

2008-CA

Test Paper Code: CA

Time: 3 Hours

Max. Marks: 300

INSTRUCTIONS

A. General:

1. This Question Booklet is your Question Paper.
2. This Question Booklet contains **24** pages and has 100 questions.
3. The Question Booklet **Code** is printed on the right-hand top corner of this page.
4. The Question Booklet contains blank spaces for your rough work. No additional sheets will be provided for rough work.
5. **Clip board, log tables, slide rule, calculator, cellular phone, pager and electronic gadgets in any form are NOT allowed.**
6. Write your **Name** and **Roll Number** in the space provided at the bottom.
7. All answers are to be marked only on the machine gradable Objective Response Sheet (**ORS**) provided along with this booklet, as per the instructions therein.
8. The Question Booklet along with the Objective Response Sheet (**ORS**) must be handed over to the Invigilator before leaving the examination hall.
9. Refer to **Special Instructions** on reverse of this sheet.

B. Filling-in the ORS:

10. Write your Roll Number in the boxes provided on the upper left-hand-side of the **ORS** and darken the appropriate bubble under each digit of your Roll Number using a **HB pencil**.
11. Ensure that the **code** on the **Question Booklet** and the **code** on the **ORS** are the same. If the codes do not match, report to the Invigilator immediately.
12. On the lower-left-hand-side of the **ORS**, write your Name, Roll Number, Name of the Test Centre and put your signature in the appropriate box with ball-point pen. Do not write these anywhere else.

C. Marking of Answers on the ORS:

13. Each question has **4 choices** for its answer: (A), (B), (C) and (D). Only **ONE** of them is the correct answer.
14. On the right-hand-side of **ORS**, for each question number, darken with a **HB Pencil** **ONLY** one bubble corresponding to what you consider to be the most appropriate answer, from among the four choices.
15. There will be **negative marking** for wrong answers.

MARKING SCHEME:

- (a) For each correct answer, you will be awarded **3 (Three)** marks.
- (b) For each wrong answer, you will be awarded **-1 (Negative one)** mark.
- (c) Multiple answers to a question will be treated as a wrong answer.
- (d) For each un-attempted question, you will be awarded **0 (Zero)** mark.

Name							
Roll Number							

Special Instructions

N denotes the set of natural numbers.

R denotes the set of real numbers.

P^T denotes the transpose of a matrix P .

\bar{x} denotes the complement of a Boolean variable x .

f' denotes the derivative of a function f .

$f_x(x, y)$ denotes the partial derivative of $f(x, y)$ with respect to x .

Z_n denotes the set of integers congruent modulo n .

$\lfloor x \rfloor$ denotes the greatest integer less than or equal to x .

${}^n P_r$ denotes the number of permutation of n objects taken r at a time.

${}^n C_r$ denotes the number of combination of n objects taken r at a time.

I denotes the identity matrix.

\oplus denotes the XOR Boolean operator.

$|x|$ denotes the absolute value of x .

$n!$ denotes $1 \times 2 \times \dots \times n$ where n is a positive integer.

LPP stands for linear programming problem.

All **bold faced** vectors are column vectors.

For all C programs assume that all standard library functions are accessible.

Q.1 Let $S_n = \frac{1^2}{n^3} + \frac{2^2}{n^3} + \frac{3^2}{n^3} + \dots + \frac{(n-1)^2}{n^3}$. Then $\lim_{n \rightarrow \infty} S_n$ is

- (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) 0 (D) 1

Q.2 The value of the integral $\int_0^{\pi/2} \frac{\tan x}{\tan x + \cot x} dx$ is

- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{2}$ (C) 0 (D) $\frac{\pi}{4}$

Q.3 Suppose that f is continuous and differentiable on $[a, b]$. If $f'(x) \leq 0$ on $[a, c]$ and $f'(x) \geq 0$ on $(c, b]$, $a < c < b$ then on $[a, b]$

- (A) $f(x)$ is never less than $f(c)$ (B) $f(x)$ is always less than $f(c)$
 (C) $f(x)$ is always less than $f(a)$ (D) $f(x)$ is always greater than $f(b)$

Q.4 A company produces two products A and B whose monthly production amount does not exceed 70 units of A and 90 units of B. To produce a unit of A requires 15 man-hours and a unit of B requires 8 man-hours. The company has a total of 400 man-hours available in a month. The profit margins on sale are Rs.120 per unit of A and Rs.85 per unit of B. Let x and y be respectively the number of units of A and B produced per month. The correct LPP formulation of this problem is

- (A) Maximize $400x + 85y$
 Subject to $8x + 15y \leq 120$
 $x \leq 70$
 $y \leq 90$
 $x, y \geq 0$
- (B) Maximize $120x + 85y$
 Subject to $15x + 8y \leq 400$
 $x \leq 70$
 $y \leq 90$
 $x, y \geq 0$
- (C) Maximize $120x + 85y$
 Subject to $15x + 8y \leq 400$
 $x \leq 90$
 $y \leq 70$
 $x, y \geq 0$
- (D) Maximize $120x + 85y$
 Subject to $8x + 15y \leq 400$
 $x \leq 70$
 $y \leq 90$

Q.5 Consider the following regions

$$P = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \geq 1 \text{ and } x + y \leq 2\}$$

$$Q = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1 \text{ and } x + y \leq 2\}$$

$$R = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \geq 1 \text{ and } x + y \geq 2\}$$

$$S = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1 \text{ and } x + y \leq 2\}$$

Which one of the following pair of regions is convex?

