

# Mental Math with Tricks and Shortcuts

## Addition

**Technique:** Add left to right

$$326 + 678 + 245 + 567 = 900, 1100, 1600, 1620, 1690, 1730, 1790, 1804, \text{ \& } \mathbf{1816}$$

**Note:** Look for opportunities to combine numbers to reduce the number of steps to the solution. This was done with  $6+8 = 14$  and  $5+7 = 12$  above. Look for opportunities to form 10, 100, 1000, and etc. between numbers that are not necessarily next to each other. Practice!

## Multiplication & Squaring

Some useful formulae

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

$$(a+b)(c+d) = (ac + ad) + (bc + bd)$$

$$(a+b)(c-d) = (ac - ad) + (bc - bd)$$

$$(a-b)(c-d) = (ac - ad) - (bc - bd)$$

**$X = 1 \text{ to } 9$  &  $Y = \text{Any Number}$**

$$(X5)^2 = 100X(X+1) + 25$$

$$25 \times Y = (Y \times 100)/4$$

$$50 \times Y = (Y \times 100)/2$$

$$75 \times Y = 3(Y \times 100)/4$$

### Examples

$$49^2 = (40 + 9)^2 = 1600 + 720 + 81 = \mathbf{2401}$$

$$56^2 = (60 - 4)^2 = 3600 - 480 + 16 = \mathbf{3136}$$

$$64 \times 56 = (60 - 4)(60 + 4) = 3600 - 16 = \mathbf{3584}$$

$$23 \times 34 = (20 + 3)(30 + 4) = 600 + 80 + 90 + 12 = \mathbf{782}$$

$$34 \times 78 = (30 + 4)(80 - 2) = 2400 - 60 + 320 - 8 = \mathbf{2652}$$

$$67 \times 86 = (70 - 3)(90 - 4) = 6300 - 280 - 270 + 12 = \mathbf{5762}$$

$$65^2 = 600(7) + 25 = 4200 + 25 = \mathbf{4225}$$

$$25 \times 76 = 7600/4 = \mathbf{1900}$$

$$50 \times 67 = 6700/2 = \mathbf{3350}$$

$$75 \times 58 = (5800 \times 3)/4 = 17400/4 = \mathbf{4350}$$

Square any Two Digit Number ( $a = 10$ 's digit &  $b = 1$ 's digit)

$$(ab)^2 = 100a^2 + 20(a \times b) + b^2$$

$$67^2 = 100(36) + 20(42) + 49 = \mathbf{4489}$$

Multiply any Two 2 Digit Numbers ( $a$  &  $c = 10$ 's digit,  $b$  &  $d = 1$ 's digit)

$$ab \times cd = 100(a \times c) + 10[(b \times c) + (a \times d)] + (b \times d)$$

$$53 \times 68 = 3000 + 580 + 24 = \mathbf{3604}$$

Tricks using  $(X5)^2$

$$(X5 - a)^2 = (X5)^2 - X5(2a) + a^2$$

$$(X5 + a)^2 = (X5)^2 + X5(2a) + a^2$$

$$63^2 = (65 - 2)^2 = 4225 - 260 + 4 = \mathbf{3969}$$

$$67^2 = (65 + 2)^2 = 4225 + 260 + 4 = \mathbf{4489}$$

Squaring Numbers 52 to 99

$$a^2 = [a - (100 - a)]100 + (100 - a)^2$$

$$93^2 = (93 - 7)100 + 7^2 = \mathbf{8649}$$

Squaring Numbers 101 to 148

$$a^2 = [a + (a - 100)]100 + (a - 100)^2$$

$$108^2 = (108 + 8)100 + 8^2 = \mathbf{11664}$$

## Mental Math with Tricks and Shortcuts (continued)

Squaring Numbers near 1000

$$a^2 = [a - (1000 - a)]1000 + (1000 - a)^2$$
$$a^2 = [a + (a - 1000)]1000 + (a - 1000)^2$$

