M. A. / M. Sc. Part I Semester I Mathematics 104

Topology I

PAPER IV (2004)

P. Pages : 4

Time : Three Hours Max. Marks : 60

Note : 1. All questions are compulsory.

UNIT - I

1. A) a) Define denumerable set. Show that the set of all rational numbers is denumerable. **6**

OR

b) State and Prove Burali - Forti Paradox. **6**

B) c) Show that $\alpha \leq \beta$ or $\beta \leq \alpha$. if α and β are ordinal numbers. **6**

OR

d) Explain cardinal numbers and show that $CC = C$. **6**

UNIT - II

2. A) a) Show that, If $x \notin F$, where F is a closed subset of a topological space $(X, \mathcal{J})$, then there exists an open set G such that $x \in G \subseteq CF$. **6**

OR

b) If A, B are subsets of the topological space $(X, \mathcal{J})$. then show that derived set has the following property.

i) If $A \subseteq B$ then $d(A) \subseteq d(B)$.

ii) $d(A \cup B) = d(A) \cup d(B)$. **6**

B) c) If E is a subset of a subspace $(X^*, \mathcal{J}^*)$ of a topological space $(X, \mathcal{J})$ then show that $C^*(E) = X^* \cap C(E)$. **6**

OR

d) Show that every family of subsets of a set X whose union is X is a subspace for a topology for X. **6**

UNIT - III

3. A) a) Show that E is connected if C is a connected set and $C \subseteq E \subseteq C(C)$. **6**

OR

b) If f is a homeomorphism of X onto X^* then show that f maps every isolated subset of X onto an isolated subset of X^*. **6**

B) c) If E is a subset of a subspace $(X^*, \mathcal{J}^*)$ of a topological space $(X, \mathcal{J})$ then show that E is $\mathcal{J}^*$ compact if and only if it is $\mathcal{J}$ - compact. **6**

OR

d) Show that an open connected subset of the plane is arcwise connected. **6**

UNIT - IV

4. A) a) Show that a T_1-space X is countably compact if and only if every countable open covering of X is reducible to a finite sub cover. **6**

OR

b) Show that the one point compactification X^* of a topological space X is a Hausdorff space if and only if X is a locally compact Hausdorff space. **6**

B) c) Define condensation point. Show that every uncountable subset of a second axiom space contain a condensation point. **6**

OR

d) Show that A topological space X satisfying the first axiom of countability is a Hausdorff space if and only if every convergent sequence has a unique limit. **6**

UNIT - V

5. A) a) Show that a topological space X is regular if and only if for every point $x \in X$ and open set G containing x there exists an open set G^* such that $x \in G^*$ and $C(G^*) \subseteq G$. **6**

OR

b) Show that a normal space is completely regular if and only if it is regular. **6**

B) c) Show that a topological space X is completely normal if and only if every subspace of X is normal. **6**

OR

d) Show that, If x & y are two distinct points in a Tichonov space X then there exists a real valued continuous mapping f of X such that $f(x) \neq f(y)$. **6**

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