USN

First Semester B.E. Degree Examination, May/June 2010
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: *Answer any FIVE full questions, selecting at least ONE question from each part.*

PART – A

1 a. Find the acute angle between the two lines whose direction cosines are related by $\ell + m + n = 0;\ 2\ell m + 2\ell n - mn = 0$. **(06 Marks)**

b. A line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube. Show that : $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$. **(07 Marks)**

c. Find the equation of the plane passing through (2, 3, 4), (-3, 5, 1), (4, -1, 2). **(07 Marks)**

2 a. Find the perpendicular distance from (3, 9, -1) to the line $\frac{x+8}{-8} = \frac{y-13}{1} = \frac{z-13}{5}$. Find the equation of the perpendicular. **(06 Marks)**

b. Find the equation of the right circular cone when the straight line $2y + 3z = 6$, $x = 0$ revolves about z – axis. **(07 Marks)**

c. Find the equation of the circular cylinder, having for its base the circle $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$. **(07 Marks)**

PART – B

3 a. Find n^{th} derivative of $\sin(ax + b)$. **(06 Marks)**

b. If $y = \sin(m \sin^{-1}x)$, prove that $(1 - x^2)y_{n+2} - (2n + 1)\,xy_{n+1} + (m^2 - n^2)y_n = 0$. **(07 Marks)**

c. Find the pedal equation of $r = 2a\cos\theta$. **(07 Marks)**

4 a. If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. **(06 Marks)**

b. If $u = \sin\left(\frac{x}{y}\right)$, $x = e^t$, $y = t^2$, find $\frac{du}{dt}$ as a function of t. Verify from answer by direct substitution. **(07 Marks)**

c. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\sin\theta$, find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$. **(07 Marks)**

PART – C

5 a. Obtain a reduction formula for $\int \cos^n x \cdot dx$. **(06 Marks)**

b. Evaluate $\int_0^a \frac{x^4}{\sqrt{a^2 - x^2}}\,dx$. **(07 Marks)**

c. Trace the curve $y^2 = \frac{4a^2(2a - x)}{x}$. **(07 Marks)**

6 a. Find the area of $r = a(1 + \cos\theta)$. **(06 Marks)**

b. Find the length of one arch of cycloid $x = a(t - \sin t)$ $y = a(1 - \cos t)$. **(07 Marks)**

c. The area bound by $y^2 = 4ax$ and $x = a$ revolves about x-axis. Find the volume generated. **(07 Marks)**

PART – D

7 a. Solve $x\frac{dy}{dx} + \cot y = 0$, given $y = \frac{\pi}{4}$ at $x = \sqrt{2}$. **(05 Marks)**

b. Solve $\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy}$. **(05 Marks)**

c. Solve $\frac{dy}{dx} = \frac{x - y + 1}{2x - 2y + 3}$. **(05 Marks)**

d. Find the orthogonal trajectories of $r^2 = a^2 \cos 2\theta$. **(05 Marks)**

8 a. Test for convergence of the series :

$$\Sigma\left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}}.$$

(06 Marks)

b. Test for convergence of the series $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + ---- + ----$. **(07 Marks)**

c. Define :

i) Absolute convergence

ii) Conditional convergence, with an example each.

Test $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + ----$ for convergence. **(07 Marks)**

* * * * *