- (A) (P, Q) (B) (Q, R) (C) (R, S) (D) (S, P)

Q.6 Consider the following LPP

$$\begin{aligned} &\text{Maximize } ax_1 + x_2 \\ &\text{Subject to } -x_1 + x_2 \leq 5 \\ &\quad \quad \quad x_1 + x_2 \leq 10 \\ &\quad \quad \quad x_1, x_2 \geq 0 \end{aligned}$$

Which one of the following statements is true?

- (A) $x_1 = 5, x_2 = 0$ is the optimal solution if $a = 1$
 (B) $x_1 = 10, x_2 = 0$ is the optimal solution if $a < 1$.
 (C) $x_1 = \frac{5}{2}, x_2 = \frac{15}{2}$ is the optimal solution if $-1 < a < 1$.
 (D) $x_1 = 0, x_2 = 5$ is the optimal solution if $-1 < a < 1$.

Q.7 Consider the following LPP

$$\begin{aligned} &\text{Maximize } 10x - 7y \\ &\text{Subject to } x - y \leq 1 \\ &\quad \quad \quad x + y \geq 3 \\ &\quad \quad \quad x, y \geq 0 \end{aligned}$$

Which of the following is true?

- (A) The LPP has unique solution.
 (B) The LPP has infinite number of optimal solutions.
 (C) The LPP has no feasible solution.
 (D) The LPP is unbounded.

Q.8 Consider the double integral $\int_0^1 \int_x^{2+x} f(x, y) dy dx$. After reversing the order of the integration, the integral becomes

- (A) $\int_0^1 \int_0^{y-2} f(x, y) dx dy + \int_1^2 \int_0^1 f(x, y) dx dy + \int_2^3 \int_y^1 f(x, y) dx dy$
- (B) $\int_0^1 \int_0^y f(x, y) dx dy + \int_1^2 \int_0^1 f(x, y) dx dy + \int_2^3 \int_{y-2}^1 f(x, y) dx dy$
- (C) $\int_0^1 \int_0^y f(x, y) dx dy + \int_1^2 \int_0^y f(x, y) dx dy + \int_2^3 \int_{y-2}^1 f(x, y) dx dy$
- (D) $\int_0^1 \int_0^{y-2} f(x, y) dx dy + \int_1^2 \int_0^y f(x, y) dx dy + \int_2^3 \int_y^1 f(x, y) dx dy$

Q.9 The volume of the closed region bounded by the planes $x=0$, $y=0$, $z=0$ and $2x+5y+10z=10$ is

- (A) $\frac{20}{3}$ (B) 5 (C) $\frac{10}{3}$ (D) $\frac{5}{3}$

Q.10 The area of the surface bounded by $z+x^2+y^2-1=0$, $0 \leq x \leq 1$, $0 \leq y \leq 1$, $z \geq 0$ is

- (A) $\frac{\pi}{12}(5^{\frac{3}{2}}-1)$ (B) $\frac{\pi}{24}(5^{\frac{3}{2}}-1)$ (C) $\frac{\pi}{12}(5^{\frac{3}{2}}+1)$ (D) $\frac{\pi}{24}(5^{\frac{3}{2}}+1)$

Q.11 Let $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ and $P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ -1 & 1 & 0 \end{bmatrix}$. If $A = P^{-1}DP$, then the matrix D is equal to

- (A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Q.12 The double integral $\int_0^2 \int_x^{4-x} f(x, y) dy dx$ under the transformation $u = x + y, v = y - 2x$, is transformed into

- (A) $\int_0^4 \int_{\frac{u}{2}}^u f\left(\frac{u-v}{3}, \frac{2u+v}{3}\right) dv du$ (B) $3 \int_0^4 \int_{\frac{u}{2}}^u f\left(\frac{u-v}{3}, \frac{2u+v}{3}\right) dv du$
 (C) $\frac{1}{3} \int_0^4 \int_{\frac{u}{2}}^u f\left(\frac{u-v}{3}, \frac{2u+v}{3}\right) dv du$ (D) $\frac{1}{3} \int_0^4 \int_{-\frac{u}{2}}^u f\left(\frac{u-v}{3}, \frac{2u+v}{3}\right) dv du$

Q.13 The area of the region bounded by the curves $x^2 = 2y$ and $y^2 = 2x$ is

- (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{4}{3}$ (D) 4

Q.14 The values of a and b for which the following system of linear equations

$$ax + y + 3z = a$$

$$2x + by - z = 3$$

$$5x + 7y + z = 7$$

has an infinite number of solutions, are

- (A) $a = 1, b = 1$ (B) $a = 1, b = 3$ (C) $a = 2, b = 3$ (D) $a = 2, b = 1$

Q.15 Let A and B be any arbitrary square matrices of order 3. Then AB and BA have

- (A) the same eigen values and the same eigen vectors.
 (B) the same eigen values but may have different eigen vectors.
 (C) different eigen values but the same eigen vectors.
 (D) different eigen values and different eigen vectors.

Q.16

The multiplicative inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, is given by

- (A) $A + 4I$ (B) $A - 4I$ (C) $\frac{1}{5}(A + 4I)$ (D) $\frac{1}{5}(A - 4I)$

Q.17

The system of linear equations

$$\begin{aligned} 9x + 3y + z &= 0 \\ 3x + z - 6w &= 0 \\ x + y + z + w &= 0 \\ -6y + z + 9w &= 0 \end{aligned}$$

has

- (A) no solution. (B) infinite number of solutions.
 (C) only one solution. (D) more than one but finite number of solutions.

