Test Code MS (Short answer type) 2007
Syllabus for Mathematics

Permutations and Combinations. Binomial and multinomial theorem. Theory of equations. Inequalities.

Determinants, matrices, solution of linear equations and vector spaces.

Trigonometry, Coordinate geometry of two and three dimensions.

Geometry of complex numbers and De Moivre's theorem. Elements of set theory.

Convergence of sequences and series. Power series. Functions, limits and continuity of functions of one or more variables.

Differentiation, Leibnitz formula, maxima and minima, Taylor's theorem. Differentiation of functions of several variables. Applications of differential calculus.

Indefinite integral, Fundamental theorem of calculus, Riemann integration and properties. Improper integrals. Differentiation under the integral sign. Double and multiple integrals and applications.

Syllabus for Statistics

Probability and Sampling Distributions

Notions of sample space and probability, combinatorial probability, conditional probability and independence, random variable and expectations, moments, standard discrete and continuous distributions, sampling distributions of statistics based on normal samples, central limit theorem, approximation of binomial to normal or Poisson law. Bivariate normal and multivariate normal distributions.

Descriptive Statistics

Descriptive statistical measures, graduation of frequency curves, product-moment, partial and multiple correlation, Regression (bivariate and multivariate).

Inference

Elementary theory and methods of estimation (unbiasedness, minimum variance, sufficiency, maximum likelihood method, method of moments). Tests of hypotheses (basic concepts and simple applications of Neyman-Pearson Lemma). Confidence intervals. Tests of regression. Elements of non-parametric inference.

Design of Experiments and Sample Surveys

Basic designs (CRD/RBD/LSD) and their analyses. Elements of factorial designs. Conventional sampling techniques (SRSWR/SRSWOR) including stratification; ratio and regression methods of estimation.

Sample Questions :
Section A

1. Let A be a 2×2 matrix with real entries such that $A^2 = 0$. Find the determinant of $I + A$ where I denotes the identity matrix.

2. Let A and B be $n \times n$ real matrices such that $A^2 = A$ and $B^2 = B$. Suppose that $I - (A + B)$ is invertible. Show that $\text{rank}(A) = \text{rank}(B)$.

3. Let f be a function such that $f(0) = 0$ and f has derivatives of all order. Show that
$$\lim_{h \to 0} \frac{f(h) + f(-h)}{h^2} = f''(0)$$
where $f''(0)$ is the second derivative of f at 0.

4. Let $f : \mathbb{R} \to \mathbb{R}$ be a bounded continuous function. Define $g : [0, \infty) \to \mathbb{R}$ by,
$$g(x) = \int_{-x}^{x} (2xt + 1) f(t) dt.$$
Show that g is differentiable on $(0, \infty)$ and find the derivative of g.

5. Let X and Y be i.i.d. random variables, with $P(X = k) = 2^{-k}$ for $k = 1, 2, 3, \ldots$. Find $P(X > Y)$ and $P(X > 2Y)$.

6. 18 boys and 2 girls are made to stand in a line in a random order. Let X be the number of boys standing in between the girls. Find

   (a) $P(X = 5)$,

   (b) $E(X)$.

Section B

7. Let X and Y be i.i.d. exponentially distributed random variables with mean $\lambda > 0$. Define Z by:
$$Z = \begin{cases} 1 & \text{if } X < Y \\ 0 & \text{otherwise.} \end{cases}$$

Find the conditional mean $E(X \mid Z = 1)$.

8. Let $U_1, U_2, \ldots, U_n$ be i.i.d. uniform $(0,1)$ random variables and suppose

$$X = \max(U_1, U_2, \ldots, U_n) \text{ and } Y = \min(U_1, U_2, \ldots, U_n).$$

Find the distribution of $Z = X - Y$.

9. Let X_1 and X_2 be i.i.d. random variables from Bernoulli(θ) distribution. Verify if the statistic $X_1 + 2X_2$ is sufficient for θ.

10. Suppose X takes three values $1, 2$ and 3 with

$$P(X = k) = \begin{cases} (1-\theta)/2 & \text{if } k = 1 \\ 1/2 & \text{if } k = 2 \\ \theta/2 & \text{if } k = 3 \end{cases}$$

where $0 < \theta < 1$. Suppose that the following random sample of size 10 was drawn from the above distribution :

$$1, 3, 1, 2, 3, 1, 2, 2, 1, 1.$$

Find the m.l.e. of θ based on the above sample.

11. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables from the exponential distribution with mean $\theta > 0$. Find the most powerful test for testing $H_0 : \theta = 2$ against $H_1 : \theta = 1$. Find the power of the test.

12. Let Y_1, Y_2 and Y_3 be uncorrelated random variables with common variance $\sigma^2 > 0$ such that

$$E(Y_1) = \beta_1 + \beta_2, E(Y_2) = 2\beta_1 \text{ and } E(Y_3) = \beta_1 - \beta_2$$

where β_1 and β_2 are unknown parameters. Find the residual (error) sum of squares under the above linear model.