Subject: DISCRETE STRUCTURES

Time: 3 Hours **DECEMBER 2010** **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after half an hour of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. The probability that a leap year selected at random will have 53 Mondays or 53 Tuesdays is

(A) $1/7$ **(B)** $2/7$
(C) $3/7$ **(D)** 0

b. A function $f : A \to B$ is invertible if and only if f is

(A) one – one **(B)** onto
(C) Both one-one and onto **(D)** All are correct

c. In a group G which of the following is not correct

(A) Identity element is unique **(B)** Inverse is unique
(C) $\left(a^{-1}\right)^{-1} = a$ **(D)** $(ab)^{-1} = a^{-1}b^{-1}$

d. The distance between x = 110110 and y = 000101 is

(A) 3 **(B)** 4
(C) 5 **(D)** None of the above

e. If n is an integer and n^2 is odd, then n is

(A) Odd **(B)** Even
(C) Prime **(D)** Even or odd

f. The Bi-conditional $p \leftrightarrow q$ will have a truth value of T whenever

(A) Both p and q have identical Truth values
(B) p is True and q is False
(C) p is False and q is True
(D) p is False and q is False

g. If A and B are any two sets such that $A - B = \{1,2,4\}, B - A = \{7,8\}$ and $A \cup B = \{1,2,4,5,7,8,9\}$, then the set A is

(A) $A = \{1,2,5,9\}$ **(B)** $A = \{1,2,4,5,9\}$
(C) $A = \{1,2,4,5,7\}$ **(D)** $A = \{2,4,5,7,9\}$

h. An Equivalence relation is

(A) Reflexive, Symmetric and Transitive
(B) Reflexive, Symmetric but not Transitive
(C) Reflexive, Antisymmetric and Transitive
(D) Irreflexive, Antisymmetric and Transitive

i. If $f : G_1 \to G_2$ is a homomorphism. Then which of the statement is true

(A) If e_1 is the identity in G_1 and G_2 is the identity in G_2, then $f(e_1) = f(e_2)$
(B) $f(a^{-1}) = [f(a)]^{-1} \forall a \in G$
(C) Both the above are correct
(D) Both are incorrect

j. If A and B are Mutually Exclusive Events then

(A) $A \cap B = \phi$ **(B)** $A \cap B \neq \phi$
(C) $A \cup B = \phi$ **(D)** $A \cup B \neq \phi$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. A computer company requires 30 programmers to handle system programming Jobs and 40 programmers for application programming. If the company appoints 55 programmers to carryout these Jobs, how many of these perform jobs of both types? How many handle only system programming Jobs? How many can handle application programming. **(8)**

b. If A and B are any two events prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and hence prove that $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$ **(8)**

Q.3 a. Construct the truth table for the following compound proposition.
$(p \to q) \leftrightarrow (\neg q \vee r)$ **(8)**

b. Show that $\neg \forall x(P(x) \to Q(x))$ and $\exists x(P(x) \wedge \neg Q(x))$ are logically equivalent. **(8)**

Q.4 a. State any Four rules of Inference. Explain them with an example. **(8)**

b. Consider the following argument: I will get grade A in this course or I will not graduate. If I donot graduate, I will join the army. I got grade A. Therefore I will not join the army. Is this a valid argument? **(4)**

c. Give an example of the following:-
   (i) Direct proof (ii) Indirect proof **(4)**

Q.5 a. Prove the following statement by Mathematical Induction. If a set has n elements, then its power set has 2^n elements. **(8)**

b. Suppose U is a universal set and $A, B_1, B_2 \ldots\ldots\ldots B_n \subseteq U$ prove that
$A \cap (B_1 \cup B_2 \cup \ldots\ldots\ldots \cup B_n) = (A \cap B_1) \cup (A \cap B_2) \cup \ldots. \cup (A \cap B_n)$ **(8)**

Q.6 a. If A = {1,2,3,4,6,12} then **A** define the relation **R** by aRb if and only if **a** divides **b**. Prove that **R** is a partial order on **A**. Draw the Hasse diagram for this relation. **(8)**

b. Define
   (i) Partial order on a set A
   (ii) Total order on a set A
   (iii) Digraph of a Relation **(8)**

Q.7 a. Define Homomorphism and Isomorphism. If f is a homomorphism from group G_1 to G_2 then $f(a^{-1}) = [f(a)]^{-1} \forall a \in G$. **(8)**

b. Find all the cyclic subgroups of $(Z^*, \otimes_7)$. **(8)**

Q.8 a. Let $f : A \to B$ and $g : B \to C$ prove that (i) If both f and g are one-one functions then g*o*f is also one-one. (ii) If both f and g are onto functions then g*o*f is also onto. **(8)**

b. Function f,g,h are defined on a set X={1,2,3} as
   f={(1,2),(2,3),(3,1)}
   g={(1,2),(2,1),(3,3)}
   h={(1,1),(2,2),(3,1)}.
   (i) Find f*o*g, g*o*f. Are they equal?
   (ii) Find f*o*g*o*h and f*o*h*o*g. **(8)**

Q.9 a. The parity-check matrix for an encoding function $E : Z_2^3 \to Z_2^6$ is given by

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

   (i) Determine the associated generator matrix
   (ii) Does this code correct all single errors in transmission. **(8)**

b. Define a Ring. Find all integers K and m for which $(Z, \oplus, \odot)$ is a Ring under the binary operations $x \oplus y = x + y - k$, $x \odot y$ mxy **(8)**