Q.18 Let u be a unit column vector and $A = I - 2uu^T$. Then A^{-1} is

- (A) $I - 2uu^T$ (B) $I + 2uu^T$ (C) $2uu^T - I$ (D) $4uu^T$

Q.19 The value of the integral $\int_0^3 \int_0^{\sqrt{3}x} \frac{dydx}{\sqrt{x^2 + y^2}}$ is

- (A) $3 \log(2 + \sqrt{3})$ (B) $3 \log(2 - \sqrt{3})$ (C) $3 \log 2$ (D) $\frac{3}{2} \log(2 + \sqrt{3})$

Q.20 Projection of $a + b$ in the direction of c where $a = \vec{i} + 2\vec{j} + 3\vec{k}$, $b = 2\vec{i} - \vec{j} + 3\vec{k}$ and $c = 2\vec{i} - \vec{j} + 2\vec{k}$ is

- (A) $\frac{8}{3}$ (B) $\frac{17}{3}$ (C) $\frac{11}{3}$ (D) 1

Q.21 Let u and v be two non zero and non parallel vectors with $w = \vec{i} + \vec{j} - 2\vec{k}$. Then $(u \times v) \times w = 0$ if

- (A) $u = a\vec{i} + b\vec{j} + c\vec{k}$, $a + b = 2c$ and v is any vector orthogonal to u .
 (B) u and v are orthogonal to each other but not orthogonal to w .
 (C) u and v both are of the form $a\vec{i} + b\vec{j} + c\vec{k}$ where $a + b = 2c$.
 (D) v is orthogonal to u but not orthogonal to w .

Q.22 For what values of a and b , the directional derivative of $u(x, y, z) = ax^2yz + bxy^2z$ at $(1, 1, 1)$ along $\vec{i} + \vec{j} - 2\vec{k}$ is $\sqrt{6}$ and along $\vec{i} - \vec{j} + 2\vec{k}$ is $3\sqrt{6}$?

- (A) $a = 6, b = 0$ (B) $a = 0, b = 6$ (C) $a = 6, b = 6$ (D) $a = -6, b = 6$

Q.23 The solution of the differential equation $yy' + y^2 - x = 0$, where c is a constant, is

- (A) $y^2 = x + ce^{-2x}$ (B) $y^2 = x + ce^{-2x} - \frac{1}{2}$
 (C) $y^2 = x + ce^{-2x} + \frac{1}{2}$ (D) $y^2 = x + ce^{-2x} + 1$

Q.24 Let k be a real constant. The solution of the differential equations $\frac{dy}{dx} = 2y + z$ and $\frac{dz}{dx} = 3y$ satisfies the relation

- (A) $y - z = ke^{3x}$ (B) $y + z = ke^{-x}$ (C) $3y + z = ke^{3x}$ (D) $3y - z = ke^{-3x}$

Q.25 Consider the two sub-spaces $U = \{p(x) \in P_3(x) \mid p'(1) = 0\}$ and $W = \{p(x) \in P_3(x) \mid p(1) = 0, p''(1) = 0\}$ of $P_3(x)$, where $P_3(x)$ is the set of real polynomials of degree at most 3. Then which one of the following polynomials is in $U \cap W$

- (A) $x^3 + 3x^2 - 3x - 1$ (B) $x^3 - 3x^2 - 3x + 1$
 (C) $x^3 - 3x^2 + 3x - 1$ (D) $x^3 + 3x^2 - 3x + 1$

Q.26 Let G be a group with identity e such that for some $a \in G$, $a^2 \neq e$ and $a^6 = e$. Then which of the following is true?

- (A) $a^3 = e, a^4 \neq e$ (B) $a^4 \neq e, a^5 \neq e$ (C) $a^4 \neq e, a^5 = e$ (D) $a^3 = e, a^4 = e$

Q.27 Consider the group of 2×2 non-singular matrices under matrix multiplication over Z_5 . The inverse of $\begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}$ is

- (A) $\begin{bmatrix} 3 & -3 \\ 0 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 3 & 5 \\ 5 & 4 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & 2 \\ 3 & 4 \end{bmatrix}$

Q.28 G is a group of order 51. Then which one of the following statements is false?

- (A) All proper subgroups of G are cyclic.
 (B) If G has only one subgroup of order 3 and only one subgroup of order 17, then G is cyclic.
 (C) G must have an element of order 17.
 (D) If G is abelian then there exists no proper subgroup H of G such that product of all elements of H is identity.

