ROLL NO. ___________________

Code: AC65 Subject: DISCRETE STRUCTURES

# AMIETE – CS (NEW SCHEME)

Time: 3 Hours **JUNE 2012** Max. Marks: 100

***PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.***

**NOTE: There are 9 Questions in all.**

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

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**Q.1 Choose the correct or the best alternative in the following: (2×10)**

a. Let A be a finite set of size *n*. The number of elements in the power set of A×A is

**(A)** $2^{2^n}$ **(B)** $2^{n^2}$
**(C)** $(2^n)^2$ **(D)** $(2^2)^n$

b. Two events A and B are mutually exclusive events if

**(A)** $A \cap B = \phi$ **(B)** $A \cap B = A - B$
**(C)** $A \cup B = S$ (sample space) **(D)** None of these

c. The contra-positive of $P \rightarrow \sim Q$ is

**(A)** $\sim P \rightarrow \sim Q$ **(B)** $P \rightarrow Q$
**(C)** $\sim P \rightarrow Q$ **(D)** $P \rightarrow \sim Q$

d. Find the negation of "There exists a dog that is 25 years old".

**(A)** Some dog is not 25 years old **(B)** All dog is 25 years old
**(C)** Every dog is 25 years old **(D)** Every dog is not 25 years old

e. Which of the following is correct?

**(A)** $(\forall X)(\exists Y)\{X+Y = 100\}$ **(B)** $(\exists Y)(\forall X)\{X+Y = 100\}$
**(C)** $(\forall X)(\forall Y)\{X+Y = 100\}$ **(D)** None of these

f. A relation R is reflexive on a set A if

**(A)** Domain (R) $\subseteq$ Range (R) **(B)** Domain (R) $\subset$ Range (R)
**(C)** Domain (R) = Range (R) $\neq$ A **(D)** Domain (R) = Range (R) = A

g. Relation $a_n = 3a_{n-1} + n$ is

**(A)** homogeneous recurrence relation
**(B)** non homogeneous recurrence relation
**(C)** generating function
**(D)** None of these

h. The value of the parameter $\alpha$, for which the function $f(x) = 1 + \alpha x$, $\alpha \neq 0$ is the inverse of itself, is

**(A)** –2 **(B)** –1
**(C)** 1 **(D)** 2

i. Which of the following is not true?

**(A)** Every group of prime order is cyclic
**(B)** Every subgroup of a cyclic group is cyclic
**(C)** Every proper subgroup of an infinite cyclic group is infinite
**(D)** Every cyclic group may not be abelian

j. Which of the following is not a ring with respect to addition and multiplication?

**(A)** Set of all natural numbers
**(B)** Set of all integers
**(C)** Set of all rational numbers
**(D)** Set of all $n \times n$ matrices with their elements as real numbers

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**Answer any FIVE Questions out of EIGHT Questions.**
**Each question carries 16 marks.**

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**Q.2** a. If A, B and C are any sets, prove that $A \cap (B - C) = (A \cap B) - C$ **(5)**

b. Write all the subsets of the set $S = \{\phi, \{\phi\}\}$ where $\phi$ is null set. **(5)**

c. A problem is given to four students A, B, C, D whose chances of solving it are $1/2, 1/3, 1/4, 1/5$ respectively. Find the probability that the problem is solved. **(6)**

**Q.3** a. Simplify $[\sim (P \vee Q) \wedge R] \vee [R \wedge (Q \vee P)]$. **(5)**

b. Prove that $[(P \vee \sim Q) \rightarrow P]$ and $(P \vee Q)$ are equivalent. **(5)**

c. Prove that $P \rightarrow (Q \rightarrow R) = (P \wedge Q) \rightarrow R)$ **(6)**

**Q.4** a. Write the predicate calculus (using quantifiers) representation of the following sentence:
"Some baby boys are more mischievous than all baby girls".
Use B(X): X is a baby boy, G(X): X is a baby girl, N(X, Y): X is mischievous than Y. **(7)**

b. Consider the following knowledge base:
$\forall X \forall Y\ cat(X) \wedge fish(Y) \Rightarrow likes\_to\_eat(X, Y)$
$\forall X\ calico(X) \Rightarrow cat(X)$
$\forall X\ tuna(X) \Rightarrow fish(X)$
Convert these into Horn clauses (convert into conjunction of disjuncts by removing all quantifiers and implications). **(9)**

**Q.5** a. Solve the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2$ and $a_1 = 7$. **(8)**

b. Prove the following by mathematical induction:
$(1 + x)^n > 1 + nx$ for natural number $n \geq 2$, $x > -1$ and $x \neq 0$. **(8)**

**Q.6** a. Let A denotes the set of ordered pair of all natural numbers (N×N). Relation R is defined as follows:
for $(a, b), (c, d) \in$ A, $(a, b)$ R $(c, d)$ iff $(a + d) = (b + c)$.
Prove that the relation R is equivalence relation. **(8)**

b. If (L, ≤) is a lattice, then prove that the dual of the poset (L, ≤) is also a lattice. **(8)**

**Q.7** a. Let A : $\{a, b, c, d\}$, B = $\{p, q, r, s\}$ denotes two sets. Identify which of the following is (are) a function/ not a function from A to B? Give reasons for each.
(i) $\{(a, p), (b, q), (c, r)\}$
(ii) $\{(a, p), (b, q), (c, s), (d, r)\}$
(iii) $\{(a, p), (b, s), (b, r), (c, q)\}$
(iv) $\{(a, p), (b, r), (d, r), (c, q)\}$ **(8)**

b. Let $f$ : R → R be a mapping defined as $f(x) = ax + b$; $a, b \in$ R, $a \neq 0$. Is $f$ invertible? Justify. **(8)**

**Q.8** a. Prove that fourth roots of unity form a cyclic group. Find the order of each element of the group. **(8)**

b. Let G be a multiplicative group and $ba = a^p b^q$ where $a, b \in$ G and $p, q \in Z^+$. Prove that order of $a^p b^{q-2}$ is equal to order of $a b^{-1}$. **(8)**

**Q.9** a. Show that the set of numbers of the form $a + b\sqrt{2}$, with $a$ and $b$ as rational numbers is a ring with respect to addition and multiplication. **(8)**

b. Write a note on Generator matrix of an encoding function E and parity-check matrix associated with it. **(8)**