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06MAT11

First Semester B.E. Degree Examination, Dec. 07 / Jan. 08
Engineering Mathematics I

Time: 3 hrs. Max. Marks:100

Note : *Answer any FIVE full questions choosing at least two questions from each part.*

Part A

1 a. Find the n^{th} derivatives of,

i) $e^{-x}\sin^2 x$.

ii) $\dfrac{x}{(x-1)(2x+3)}$ **(07 Marks)**

b. Prove that

$D^n\left[\dfrac{\log x}{x}\right]=\dfrac{(-1)^n n!}{x^{n+1}}\left[\log x-1-\dfrac{1}{2}-\dfrac{1}{3}\cdots\cdots-\dfrac{1}{n}\right]$. **(07 Marks)**

c. With the usual notation, prove that

$\dfrac{1}{P^2}=\dfrac{1}{r^2}+\dfrac{1}{r^4}\left(\dfrac{dr}{d\theta}\right)^2$. **(06 Marks)**

2 a. If $u=\sin^{-1}\left(\dfrac{3x^2+4y^2}{3x+4y}\right)$, prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\tan u$. **(07 Marks)**

b. If $u=f(x-y,\, y-z,\, z-x)$, prove that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z}=0$. **(07 Marks)**

c. If $x=e^u\cos v$ and $y=e^u\sin v$, show that $J\cdot J'=1$. **(06 Marks)**

3 a. Obtain the reduction formula for $I_n=\int_0^{\pi/2}\cos^n x\,dx$, where n is a positive integer and hence evaluate I_5. **(07 Marks)**

b. Evaluate : $\int_0^{2a} x^2\sqrt{2ax-x^2}\cdot dx$. **(07 Marks)**

c. Trace the curve $y^2(a-x)=x^3$, where $a>0$. **(06 Marks)**

4 a. For the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$, find $\dfrac{ds}{dx}$ and $\dfrac{ds}{dy}$. **(07 Marks)**

b. Find the area of the cardioid $r=a(1+\cos\theta)$. **(07 Marks)**

c. By the differentiation under integral sign, evaluate $\int_0^1 \dfrac{x^\alpha-1}{\log x}dx$, given $\alpha\geq 0$. **(06 Marks)**

Part B

5 a. Solve :

i) $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$.

ii) $(1+y^2)dx = (\tan^{-1} y - x)dy$.

iii) $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$ (15 Marks)

b. Find the orthogonal trajectories of the family $\frac{2a}{r} = 1 - \cos\theta$. (05 Marks)

6 a. Test for convergence of the series,

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + \sqrt{n+1}}$. (07 Marks)

b. Test for convergence of the series,

$\frac{x}{1\cdot 2} + \frac{x^2}{2\cdot 3} + \frac{x^3}{3\cdot 4} + \cdots\cdots + \infty$. (07 Marks)

c. Test the following series for convergence and absolute convergence,

$1 - \frac{1}{5} + \frac{1}{9} - \frac{1}{13} + \cdots\cdots$ (06 Marks)

7 a. If (l_1, m_1, n_1) and (l_2, m_2, n_2) are the direction cosines of two lines subtending an angle θ between them. Then prove that $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$. (07 Marks)

b. Find the image of the point (1, -1, 2) in the plane $2x + 2y + z = 1$. (07 Marks)

c. Find the magnitude and equations of the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$. (06 Marks)

8 a. A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is time. Find the components of velocity and acceleration at time $t = 1$ in the direction of $i - 3j + 2k$. (07 Marks)

b. If $\vec{F} = grad(x^3 + y^3 + z^3 - 3xyz)$. Then find $div\,\vec{F}$ and $curl\,\vec{F}$. (07 Marks)

c. Prove that $\nabla \times (\phi \vec{A}) = \nabla\phi \times \vec{A} + \phi(\nabla \times \vec{A})$. (06 Marks)

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