- Q.29 Which of the following is a field?
- (A) Z_{2p} , p is prime. (B) $\left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \mid a, b \in Z_3 \right\}$
- (C) $\left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \mid a, b \in Z_4 \right\}$ (D) $\{a+bx \mid a, b \in Z_3\}$
- Q.30 If general solution of the differential equation $ay''' + by'' + cy' + dy = 0$ is linearly spanned by e^x , $\sin x$ and $\cos x$, then which one of the following holds?
- (A) $a+b-c-d=0$ (B) $a+b+c+d=0$
 (C) $a-b+c-d=0$ (D) $a-b-c-d=0$
- Q.31 Two linearly independent solutions of the differential equation $y'' - 2y' + y = 0$ are $y_1 = e^x$ and $y_2 = xe^x$. Then a particular solution of $y'' - 2y' + y = e^x \sin x$ is
- (A) $y_1 \cos x + y_2(\sin x - x \cos x)$ (B) $y_1 \sin x + y_2(x \cos x - \sin x)$
 (C) $y_1(x \cos x - \sin x) - y_2 \cos x$ (D) $y_1(x \sin x - \cos x) + y_2 \cos x$
- Q.32 Orthogonal trajectories of the family of curves $(x-1)^2 + y^2 + 2ax = 0$ are the solutions of the differential equation
- (A) $x^2 - y^2 - 1 + 2xy \frac{dy}{dx} = 0$ (B) $x^2 + y^2 - 1 + 2xy \frac{dy}{dx} = 0$
 (C) $x^2 - y^2 - 1 - 2xy \frac{dx}{dy} = 0$ (D) $x^2 + y^2 + 1 - 2xy \frac{dx}{dy} = 0$
- Q.33 Which one of the following differential equations represents all circles with radius a ?
- (A) $1 + \left(\frac{dy}{dx}\right)^2 + \sqrt{a^2 - x^2} \frac{d^2y}{dx^2} = 0$ (B) $1 + \left(\frac{dy}{dx}\right)^2 + \sqrt{a^2 - y^2} \frac{d^2y}{dx^2} = 0$
 (C) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 + a^2 \left(\frac{d^2y}{dx^2}\right)^2 = 0$ (D) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = a^2 \left(\frac{d^2y}{dx^2}\right)^2$
- Q.34 An iterative process $x_{n+1} = f(x_n)$ is convergent when
- (A) $|x_{n+1}| < |x_n|$ (B) $|x_{n+1} - x_n| < |x_n - x_{n-1}|$
 (C) $|x_n - x_{n-1}| < |x_{n+1} - x_n|$ (D) $x_{n+1} - x_n < x_n - x_{n-1}$

- Q.35 The function $f(x) = \sin\left(\frac{x}{2} - \sin x\right)$ has
- (A) no zero. (B) only zero at $x=0$
 (C) zeros at $x = n\pi, n = 0, \pm 1, \pm 2, \dots$ (D) zeros at $x = 2n\pi, n = 0, \pm 1, \pm 2, \dots$
- Q.36 What is the degree of the interpolated polynomial for the data (1,5), (2,18), (3,37), (4,62) and (5,93)?
- (A) 3 (B) 4 (C) 5 (D) 2
- Q.37 The real positive root of the equation $x^3 - x - 1 = 0$ to two significant places is
- (A) 1.34 (B) 0.291 (C) 2.32 (D) 1.244
- Q.38 The value of the integral $\int_0^1 \frac{\log(1+x)}{x} dx$ with step length 0.5 by Simpson's rule is
- (A) $\frac{1}{6}(1 + 8\log(1.5) + \log 2)$ (B) $\frac{1}{6}(4\log(1.5) + \log 2)$
 (C) $\frac{1}{6}(-4\log(1.5) + \log 2)$ (D) $\frac{1}{6}(1 - 8\log(1.5) + \log 2)$
- Q.39 The integral $\int_0^1 f(x) dx$ is computed by trapezoidal rule with step length $h=0.1$. Given that the value of second derivative of $f(x)$ lies between 0 and 8 for $x \in [0,1]$, the maximum absolute error in the computed value is
- (A) 0.05 (B) 0.007 (C) 0.0001 (D) 0.7
- Q.40 The solution of the differential equation $\left(x \sin \frac{y}{x} - y \cos \frac{y}{x}\right) dx + x \cos \frac{y}{x} dy = 0$ with initial condition $y(0) = 0$ is
- (A) $x \left| \sin \frac{y}{x} \right| = 1$ (B) $y = \pm n\pi x$ (C) $y = x \left| \sin \frac{y}{x} \right|$ (D) $x = y$
- Q.41 The differential equation $(2x^2 + by^2)dx + cxydy = 0$ is made exact by multiplying the integrating factor $\frac{1}{x^2}$. Then the relation between b and c is
- (A) $2c = b$ (B) $b = c$ (C) $2b + c = 0$ (D) $b + 2c = 0$

Q.42

Let $F(x) = \int_0^x (t^2 - 3t + 2) dt$. Then F has

- (A) a local maximum at $x = 1$ and a local minimum at $x = 2$.
 (B) a local minimum at $x = 1$ and a local maximum at $x = 2$.
 (C) local maxima at $x = 1$ and $x = 2$.
 (D) local minima at $x = 1$ and $x = 2$.

Q.43 Which one of the following is false?

- (A) A continuous function that is never zero on an interval, never changes sign on that interval.
 (B) The function $f(x) = 1$ when x is rational and $f(x) = 0$ when x is irrational is always continuous.
 (C) If the product function $h(x) = f(x)g(x)$ is continuous at $x = 0$, then $f(x)$ or $g(x)$ may not be continuous at $x = 0$.
 (D) A function $f(x)$ is continuous in $[0, 1]$ such that $f(x) \in [0, 1]$. Then there exists a point c in $[0, 1]$ such that $f(c) = c$.

Q.44 Consider the function $f(x) = \min\{x+1, |x+1|\}$. Then $f(x)$ is

- (A) always continuous and differentiable.
 (B) always continuous but not differentiable at all points.
 (C) always continuous but not differentiable at $x = -1$.
 (D) not always continuous.

