First Semester B.E. Degree Examination, December 2010
Engineering Mathematics – I

Time: 3 hrs.
Max. Marks:100

Note: *Answer any FIVE full questions, selecting*
at least ONE question from each part.

PART – A

1 a. A line makes angles α, β, γ and δ with the four diagonals of a cube, prove that :
   $Cos^2\alpha + Cos^2\beta + Cos^2\gamma + Cos^2\delta = 4/3$. **(07 Marks)**

   b. Find the equation of the plane through the point (-1, 2, 4) and parallel to the plane
   $2x - 3y - 5z + 6 = 0$. **(07 Marks)**

   c. Find the equation to the line through the point (1, 2, 3) and parallel to the line whose
   equations are $x - y + 3z = 5$, $3x + y + z = 6$. **(06 Marks)**

2 a. Prove that the lines $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$ and $\dfrac{x-2}{3} = \dfrac{y-3}{4} = \dfrac{z-4}{5}$ are coplanar. **(07 Marks)**

   b. Find the shortest distance between the lines $\dfrac{x-3}{3} = \dfrac{y-8}{-1} = \dfrac{z-3}{1}$ and $\dfrac{x+3}{-3} = \dfrac{y+7}{2} = \dfrac{z-6}{4}$.
   (07 Marks)

   c. Find the equation of the right circular cone whose vertex is the origin, semi vertical angle is
   45° and whose axis is the line $x = 2y = z$. **(06 Marks)**

PART – B

3 a. Find the nth derivative of $\dfrac{x^2}{(x+2)(2x+3)}$. **(07 Marks)**

   b. Find the angle between radius vector and tangent for the curve $r = a(1 - Cos\,\theta)$. **(07 Marks)**

   c. Prove that with usual notations $\dfrac{1}{p^2} = u^2 + \left(\dfrac{du}{d\theta}\right)^2$. **(06 Marks)**

4 a. If $z = f(x, y)$ is a homogeneous function of x and y of degree n, then prove that
   $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = nz$. **(07 Marks)**

   b. If $H = f(y - z, z - x, x - y)$, prove that $\dfrac{\partial H}{\partial x} + \dfrac{\partial H}{\partial y} + \dfrac{\partial H}{\partial z} = 0$. **(07 Marks)**

   c. Find the percentage error in the area of an ellipse when an error of $+1\%$ is made by
   measuring the major and minor axis. **(06 Marks)**

PART – C

5 a. Obtain reduction formula for $\int Sin^n x\,dx$, $x \geq 0$.   (07 Marks)

    b. Trace the curve $y^2(a-x)=x^3$.   (07 Marks)

    c. Evaluate $\int_0^\pi x\,Sin^5 x\,dx$.   (06 Marks)

6 a. Trace the curve $r = a\,Cos\,2\theta$.   (07 Marks)

    b. Show that the area enclosed between parabolas $y^2 = 4a\,(x+a)$ and $y^2 = -4a\,(x-a)$ is $\dfrac{16}{3}a^2$.   (07 Marks)

    c. Find the volume of the solid formed by the revoluation of one arch of the cycloid $x = a\,(\theta - Sin\,\theta)$, $y = a\,(1 - Cos\,\theta)$ about its base.   (06 Marks)

PART – D

7 a. Solve $\dfrac{dy}{dx} = \dfrac{x + 2y + 3}{2x - y + 1}$.   (07 Marks)

    b. Solve $\dfrac{dy}{dx} + y\sec x = \tan x$.   (07 Marks)

    c. Solve $(1 + e^{x/y})\,dx + (1 - x/y)\,e^{x/y}\,dy = 0$.   (06 Marks)

8 a. Test the convergence of $\dfrac{x}{1.2} + \dfrac{x^2}{3.4} + \dfrac{x^3}{5.6} + \ldots\ldots(x > 0)$.   (07 Marks)

    b. Test the convergence of the series $\dfrac{x}{3} + \dfrac{1.2}{3.5}x^2 + \dfrac{1.2.3}{3.5.7}x^3 + \ldots$ (07 Marks)

    c. Discuss the convergence of the series $1 - 2x + 3x^2 - 4x^3 + \ldots$ $(0 < x < 1)$.   (06 Marks)

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