, 3) and Q(5, 7) is perpendicular to the line joining A (5, k) and B(3, 2).   (5+6+4)

OR

7. a) Prove that the points (2, -1), (3, 4), (-2, 3) and (-3, -2) taken in order are the vertices of rhombus. Also find its area.

b) The centroid of a triangle ABC is at (3, 2). If A (-3, 4) and B and C lie on the X-axis and Y-axis respectively, find the coodinates of B and C.

c) Find the equations of the straight lines passing through the point (-1, -5) and (i) Parallel (ii) Perpendicular to 2x + 3y = 5.   (4+5+6)

UNIT-IV

8. a) Define circle. What is the equation of the circle if centre and radius are given ? Show that the equation $x^2 + y^2 + 4x + 6y + 13 = 0$ represent a point circle.

b) Find the focus, directix, vertex, axis, the end points of latus rectum and the length of the latus rectum of the parabola $y^2 = 12x$.

c) Evaluate (i) prove that $\underset{x \to a}{Lt} \dfrac{x^n - a^n}{x - a} = n.a^{n-1}$ for all positive integers n

(ii) Evaluate $\underset{x \to b}{Lt} \dfrac{x^5 - b^5}{x^4 - b^4}$ (4+6+5)

OR

9. a) (i) Find the length of tangent drawn from (1, 0) to the circle

$x^2 + y^2 + 8x - 8y = 0$.

(ii) Find the centre and radius of the circle

$4x^2 + 4y^2 + 80x + 12y + 265 = 0$

b) Find the centre, vertices, lengths of major and minor axes, the length of LR, foci, End points of LR of the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

c) Evaluate (i) $\underset{x \to 0}{Lt} \dfrac{\tan(2x^4) . \sin^2(4x)}{x^6}$

(ii) $\underset{x \to 3}{Lt} \dfrac{\sin 2x . \tan 4x}{x^2}$ (4+6+5)

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