Q.45

Given that $\frac{dy}{dx} \frac{dx}{dy} = 1$. Which one of the following is always true?

- (A) $\frac{d^2y}{dx^2} \frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2 \frac{d^2x}{dy^2} = 0$ (B) $\frac{d^2y}{dx^2} \frac{dx}{dy} + \left(\frac{dy}{dx}\right)^2 \frac{d^2x}{dy^2} = 0$
 (C) $\frac{d^2y}{dx^2} \left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} \frac{d^2x}{dy^2} = 0$ (D) $\frac{d^2y}{dx^2} \frac{dx}{dy} + \frac{dy}{dx} \frac{d^2x}{dy^2} = 0$

Q.46 The number of local minimum points of $f(x) = |x| + |x-1|$, $x \in R$ is

- (A) 0 (B) 1 (C) 2 (D) infinitely many

Q.47 Let $A_n = \{1, 2, 3, \dots, n\}$ and $A_n^c = N - A_n$ where $n \geq 1$ is a natural number and N is the set of all natural numbers. Which one of the following sets is a finite set?

- (A) $\bigcup_{i=1}^n A_i^c$ (B) $\bigcap_{i=1}^n A_i^c$ (C) $\bigcup_{i=n}^{\infty} A_i^c$ (D) $\bigcap_{i=n}^{\infty} A_i$

- Q.48 Let A and B be any two arbitrary sets. If $P(X)$ and ϕ denote the set of all subsets of a set X and the empty set respectively, then which one of the following is not true ?
- (A) $P(A \cup B) = P(A) \cup P(B)$ (B) $P(A \cap B) = P(A) \cap P(B)$
 (C) $\{\phi\} \subseteq P(A)$ (D) $\phi \in P(A)$
- Q.49 Out of 120 students, 80 students have taken mathematics, 60 students have taken physics, 40 students have taken chemistry, 30 students have taken both physics and mathematics, 20 students have taken both chemistry and mathematics and 15 students have taken both physics and chemistry. If every student has taken at least one course, then how many students have taken all the three courses?
- (A) 5 (B) 25 (C) 15 (D) 10
- Q.50 Consider the experiment of throwing two fair dice. What is the probability that the sum of the numbers obtained in these dice is even?
- (A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{6}$
- Q.51 Two dice are rolled until the sum of the numbers appearing on these dice is either 7 or 8. What is the probability that the sum is 7?
- (A) $\frac{5}{11}$ (B) $\frac{6}{11}$ (C) $\frac{7}{11}$ (D) $\frac{8}{11}$
- Q.52 Let X be the random variable giving the number of heads obtained in 162 successive tosses of a biased coin with probability of getting head in a toss is $\frac{1}{3}$. Assume that the tosses are independent. The standard deviation of X is
- (A) 6 (B) 8 (C) 7 (D) 9
- Q.53 Assume that the number of buses arriving at a bus-stop in an interval of t minutes is Poisson distributed with parameter $0.2t$. The probability that at least three buses will arrive during a 10 minute interval is
- (A) $\frac{e^2 - 5}{e^2}$ (B) $\frac{e^2 - 3}{e^2}$ (C) $\frac{3e^2 - 19}{3e^2}$ (D) $\frac{5}{e^2}$

Q.54 Which one of the following statements is true for the function $f(x, y)$, where

$$f(x, y) = \begin{cases} \frac{xy + y^2}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

- (A) $f(x, y)$ is continuous at $(0, 0)$ (B) $\lim_{(x, y) \rightarrow (0, 0)} f(x, y)$ does not exist
 (C) $\lim_{(x, y) \rightarrow (0, 0)} f(x, y) = 1$ (D) $\lim_{(x, y) \rightarrow (0, 0)} f(x, y) = 2$

Q.55

If $f(x, y) = \begin{cases} \frac{2x^3 + 3y^3}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ Then $(f_x(0, 0), f_y(0, 0))$ is

- (A) (2, 3) (B) (6, 9) (C) (3, 4.5) (D) (0, 0)

Q.56

If $f(x, y) = \frac{1}{64} x^3 y^5$, $x = u + v$, $y = u^2 + v^2$, then $(f_u(x, y), f_v(x, y))$ at $(u, v) = (1, 1)$ is

- (A) (26, 26) (B) (26, 27) (C) (27, 26) (D) (27, 27)

Q.57 The maximum value of $f(x, y, z) = xyz$ along all points lying on the intersection of the planes $x + y + z = 40$ and $z = x + y$ is

- (A) 4000 (B) 3000 (C) 2000 (D) 1000

Q.58 The 2's complement representation of -1 in 16 bit in hexadecimal form is

- (A) $(FFFF)_{16}$ (B) $(FFFE)_{16}$ (C) $(ABCD)_{16}$ (D) $(BCDA)_{16}$

Q.59 The quotient obtained by dividing $(1000100110)_2$ by $(11001)_2$ is

- (A) $(16)_{16}$ (B) $(10)_{10}$ (C) $(14)_{14}$ (D) $(19)_{19}$

Q.60 $(1234)_b + (5432)_b = (6666)_b$ is true

- (A) only when $b > 7$ (B) only when $b < 7$ (C) whenever $b \geq 7$ (D) whenever $b \leq 7$

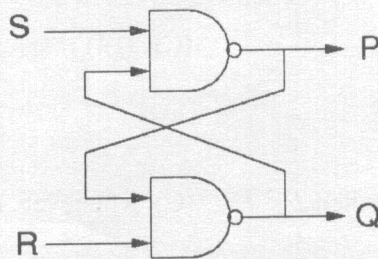
Q.61 The following are the same number represented in different bases.

