USN

First Semester B.E. Degree Examination, Dec.08/Jan.09
Engineering Mathematics - I

Time: 3 hrs. Max. Marks:100

Note:1. ***Answer any FIVE full questions selecting at least two questions from each part.***
2. ***Answer all objective type questions only in first and second writing pages.***
3. ***Answer for Objective type questions shall not be repeated.***

1 a. i) If $y = x^{2n}$ then y_{n+1} is

A) $\frac{(2n)!}{(n-1)!}x^{n-1}$ B) $\frac{(2n)!}{n!}x^{n-1}$ C) $\frac{(n-1)!}{(2n)!}x^{n-1}$ D) Zero

ii) If two curves intersect orthogonally in Cartesian form, the angle between the same two curves in polar form is,

A) $\pi/4$ B) Zero C) 1 radian D) None of these

iii) If the angle between the radius vector and the tangent is constant, then the curve is,

A) $r = ae^{b\theta}$ B) $r = a\cos\theta$ C) $r^2 = a^2\cos(2\theta)$ D) $r = a\theta$

iv) The n^{th} derivative of a constant function is,

A) n B) 1 C) Zero D) ∞ **(04 Marks)**

b. Find the nth derivative of $\frac{x+3}{(x-1)(x+2)}$. **(04 Marks)**

c. If $y = \sin(m\sin^{-1}x)$ express $(1-x^2)y_{n+2} - (2n+1)xy_{n+1}$ in terms n^{th} derivative of y. **(06 Marks)**

d. Find the pedal equation of the polar curve $r = a(1+\cos\theta)$. **(06 Marks)**

2 a. i) If $u = x^n + y^n$ then $\frac{\partial^n u}{\partial x^{n-1}\partial y}$ is equal to $((n \geq 2)$

A) Zero B) $(n!)x + ny^{n-1}$ C) $(n!)x$ D) $(2n)!$

ii) If $u = \sin(x+ay) + g(x-ay)$ then the value of $\frac{\partial^2 u}{\partial^2 y}$ is

A) $\frac{\partial^2 u}{\partial x^2}$ B) $a\frac{\partial^2 u}{\partial x^2}$ C) $a^2\frac{\partial^2 u}{\partial x^2}$ D) $-a^2\frac{\partial^2 u}{\partial x^2}$

iii) If $u = f(x^2+y^2+z^2)$ and $\frac{\partial u}{\partial x} = 2xf'$ then f' is derivative with respect to

A) x B) y C) z D) $x^2+y^2+z^2$

iv) If u and v are the two functions depending on the independent variables x and y then u and v are independent of each other if and only if, for $J = J\left(\frac{u,v}{x,y}\right)$

A) $J = 0$ B) $J \neq 0$ C) $J = 1$ D) $J = -1$ **(04 Marks)**

b. If $u = x^2y + y^2z + z^2x$ show that $u_x + u_y + u_z = (x+y+z)^2$. **(04 Marks)**

c. If $u = x\log(xy)$ where the implicit relation between x and y is $x^3 + y^3 + 3xy = 1$ find $\frac{du}{dx}$. **(06 Marks)**

d. Define 'relative error' and 'percentage error'. Find the error in calculating the power $\omega = \frac{V^2}{R}$ due to errors h and k respectively in measuring voltage V and reistance R. **(06 Marks)**

3 a. i) The value of $\int_0^{\pi} \sin^4 x dx$ is

A) $\frac{3\pi}{8}$ B) $\frac{3\pi}{16}$ C) $\frac{3\pi^2}{8}$ D) zero

ii) The value of $\int_0^{\pi/2} \sin^{99}(x)\cos(x)dx$ is

A) $\frac{1}{99}$ B) $\frac{\pi}{100}$ C) $\frac{99}{100}$ D) None of these

iii) The tangents to the curve $x^3 + y^3 = 3axy$ at origin are

A) $y = x$ and $y = -x$ B) $x = 0,\ y = 0$

C) Line perpendicular to $y = x$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ D) Do not exist

iv) If the equation of the curve remains unchanged after changing r to –r the curve $r = f(\theta)$ is symmetric about

A) Initial line B) A line perpendicular to initial line through pole

C) Radially symmetric about the point pole D) Symmetry does not exist.

