First Semester M. A. / M. Sc. I Examination

MATHEMATICS

Paper - IV

(Topology - I) (2004)

Time : Three Hours]   [Max. Marks : 60

N. B. : All questions are compulsory.

UNIT I

1. (A) (a) Define denumerable set. Show that Every infinite set contains a denumerable subset. 6

OR

(b) Show that addition of order types is not commutative. 6

(B) (c) State and prove Cantor theorem. 6

OR

(d) Show that Every set of ordinal numbers is well-ordered set. 6

Contd.

UNIT II

2. (A) (a) Show that the Union of two topologies for X may not be a topology for X. 6

OR

(b) Show that $\mathfrak{J}^*$ is a topology for X^*, if X^* is a subset of a topological space $(X, \mathfrak{J})$. 6

(B) (c) Show that $i(E) = Cc(CE)$, For any set E in a topological space $(X, \mathfrak{J})$. 6

OR

(d) Let C^* be a closure operator defined on a set X. Let $\mathfrak{F}$ be the family of all subsets F of X for which $C^*(F) = F$ and let $\mathfrak{J}$ be the family of all complements of members of $\mathfrak{J}$ then show that :—

(i) $\mathfrak{J}$ is a topology for X.

(ii) If C is the closure operator defined by the topology $\mathfrak{J}$, then $C^*(E) = C(E)$ for all subsets $E \subseteq X$. 6

UNIT III

3. (A) (a) If C is a connected subset of a topological space $(x, \mathfrak{J})$ which has a separation $X = A \mid B$ then show that either $C \subseteq A$ or $C \subseteq B$. 6

OR

(b) If f is one to one continuous mapping of $(x, \mathfrak{J})$ into $(x^*, \mathfrak{J}^*)$ then show that f map every dense-in itself subset of X onto a dense-in-itself subset of X^*. 6

(B) (c) Show that the union E of any family $\{C_\lambda\}$ of connected sets having a non empty intersection is a connected set. 6

OR

(d) Define compact set. Show that a continuous image of a compact set is compact. 6

UNIT IV

4. (A) (a) Show that a topological space X is a T_1 – space if and only if every subset consisting of exactly one point is closed. 6

OR

(b) Show that every convergent sequence in a Hausdorff space has a unique Limit. 6

(B) (c) Show that Every second axiom space is hereditarily separable. 6

OR

(d) Show that every subspace of T_0 – space is a T_0 – space. 6

UNIT V

5. (A) (a) Show that every regular Lindelof space is normal. 6

OR

(b) Show that a Hausdorff space $(x, \mathfrak{I})$ is completely regular if and only if the family of all cozero sets of real valued continuous mapping of X is a base for the topology $\mathfrak{I}$. 6

(B) (c) State and prove Urysohn's lemma. 6

OR

(d) Show that Every T_4 – space is a T_3 – space but a normal space need not be regular. 6