**Examples**

$$994^2 = (994 - 6)1000 + 6^2 = \mathbf{988036}$$
$$1007^2 = (1007 + 7)1000 + 7^2 = \mathbf{1014049}$$

Squaring Numbers that end in 1

$$a^2 = (a - 1)^2 + 2a - 1$$

$$61^2 = 60^2 + 122 - 1 = 3600 + 121 = \mathbf{3721}$$

Squaring Numbers that end in 4

$$a^2 = (a + 1)^2 - (2a + 1)$$

$$44^2 = 45^2 - (88 + 1) = 2025 - 89 = \mathbf{1936}$$

Squaring Numbers that end in 6

$$a^2 = (a - 1)^2 + (2a - 1)$$

$$56^2 = 55^2 + 112 - 1 = 3025 + 111 = \mathbf{3136}$$

Squaring Numbers that end in 9

$$a^2 = (a + 1)^2 - (2a + 1)$$

$$79^2 = 80^2 - (158 + 1) = 6400 - 159 = \mathbf{6341}$$

### Using Squares to Help Multiply

Two Numbers that Differ by 1

$$\text{If } a > b \text{ then } a \times b = a^2 - a$$

$$35 \times 34 = 1225 - 35 = \mathbf{1190}$$

$$\text{If } a < b \text{ then } a \times b = a^2 + a$$

$$35 \times 36 = 1225 + 35 = \mathbf{1260}$$

Two Numbers that Differ by 2

$$a \times b = [(a + b)/2]^2 - 1$$

$$26 \times 28 = 27^2 - 1 = 729 - 1 = \mathbf{728}$$

Two Numbers that Differ by 3 ( $a < b$ )

$$a \times b = (a + 1)^2 + (a - 1)$$

$$26 \times 29 = 27^2 + 25 = 729 + 25 = \mathbf{754}$$

Two Numbers that Differ by 4

$$a \times b = [(a + b)/2]^2 - 4$$

$$64 \times 68 = 66^2 - 4 = 4356 - 4 = \mathbf{4352}$$

Two Numbers that Differ by 6

$$a \times b = [(a + b)/2]^2 - 9$$

$$51 \times 57 = 54^2 - 9 = 2916 - 9 = \mathbf{2907}$$

Two Numbers that Differ by an Even Number:  $a < b$  and  $c = (b - a)/2$

$$a \times b = [(a + b)/2]^2 - c^2$$

$$59 \times 67 = 63^2 - 4^2 = 3969 - 16 = \mathbf{3953}$$

## Mental Math with Tricks and Shortcuts (continued)

Two Numbers that Differ by an Odd Number:  $a < b$  and  $c = [1 + (b - a)]/2$

### Examples

$$a \times b = (a + c)^2 - [b + (c - 1)^2]$$

$$79 \times 92 = 86^2 - (92 + 36) = 7396 - 128 = \mathbf{7268}$$

### Other Multiplying Techniques

Multiplying by 11

$$a \times 11 = a + 10a$$

$$76 \times 11 = 76 + 760 = \mathbf{836}$$

$a \times 11$  = If  $a > 9$  insert a 0 between digits and add sum of digits  $\times 10$

$$76 \times 11 = 706 + 130 = \mathbf{836}$$

Multiplying by Other Two Digit Numbers Ending in 1 ( $X = 1$  to 9)

$$a \times X1 = a + X0a$$

$$63 \times 41 = 63 + (40 \times 63) = 63 + 2520 = \mathbf{2583}$$

Multiplying with Numbers Ending in 5 ( $X = 1$  to 9)

$$a \times X5 = a/2 \times 2(X5)$$

$$83 \times 45 = 41.5 \times 90 = 415 \times 9 = \mathbf{3735}$$

Multiplying by 15

$$a \times 15 = (a + a/2) \times 10$$

$$77 \times 15 = (77 + 38.5) \times 10 = \mathbf{1155}$$

Multiplying by 45

$$a \times 45 = 50a - 50a/10$$

$$59 \times 45 = 2950 - 295 = \mathbf{2655}$$

Multiplying by 55

$$a \times 55 = 50a + 50a/10$$

$$67 \times 55 = 3350 + 335 = \mathbf{3685}$$

Multiplying by Two Digit Numbers that End in 9 ( $X = 1$  to 9)

$$a \times X9 = (X9 + 1)a - a$$

$$47 \times 29 = (30 \times 47) - 47 = 1410 - 47 = \mathbf{1363}$$

Multiplying by Multiples of 9 ( $b =$  multiple of 9 up to  $9 \times 9$ )

