Subject: DISCRETE STRUCTURES

Time: 3 Hours | **JUNE 2011** | **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions, answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. If R is a relation "Less Than" on A = {1, 2, 3, 4} then which of the following is not a partial order relation?

(A) {(1, 1), (2, 2), (3, 3), (4, 4), (3, 4)}
(B) {(1, 1), (2, 2), (3, 3), (4, 4), (3, 4), (4, 3)}
(C) {(1, 1), (2, 2), (3, 3), (4, 4), (3, 4), (1, 2)}
(D) {(1, 1), (2, 2), (3, 3), (4, 4), (1, 4), (4, 3), (1, 3)}

b. Number of reflexive relations that can be defined on a set having 4 elements is

(A) 16 **(B)** 10
(C) 12 **(D)** None of these

c. Composition of functions follows the following rules:

(A) Commutative but not Associative
(B) Commutative and Associative
(C) Not Commutative and Associative
(D) None of the above

d. The n^{th} term of the sequence 5, 13, 25, 41,….. is given by the recursive formula

(A) $f(n) = 4n + f(n-1)$ **(B)** $f(n) = 4n + f(n-1);\ f(0) = 1$
(C) $f(n) = 4n + f(n-1);\ f(1) = 1$ **(D)** $f(n+1) = 4n + f(n-1);\ f(0) = 1$

e. Which one is the contra positive of $q \rightarrow p$?

(A) $p \rightarrow q$ **(B)** $\neg p \rightarrow \neg q$
(C) $\neg q \rightarrow \neg p$ **(D)** None of these

f. Ramesh and Suresh took an examination and probability of their passing is 0.12 and 0.27 respectively. What is the probability that both will not fail in the examination?

(A) 0.324 **(B)** 0.0324
(C) 0.97 **(D)** 0.39

g. If f: $G_1 \rightarrow G_2$ be a homomorphism from group G_1 to G_2 then identity element in group G_2 is given by

(A) $f(e_1)$, where e_1 is identity element in G_1.
(B) f(a), where a is any element in G_1.
(C) There is no relationship between elements of the two sets.
(D) $f(e_1)$, where e_1 is inverse of identity element in G_2.

h. Statement $p \rightarrow q \leftrightarrow \neg p \vee q$ is

(A) A tautology **(B)** A contingency
(C) A contradiction **(D)** None of the above

i. The inference "If you repent, you will go to heaven. You have repented. So you will go to heaven" is derived using

(A) Modus Ponens **(B)** Hypothetical Syllogism
(C) Modus Tollens **(D)** Theory of Attachment

j. An (m, m+1) encoding function e: $B^m \rightarrow B^{m+1}$ is even parity check code generator. The code for data 110 is then

(A) 1111 **(B)** 0110
(C) 1001 **(D)** 1100

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Prove or disprove the followings:

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $(A \cap B)^C = A^C \cup B^C$ **(4×2)**

b. An equipment has two components A and B attached in sequence. The probability that they function is 0.95 and 0.93 respectively. What is probability that the equipment will work? **(8)**

Q.3 a. Show the equivalence of the following statements:
(i) $((p \rightarrow q) \wedge p) \rightarrow q \leftrightarrow T$
(ii) $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow p \rightarrow r$ **(4×2)**

b. Convert the following statement into proposition using standard symbols
   (i) If I appear in the examination then I will pass.
   (ii) I will not pass the examination unless I appear in the examination.
   (iii) If it is Sunday then I will go to temple. Today is Sunday.
   (iv) Mohan will be late or he will not come to office today. **(2×4)**

Q.4 a. Prove that sum of a rational number and an irrational number is irrational. **(8)**

b. Verify the validity of the arguments: "X will pay Rs 1000 to Y if India defeats Sri Lanka in the Test. India won the Test against Sri Lanka. Therefore X paid Rs. 1000 to Y." **(8)**

Q.5 a. Solve the recurrence equation f(n) = f(n – 1) + 5n; f(0) =1. **(8)**

b. Use Mathematical Induction to prove that $10^{2n-1}+1$ is divisible by 11 for all natural number n. **(8)**

Q.6 a. Let R be a relation defined on set of Integer Z as for any two integer x and y, xRy iff x*y = 0. Show that R is not a transitive relation. **(8)**

b. Define lattice. Show that (R, ≤) is a lattice where R is set of real numbers and ≤ (less than equal to) is partial order relation. **(8)**

Q.7 a. Define an invertible function. Given a function f: Z→Z as f(x) = 5x – 2, determine whether the function is invertible on the set of integers Z. **(8)**

b. What is total function? Show that addition of two integers is a computable function. **(8)**

Q.8 a. Show that a subset H of group (G, *) is subgroup of G if and only if $a*b^{-1}$ is in H for all elements a, b of H. **(8)**

b. Define generator of a group. Show that the set $\{1, w, w^2\}$ forms a cyclic group under binary operation of multiplication of number, where 1, w and w^2 are cube roots of unity. **(8)**

Q.9 a. Define Group Codes. Show that (3, 6) encoding function e: $B^3 \to B^6$ defined by
e(000) = 000000, e(001) = 001100, e(010) = 010011, e(011) = 011111,
e(100) = 100101, e(101) = 101001, e(110) = 110110, e(111) = 111010
is a group code. **(8)**

b. Show that identities of both the binary operators in a ring (R, +, *) with unity are unique. **(8)**