10001, 101, 25, 21, ...

The next element of the sequence is

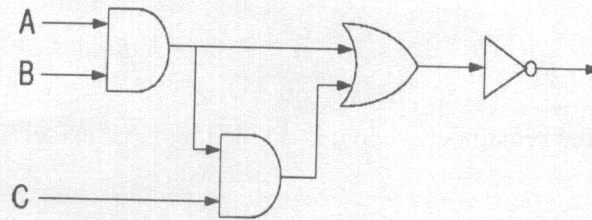
- (A) 10 (B) 17 (C) 22 (D) 33

- Q.62 If $(1494)_b = (3576)_8$, then the value of b is
 (A) 11 (B) 12 (C) 13 (D) 14
- Q.63 Which one of the following Boolean expressions is false?
 (A) $(A+B)(A+\bar{B}) = A$ (B) $A(\bar{A}+A+\bar{A}) = A$
 (C) $(A\bar{B}+AB) = A$ (D) $A+\bar{A}(B+\bar{B}) = A$
- Q.64 Let \oplus stand for the XOR operation and \bar{P} denote the complement of P . If $A \oplus B = C$, then which one of the following Boolean expressions is false?
 (A) $\bar{A} \oplus B \oplus C = 1$ (B) $A \oplus \bar{B} \oplus C = 1$ (C) $A \oplus B \oplus \bar{C} = 1$ (D) $A \oplus B \oplus C = 1$
- Q.65 Let $f(A, B, C) = \bar{A}BC + A\bar{B}C + \bar{A}\bar{B}C$ be a Boolean function. The complement of $f(A, B, C)$ is equal to
 (A) $ABC + \bar{A}BC + A\bar{B}C + \bar{A}\bar{B}C + \bar{A}BC$ (B) $ABC + \bar{A}BC + \bar{A}BC + \bar{A}BC + \bar{A}BC$
 (C) $ABC + \bar{A}BC + \bar{A}BC + \bar{A}BC + \bar{A}BC$ (D) $ABC + \bar{A}BC + \bar{A}BC + \bar{A}BC + \bar{A}BC$
- Q.66 Consider a logic circuit with three inputs A, B, C . The values assigned to A, B, C are such that $\overline{ABC} = \overline{A+B+C}$. Then $\bar{A}BC + \bar{A}BC + \bar{A}BC$ is equal to
 (A) $A+B+C$ (B) \overline{ABC} (C) 0 (D) 1
- Q.67 For which one of the following flip-flops, is the output just the input delayed until the next active clock transition?
 (A) SR (B) T (C) JK (D) D
- Q.68 The flip-flop shown in the following figure is unstable when



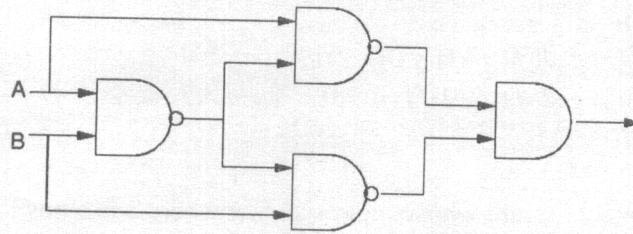
- (A) $S = 0, R = 0, P = 0, Q = 0$ (B) $S = 1, R = 1, P = 0, Q = 0$
 (C) $S = 1, R = 0, P = 0, Q = 0$ (D) $S = 0, R = 1, P = 0, Q = 0$

Q.69 The logic circuit shown in the following figure is equivalent to the Boolean expression.



- (A) $\bar{A} + \bar{B} + \bar{C}$ (B) $A + \bar{B}$ (C) $\bar{A} + B$ (D) $\bar{A} + \bar{B}$

Q.70 The logic circuit shown in the following figure computes



- (A) $A \oplus B$ (B) $\overline{A \oplus B}$ (C) $A + B$ (D) $\overline{A + B}$

Q.71 The recurrence relation $a_r = a_{r-2} + 2$ for $r \geq 2$, with initial conditions $a_0 = 0$ and $a_2 = 2$ has

- (A) no solutions (B) a unique solution
(C) two solutions (D) an infinite number of solutions

Q.72 The computation of ${}^n C_r$ for $n \geq r \geq 0$, using the recursive procedure given by ${}^n C_r = {}^{n-1} C_{r-1} + {}^{n-1} C_r$ with initial condition ${}^n C_0 = 1$ requires

- (A) ${}^n C_r$ additions (B) ${}^n P_r$ additions
(C) $(n-1) + (n-r-1)$ multiplications (D) n additions

Q.73
$$f(n) = \begin{cases} 1 & \text{if } n \leq 0 \\ f(n+1) & \text{if } n > 0 \\ n+1 & \end{cases}$$

The computation of $f(n)$ using the above definition

- (A) terminates for all positive integers n (B) terminates for all negative integers n
(C) terminates only for $n = 0$ (D) terminates for all $n \neq 0$