(04 Marks)

b. Evaluate $I = \int_0^{\pi} x\sin^7 x dx$. (04 Marks)

c. Obtain the reduction formula for $\int \tan^n x dx$ and hence find the reduction formula for $\int_0^{\pi/4} \tan^n x dx$. (06 Marks)

d. Trace the curve $r = a\sin(2\theta)$. (06 Marks)

4 a. i) If the derivative of arc length $\frac{ds}{dr} = \phi(r)$ then $\phi(r)$ is

A) $\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$ B) $\sqrt{r^2\left(\frac{d\theta}{dr}\right)^2 + 1}$ C) $\sqrt{r \Big/ \left(\frac{dr}{d\theta}\right)^2}$ D) $\sqrt{s^2 + r^2}$

ii) If S_1 and S_2 are surface areas of the solids generated by revoling the ellipses $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ about the y-axis and then

A) $S_1 > S_2$ B) $S_1 < S_2$ C) $S_1 = S_2$ D) Cant predict

iii) If V_1 = volume of the solid generated by revolving area included between x-axis and $x^2 + y^2 = a^2$ about x-axis

V_2 = volume of the solid generated by the entire area of the circle $x^2 + y^2 = a^2$ about x-axis then

A) $V_1 = V_2$ B) $V_2 = 2V_1$ C) $V_2 = 4V_1$ D) $V_2 = 16V_1$

4 iv) The length of the arc in parametric form is

A) $s=\int_{t_1}^{t_2}\sqrt{1+\left(\frac{dy}{dt}\right)^2}\,dt$ B) $s=\int_{t_1}^{t_2}\sqrt{1+\left(\frac{dx}{dt}\right)^2}\,dt$

C) $s=\int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$ D) $s=\int_{t_1}^{t_2}\sqrt{(dx)^2+(dy)^2}\,dt$ **(04 Marks)**

b. Find the volume of the solid generated by revolving the part of the parabola $y^2=4ax$ lying between the vertex and the latus-rectum, about the x-axis. **(04 Marks)**

c. Find the surface area of the solid of revolution of the curve $r=2a\cos\theta$ about the initial line. **(06 Marks)**

d. Evaluate $\int_0^1 \frac{x^\alpha-1}{\log x}dx$, $\alpha\geq 0$. **(06 Marks)**

Part B

5 a. i) The order of the differential equation $\sqrt{\frac{dy}{dx}}=(4x+y+1)$ is

A) 1 B) ½ C) zero D) does not exist

ii) The differential equation $\frac{dy}{dx}=\sin(x+y+1)$ with $y(0)=1$ is

A) zero value problem B) Infinite solution problem
C) Initial value problem D) None of these

iii) By Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$ in the differential $f\left(x,y,\frac{dy}{dx}\right)=0$ we get the differential equation of,

A) Polar trajectory B) Parametric trajectory
C) Orthogonal trajectory D) Parallel trajectory

iv) In the homogeneous differential equation $\frac{dy}{dx}=\frac{f(x,y)}{\phi(x,y)}$ the degrees of the homogeneous functions f(x, y) and $\phi(x,y)$ are,

A) Same B) Different C) Relatively prime D) Exactly one **(04 Marks)**

b. Solve $e^x\tan y\,dx+(1-e^x)\sec^2 y\,dy=0$. **(04 Marks)**

c. Solve $x\log x\frac{dy}{dx}+y=2\log x$. **(06 Marks)**

d. Find the orthogonal trajectory of $r^2=a^2\cos(2\theta)$. **(06 Marks)**

6 a. i) The sum of infinite series $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots\ldots$ is

