USN

First Semester B.E. Degree Examination, Dec. 07 / Jan. 08

Engineering Mathematics - I

Time: 3 hrs. Max. Marks:100

Note : ***Answer any Eight questions from question no. 1 to 12 and Six questions from question no. 13 to 21.***

1 Derive an expression for the angle between the lines whose direction ratios are a, b, c and a', b', c'. **(05 Marks)**

2 Find the image of the point (2, -1, 3) in the plane $2x+4y+z-24=0$. **(05 Marks)**

3 If $u=\frac{\sinh^{-1}x}{\sqrt{1+x^2}}$, show that

$(1+x^2)y_{n+2}+(2n+3)xy_{n+1}+(n+1)^2y_n=0$. **(05 Marks)**

4 If $y=(\sin^{-1}x)^2$ prove that

$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$ **(05 Marks)**

5 Obtain the reduction formula for $\int_0^{\pi/2}\sin^n x\,dx$. **(05 Marks)**

6 If $U=\left(x^2+y^2+z^2\right)^{-1/2}$ prove that $U_{xx}+U_{yy}+U_{zz}=0$. **(05 Marks)**

7 A particle moves along the curve $x=2t^2$, $y=t^2-4t$, $z=3t-5$. Find the components of velocity and acceleration at time $t=1$ in the direction $i-2j+2k$. **(05 Marks)**

8 Show that the vector $\vec{F}=2xyz^3\hat{i}+x^2z^3\hat{j}+3x^2yz^2\hat{k}$ is an irrotational vector field. Hence find the scalar ϕ such that $\nabla\phi=\vec{F}$. **(05 Marks)**

9 Examine the nature of the series $1+\frac{2!}{2^2}+\frac{3!}{3^3}+\frac{4!}{4^4}+\ldots\ldots\ldots\ldots$ **(05 Marks)**

10 Test the convergence of the series,
- i) $1+3+5+7+\ldots\ldots\ldots$
- ii) $1+½+¼+\ldots\ldots\ldots$ **(05 Marks)**

11 Solve the equation $(x+y)^2\left(x\frac{dy}{dx}+y\right)=xy\left(1+\frac{dy}{dx}\right)$. **(05 Marks)**

12 Obtain the conditions of convergence for the
- i) binomial series and
- ii) the exponential series. **(05 Marks)**

13 a. Show that the lines $\frac{x+1}{1}=\frac{y+1}{2}=\frac{z+1}{3}$ and $x+2y+3z-8=0=2x+3y+4z-11$ intersect. Find their point of intersection. **(05 Marks)**

13 b. Find the points on the lines L_1 and L_2 which are nearest to each other and hence find the shortest distance between them where,

$$L_1 : \frac{x+3}{2} = \frac{y-6}{3} = \frac{z-3}{-2} \text{ and } L_2 : \frac{x}{2} = \frac{y-6}{2} = \frac{z}{-1}$$ **(05 Marks)**

14 a. Show that the angle between the polar curves $r = a(1+\cos\theta)$ and $r^2 = a^2 \cos 2\theta$ is $3\sin^{-1}(3/4)^{1/4}$. **(05 Marks)**

b. If $u = \sin^{-1}(x-y)$ where $x = 3t$, $y = 4t^3$ show that the total derivative of u with respect to 't' is $\frac{3}{\sqrt{1-t^2}}$. Verify the result by direct substitution. **(05 Marks)**

15 a. If $u = x^2 - 2y^2$ and $v = 2x^2 - y^2$ where $x = r\cos\theta$ and $y = r\sin\theta$, show that $\frac{\partial(u,v)}{\partial(r,\theta)} = 6r^3 \sin 2\theta$. **(05 Marks)**

b. The focal length of a mirror is given by the formula $\frac{1}{v} - \frac{1}{u} = \frac{2}{f}$. If equal errors '$e$' are made in the determination of u and v, show that the resulting relative error in f is $e\left(\frac{1}{u} + \frac{1}{v}\right)$. **(05 Marks)**

16 a. Obtain the reduction formula for $I_n = \int \operatorname{cosec}^n x\,dx$ and hence evaluate I_3. **(05 Marks)**

b. Trace the curve $x^2(2a-y) = y^3$. **(05 Marks)**

17 a. Find the entire length of the cardiod $r = a(1+\cos\theta)$. **(05 Marks)**

b. Find the volume of the reel shaped solid formed by the revolution about y-axis, of the part of the parabola $y^2 = 4ax$ cut off by its latus – rectum. **(05 Marks)**

18 a. Find the angle between the normals to the surface $xy = z^2$ at the points (4, 1, 2) and (3, 3, -3). **(05 Marks)**

b. Show that the vector field,

$\overline{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$

is irrotational and find its scalar potential. **(05 Marks)**

19 a. Test for convergence of the series,

$$\frac{2}{3} + \frac{2.4}{3.5} + \frac{2.4.5}{3.5.7} + \ldots\ldots\ldots$$ **(05 Marks)**

b. Find the nature of the series

$$\frac{1}{2} + \left(\frac{2}{3}\right) + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)^3 x^3 + \ldots\ldots\ldots(x > 0)$$ **(05 Marks)**

20 a. Solve $(x^2 + y^2)dx - 2xy\,dy = 0$. **(05 Marks)**

b. Solve $(1+y^2)dx = (\tan^{-1} y - x)dy$. **(05 Marks)**

21 a. Solve $(x+y+1)dx - (2x+2y+3)dy = 0$. **(05 Marks)**

b. Solve $\left(y^2 e^{xy^2} + 4x^3\right)dx + \left(2xye^{xy^2} - 3y^2\right)dy = 0$. **(05 Marks)**

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