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06MAT11

First Semester B.E. Degree Examination, June / July 08
Engineering Mathematics - I

Time: 3 hrs. Max. Marks:100

Note : ***Answer any FIVE full questions, choosing atleast two from each part.***

PART - A

1 a. Find the n^{th} derivative of $\frac{1}{(x+2)(2x+3)} + e^{2x}\cos x$. **(07 Marks)**

b. If $y^{1/m} + y^{-1/m} = 2x$ prove that $(x^2-1)y_{n+2} - (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$. **(07 Marks)**

c. Find the angle between the curves $r = \frac{a}{1+\cos\theta}$, and $r = \frac{b}{1-\cos\theta}$. **(06 Marks)**

2 a. If $u = \log(x^3+y^3+z^3-3xyz)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$. **(07 Marks)**

b. If $u = \tan^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\sin 2u$. **(07 Marks)**

c. If $u = x^2+y^2+z^2$, $v = xy+yz+zx$, $w = x+y+z$. Find $J\left(\frac{u\ v\ w}{x\ y\ z}\right)$. **(06 Marks)**

3 a. Obtain a reduction formula for $I_n = \int \operatorname{cosec}^n x\,dx$. Hence find I_3. **(07 Marks)**

b. Evaluate $\int_0^\infty \frac{dx}{(1+x^2)^n}$, $n>1$. **(07 Marks)**

c. Trace the curve $a^2y^2 = x^2(a^2-x^2)$. **(06 Marks)**

4 a. Find the length of the curve $y^2 = 4ax$ cutoff by the line $3y = 8x$. **(07 Marks)**

b. Find the area between the curve $y^2(a+x) = x^2(a-x)$ and the asymptote. **(07 Marks)**

c. Evaluate $\int_0^1 \frac{x^\alpha - 1}{\log x}dx$, $(\alpha > -1)$ using differentiation under integral sign. **(06 Marks)**

PART - B

5 a. Solve $\frac{dy}{dx} = \frac{y}{x+\sqrt{xy}}$. **(07 Marks)**

b. Solve $x^2\frac{dy}{dx} - 2xy - x + 1 = 0;\ y(1) = 0$. **(07 Marks)**

c. For the family of curves $x^2 + 3y^2 = cy$ (C – parameter), find the orthogonal family of curves. **(06 Marks)**

6 a. Find the nature of the series, $1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + -----$. **(07 Marks)**

b. Test for convergence of the series, $\frac{1}{1+x} + \frac{1}{1+2x^2} + \frac{1}{1+3x^3} + ------$. **(07 Marks)**

c. Test the series for i) Absolute convergence ii) Conditional convergence.

$$x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + -----$$ **(06 Marks)**

7 a. Find the angle between any two diagonals of a cube. **(07 Marks)**

b. Show that the points (0, -1, 0), (2, 1, -1), (1, 1, 1) and (3, 3, 0) are coplanar. **(07 Marks)**

c. Find the shorter distance between the line $x + y + 2z - 3 = 0 = 2x + 3y + 4z - 4$ and z – axis. **(06 Marks)**

8 a. A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is time. Find the components of velocity and acceleration at time t = 1 in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$. **(07 Marks)**

b. Find a, b, c, so that the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at (1, 2, -1) has maximum magnitude of 64 in the direction of z – axis. **(07 Marks)**

c. Prove that $\text{curl}(\phi\vec{F}) = \phi(\nabla\times\vec{F}) + \nabla\phi\times\vec{F}$ **(06 Marks)**

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