Q.74 Consider the following algorithm for integers $a > 0$ and $b > 0$

$$\text{gcd}(a, b) = \begin{cases} b & \text{if } b \text{ divides } a \\ \text{gcd}(b, a \% b) & \text{otherwise} \end{cases}$$

where $a \% b$ gives the remainder when a is divided by b . Consider also the following function on integers $n \geq 0$,

$$f(n) = \begin{cases} 1 & \text{if } n \leq 1 \\ f(n-1) + f(n-2) & \text{if } n > 1 \end{cases}$$

Which one of the following statements is false for all $n > 0$

- (A) $\text{gcd}(f(n), f(n-1)) = 1$
- (B) $\text{gcd}(f(n), f(n-1)) = \text{gcd}(f(n-1), f(n-2))$, for $n > 1$
- (C) $\text{gcd}(f(n), f(n-1)) = \text{gcd}(f(n-2), f(n-3))$, for $n > 2$
- (D) $\text{gcd}(f(n), f(n-1)) = 2$

Q.75 The time taken $T(n)$ by a program whose input size is n where n is a positive integer is given by the following recurrence relation

$$T(n) = \begin{cases} T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + 1, & n > 1 \\ 1, & n = 1 \end{cases}$$

Which of the following is true given that c_1 and c_2 are constants

- (A) $T(n) = c_1 n + c_2$
- (B) $T(n) = c_1 \sqrt{n} + c_2$
- (C) $T(n) = c_1 \log_2 n + c_2$
- (D) $T(n) = c_1 n \log_2 n + c_2$

- Q.76 Consider the following method employed by Russian peasants to multiply two positive integers.

```
int prod(int a, int b)
{
    int p = 0;
    int bby2;
    bby2 = b/2;
    while(b>0){
        if(b!= bby2 * 2) p += a;
        a = 2 * a; b = bby2; bby2 = b/2;
    }
    return p;
}
```

The number of addition operations performed by the function when the function call is $\text{prod}(m, n)$ where $m, n > 0$ is

- (A) the number of 1s in the binary representation of n
 - (B) the number of 0s in the binary representation of n
 - (C) the number of 1s in the binary representation of m
 - (D) the number of 0s in the binary representation of m
- Q.77 The output of the following C program is

```
main( )
{
    int a = 1, i = -3, j = -2, k = -1;
    if(i < j <= k) a = 2;
    else a = 4;
    printf("%d", a);
}
```

- (A) 1
 - (B) 2
 - (C) 4
 - (D) -3
- Q.78 Assume that an integer (that is, a variable declared with keyword `int`) occupies 2 bytes of memory. What is the output of the following C program?

```
main( )
{
    int i;
    i = sizeof 5 * 3+4;
    printf("%d", i);
}
```

- (A) 2
- (B) 35
- (C) 38
- (D) 10

Q.79 Consider the following preprocessing directive (in C language)

```
#define CUBE(y) y * (y * y)
```

If the following segment of C code

```
int j;
j = CUBE(-2+4);
```

is executed, the value of j would be

- (A) -26 (B) 8 (C) -8 (D) -14

Q.80 Consider the statement written in C language

```
i = (j = 2, k = 3, 4);
```

What are the values of i, j, k when the above statement is executed?

- (A) i = 2, j = 2, k = 3 (B) i = 4, j = 2, k = 4
(C) i = 4, j = 2, k = 3 (D) i = 2, j = 2, k = 4

Q.81 Consider the following C program segment.

```
float i;
i = 9/2;
```

What is the value of i?

- (A) 4.0 (B) 4.5 (C) 5.0 (D) 0.0

Q.82 Consider the following declaration in C language

```
int A[10][10][10];
```

Let an integer (a variable declared with keyword `int`) occupy 2 bytes of memory. If the starting address in the memory where the array A is stored is 100, then the starting address of the element A[4][5][6] in the memory is

- (A) 912 (B) 1308 (C) 1408 (D) 1012

Q.83 What is the output of the following C program?

```
main( )
{
    int i = 4;
    switch(i)
    {
        case 0:
        case 1: printf("A");
        case 2:
        case 3: printf("B");
        case 4: printf("C");
                printf("D");
        default: printf("E");
    }
}
```

- (A) C (B) CD (C) CDE (D) E

Q.84 The output of the following C program

```
main( )
{
    int i = 2, k = 3, a;
    i ++;
    ++k;
    {
        int i = 0;
        i = k++;
        printf("%d,%d,", i, k );
    }
    printf("%d,%d", i, k);
}
```

is

- (A) 4, 5, 3, 5 (B) 4, 5, 4, 5 (C) 3, 5, 3, 5 (D) 3, 5, 4, 5

Q.85 Which one of the following is a valid IP address on a network?

- (A) 10.10.256.25 (B) 10.10.25.257 (C) 10.10.25.25 (D) 10.258.25.25

Q.86 Which of the following connections to the computer is correct for the devices *keyboard*, *mouse* and *printer* respectively.