A) 9.999... B) 99.999.... C) ∞ D) Indeterminate

ii) If the positive term infinite series $\sum_{n=1}^{\infty}u_n$ and $\sum_{n=1}^{\infty}v_n$ are divergent then $\sum_{n=1}^{\infty}u_n-\sum_{n=1}^{\infty}v_n$ is

A) Convergent B) Divergent C) Oscillatory D) Cant predict

iii) If an arbitrary term infinite series $\sum_{n=1}^{\infty}u_n$ is divergent then its absolute term series $\sum_{n=1}^{\infty}|u_n|$ is,

A) Convergent B) Divergent C) Either convergent or divergent D) Cant predict

6 iv) If $\sum u_n$ is positive term infinite series and if $\lim_{n\to\infty} u_n = 0$ then $\sum u_n$ is

A) Convergent B) Divergent C) Either convergent or divergent D) Oscillatory

(04 Marks)

b. Test the convergence of the series,

$$\frac{1}{(1)(4)(5)} + \frac{1}{(2)(9)(11)} + \frac{1}{(3)(14)(17)} + \frac{1}{(4)(19)(23)} + \ldots\ldots$$ (04 Marks)

c. Test the convergence of $\sum_{n=1}^{\infty} \frac{4.7\ldots\ldots(3n+1)}{1.2\ldots\ldots n} x^n$ (06 Marks)

d. Test the absolute and conditional convergence of the following series:

i) $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \ldots\ldots$ ii) $1 - \frac{1}{2^3} + \frac{1}{3^3} - \frac{1}{4^3} + \ldots\ldots$ (06 Marks)

7 a. i) If l, m, n are direction cosines of a straight line then,

A) l+m+n=1 B) $l^2 + m^2 + n^2 = 1$ C) $l = m = n$ D) $\frac{l}{m} = \frac{m}{n} = \frac{n}{l}$

ii) Skew lines are,

A) Intersecting B) Parallel C) Planar D) Not coplanar

iii) The angle between the two lines with direction ratios (1, 1, 2) (2, 0, -1) is

A) 0° B) 45° C) 90° D) $\cos^{-1}\frac{3}{5}$

iv) A point on the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z}{-1}$ is

A) (1, 6, 1) B) (1, 6, -1) C) (-1, 6, -1) D) (1, -6, 1) (04 Marks)

b. Find the intercept form of a plane $2x + 3y + 4z + k = 0$ passing through a point (1, 1, 1). (04 Marks)

c. Find the equation of a plane passing through the line of intersection of the planes $7x - 4y + 7z + 16 = 0$ and $4x + 3y - 2z + 13 = 0$ and perpendicular to the plane $x - y - 2z + 5 = 0$ (06 Marks)

d. Find the magnitude and the equations of the shortest distance between the lines $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$. (06 Marks)

8 a. i) If $\vec{V} = x^2 i + y^2 j + z^2 k$ then $\vec{V}$ at (x, y, z) = (1, 1, 1) becomes

A) Unit vector B) Constant vector C) Scalar D) Complex number

ii) If f is a scalar function then $\nabla f = \text{grad} f$ is

A) Scalar point function B) Vector point function

C) Both A and B D) Neither A nor B.

iii) div curl F is equal to

A) zero B) unity C) i + j + k D) does not exist

iv) If a particle moves along a curve $\vec{R}(t) = x(t)i + y(t)j + z(t)k$ then $\frac{dR}{dt}$ is

A) Radial vector B) Tangential vector C) Normal vector D) Unit vector

(04 Marks)

b. Find a unit vector normal to the surface $x^3y^3z^2 = 4$ at the point (-1, -1, 2). (04 Marks)

c. Prove that div Curl F = $\nabla \cdot \nabla \times F = 0$. (06 Marks)

d. If $\vec{V} = 3xy^2z^2 i + y^3z^2 j - 2y^2z^3 k$ and $\bar{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ then prove that $\vec{V}$ is solenoidal and F is irrotational. (06 Marks)

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