USN ☐☐☐☐☐☐☐☐☐☐ **2002 SCHEME** MAT11

First Semester B.E. Degree Examination, June-July 2009
Engineering Mathematics - I

Time: 3 hrs. Max. Marks:100

Note : Answer any FIVE full questions, choosing atleast ONE form each part..

PART - A

1 a. Show that the angle between two diagonals of a cube is $\cos^{-1}(1/3)$. **(06 Marks)**
   b. Find the equation of the plane which passes through the points (0, 1, 1), (1, 1, 2) and (-1, 2, -2). **(07 Marks)**
   c. Find the distance of the point (1, -2, 3) from the plane $x - y + z = 5$ measured parallel to the line $\dfrac{x}{2} = \dfrac{y}{3} = \dfrac{z}{-6}$. **(07 Marks)**

2 a. Find the magnitude and the equation of shortest distance between the lines $\dfrac{x}{2} = \dfrac{y}{-3} = \dfrac{z}{1}$ and $\dfrac{x-2}{3} = \dfrac{y-1}{-5} = \dfrac{z+2}{2}$. **(06 Marks)**
   b. Find the equation of the right circular cone whose vertex is the origin, whose axis is the line $\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{2}{3}$ and which has semi vertical angle of 30^0. **(07 Marks)**
   c. The radius of a normal section of a right circular cylinder is 2 units, the axis lies along the straight line. $\dfrac{x-1}{2} = \dfrac{y+3}{-1} = \dfrac{z-2}{5}$, find its equation. **(07 Marks)**

PART - B

3 a. If tan y = x, then prove that $(1 + x^2)\, y_{n+2} + 2(n+1)\, xy_{n+1} + n(n+1)\, y_x = 0$. **(06 Marks)**
   b. Find the angle between the curves $r = a(1 + \sin\theta)$ and $r = a(1 - \sin\theta)$. **(07 Marks)**
   c. Find the pedal equation for the curve $r^n = a^n \cos n\theta$. **(07 Marks)**

4 a. If $u = \tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$, then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2x$. **(06 Marks)**
   b. If u = x + y + z, uv = y + z, uvw = z, show that $\dfrac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$. **(07 Marks)**
   c. Find the percentage error in the area of a rectangle when an error of +1 percent is made in measuring its length and breadth. **(07 Marks)**

PART - C

5   a.   Derive the reduction formula for $\int \sin^n x \, dx$.   (06 Marks)

   b.   Evaluate $\int_0^1 x^{3/2}(1-x)^{3/2} dx$.   (07 Marks)

   c.   Trace the curve $a^2 y^2 = x^2(a^2 - x^2)$.   (07 Marks)

6   a.   Find the area of the cardioid $r = a(1 + \cos\theta)$.   (06 Marks)

   b.   Find the length of the parabola $y^2 = 4ax$ cut off by the Latus rectum.   (07 Marks)

   c.   Find the volume of the solid generated by revolving the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x-axis.   (07 Marks)

PART - D

7   a.   Solve $3e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0$.   (06 Marks)

   b.   Solve $\cos^2 x \dfrac{dy}{dx} + y = \tan x$.   (07 Marks)

   c.   Show that system of Confocal and Coaxial parabolas $y^2 = 4a(x + a)$ is self orthogonal.   (07 Marks)

8   a.   Test for convergence the series.
$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots\ldots$$ (06 Marks)

   b.   Discuss the convergence of the series
$$x + \frac{x^3}{2.3} + \frac{1.3}{2.4}\cdot\frac{x^5}{5} + \frac{1.3.5.x^7}{2.4.6.7} + \ldots\ldots \qquad (x > 0).$$ (07 Marks)

   c.   Discuss the convergence of the series
$$1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \ldots\ldots$$ (07 Marks)

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