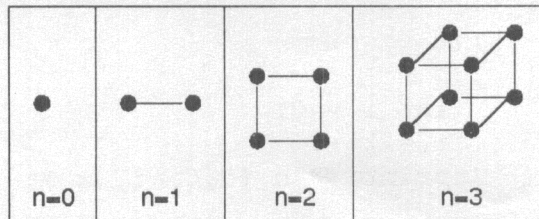
- (A) Parallel port, PS2, serial port (B) PS2, serial port, PS2
(C) PS2, PS2, parallel port (D) Serial port, parallel port, PS2

Q.87 *Noughts and crosses* is a game played on a 3x3 board of squares (labeled 1 to 9) by two players called "O" and "X" respectively who use these symbols as their marks. They play it turn by turn and each player places his mark on an empty square. The first player to place his mark on the 3 squares of a row, a column or a diagonal wins. The figure shows the game after each player has played three turns. Assuming that no player missed a chance to win which of the following is a true statement about the last turn played in this game?

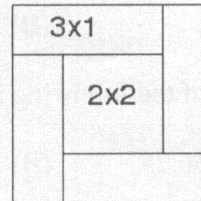
1 X	2 O	3
4 X	5	6
7 O	8 O	9 X

- (A) "X" is placed on square 9 (B) "O" is placed on square 2
(C) "X" is placed on square 1 (D) "O" is placed on square 8

- Q.88 A cube in n dimensions ($n > 0$) may be constructed by taking two cubes of $(n-1)$ dimensions each and joining the corresponding corners by an edge. See the illustration. The number of edges in a cube of 5 dimensions is



- (A) 50 (B) 64 (C) 128 (D) 80
- Q.89 A workman has tiles of sizes 3×1 and 2×2 . If he creates a pattern (as shown in the figure) in which each 2×2 tile is surrounded by four 3×1 tiles to tile a floor of size 12×16 , how many tiles of each size would he require so that he leaves no gaps and doesn't have to break any tiles.



- (A) 48 tiles of size 3×1 and 12 tiles of size 2×2
 (B) 24 tiles of size 3×1 and 30 tiles of size 2×2
 (C) 36 tiles of size 3×1 and 21 tiles of size 2×2
 (D) 16 tiles of size 3×1 and 36 tiles of size 2×2
- Q.90 Let S be a nonempty symmetric and transitive binary relation on a nonempty set A . Consider any pair $(a, b) \in S$. Since S is symmetric, $(b, a) \in S$. Further since S is transitive, $(a, a) \in S$. Which one of the following statements is true?
- (A) S is a reflexive relation since $(a, a) \in S$
 (B) S is a reflexive relation since the reasoning holds for any pair of elements in S .
 (C) S is a reflexive relation because the above reasoning is true only for the specific pair $(a, a) \in S$ that has been considered.
 (D) S need not be reflexive because there may be other elements in A which are not related to any element in A
- Q.91 Which one of the following statements is true?
- (A) A web-browser and a web-server are the same program
 (B) A web-browser can not run on the same machine that runs a web-server
 (C) If a web-browser and a web-server run on the same machine, the web-browser can not access the pages served by the web-server on the same machine.
 (D) Web browsers and web-servers are different programs and they can both run on the same machine.

Q.92 Assume that you have created a new shortcut to a program on your WINDOWS desktop so that the program may be executed by double clicking on the shortcut. Which one of the following is true?

- (A) The shortcut is another copy of the program.
- (B) The shortcut is a reference to an existing program.
- (C) Deletion of the shortcut results in deletion of the program.
- (D) The shortcut moves the existing copy of the program to the desktop.

Q.93 The next term in the sequence 1,3,6,11,20,... is

- (A) 36 (B) 38 (C) 37 (D) 39

Q.94 The digits 1, 2, 3, 4 are to be put in a 4x4 matrix in such a way that each digit appears only once in each row, each column and main diagonals. If first two rows are [1, 3, 4, 2] and [4, 2, 1, 3], then the last two columns are

- (A) $\begin{bmatrix} 1 \\ 4 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 2 \\ 4 \\ 1 \end{bmatrix}$ (B) $\begin{bmatrix} 4 \\ 1 \\ 3 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 3 \\ 1 \\ 4 \end{bmatrix}$ (C) $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \\ 4 \\ 3 \end{bmatrix}$ (D) $\begin{bmatrix} 4 \\ 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}$

Q.95 The number of 0s at the end of 95! is

- (A) 19 (B) 20 (C) 21 (D) 22

Q.96 Given 18 one rupee coins of which one is counterfeit and weighs less than any of the others. Given a two pan balance, the minimum number of weighings required to identify the counterfeit is

- (A) 4 (B) 5 (C) 3 (D) 6

Q.97 6-digit numbers are formed out of the digits 1, 2, 3, 4, 5 such that each such number satisfies the following conditions:

- (i) a digit either doesn't occur or occurs more than once, and
- (ii) all occurrences of a digit occur consecutively.

Then the total number of such numbers is

- (A) 125 (B) 145 (C) 65 (D) 85

Q.98 With respect to computer viruses which one of the following statements is true?

- (A) It is a kind of physical impurity which when present in the CPU causes it to malfunction.
- (B) It is a manufacturing defect in the motherboard.
- (C) A computer virus is a program which copies itself to infect a computer without the permission or the knowledge of the user.
- (D) It is an accumulation of dust on the hard disk causing it to malfunction.

Q.99 Pick the odd one out from the following.

(A) C++

(B) Java

(C) Oracle

(D) Visual Basic

Q.100 Which one of the following is true?

(A) Debugging is a process of identifying and correcting syntactic errors in a program.

(B) Debugging is a process of identifying and correcting logical or run-time errors in a program.

(C) Debugging is a process of downloading a large file through the Internet.

(D) Debugging is a process of de-fragmenting the hard-disk.

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