$a \times b$  = round  $b$  up to next highest 0 multiply then subtract  $1/10$  of result

$$29 \times 54 = 29 \times 60 - (29 \times 60)/10 \\ = 1740 - 174 = \mathbf{1566}$$

Multiplying by Multiples of 99 ( $b =$  multiple of 99 up to  $99 \times 10$ )

$a \times b$  = round up to next highest 0 multiply and then subtract  $1/100$  of result

$$38 \times 396 = 38 \times 400 - (38 \times 400)/100 \\ = 15200 - 152 = \mathbf{15048}$$

# Mental Math with Tricks and Shortcuts (continued)

## SUBTRACTION

### Techniques:

- 1) Learn to calculate from left to right:  $1427 - 698 = (800 - 100) + (30 - 10) + 9 = \mathbf{729}$
- 2) Think in terms of what number added to the smaller equals the larger:  $785 - 342 = \mathbf{443}$  (left to right)
- 3) Add a number to the larger to round to next highest 0; then add same number to the smaller and subtract:  $496 - 279 = (496 + 4) - (279 + 4) = 500 - 283 = \mathbf{217}$  (left to right)
- 4) Add a number to the smaller to round to the next highest 10, 100, 1000 and etc.; then subtract and add the same number to the result to get the answer:  $721 - 587 = 721 - (587 + 13) = (721 - 600) + 13 = \mathbf{134}$
- 5) Subtract a number from each number and then subtract:  $829 - 534 = 795 - 500 = \mathbf{295}$

## DIVISION

### Techniques:

### Examples

Divide by parts of divisor one at a time:

$$1344/24 = (1344/6)/4 = 224/4 = \mathbf{56}$$

Method of Short Division

$$\begin{array}{r} 340 \leftarrow \text{Remainders (3, 4, and 0 during calculations)} \\ 7 \overline{)1792} \\ \underline{256} \leftarrow \text{Answer} \end{array}$$

Divide both divisor and dividend by same number to get a short division problem

$$972/27 \text{ divide both by } 9 = 3 \overline{)108} \\ \underline{36}$$

Dividing by 5, 50, 500, and etc.: Multiply by 2 and then divide by 10, 100, 1000, and etc.

$$365/5 = 730/10 = \mathbf{73}$$

Dividing by 25, 250, 2500, and etc.: Multiply by 4 and divide by 100, 1000, 10000, and etc.

Dividing by 125: Multiply by 8 and then divide by 1000

$$36125/125 = 289000/1000 = \mathbf{289}$$

It can be divided evenly by:

- 2 if the number ends in 0, 2, 4, 6, and 8
- 3 if the sum of the digits in the number is divisible by 3
- 4 if the number ends in 00 or a 2 digit number divisible by 4
- 5 if the number ends in 0 or 5
- 6 if the number is even and the sum of the digits is divisible by 3
- 7 sorry, you must just try this one
- 8 if the last three digits are 000 or divisible by 8
- 9 if the sum of the digits are divisible by 9
- 10 if the number ends in 0

## Mental Math with Tricks and Shortcuts (continued)

11 if the number has an even number of digits that are all the same: 33, 4444, 777777, and etc.

11 if, beginning from the right, subtracting the smaller of the sums of the even digits and odd digits results in a number equal to 0 or divisible by 11:

$$\begin{array}{l} 406857/11 \text{ Even} = 15 \text{ Odd} = 15 = 0 \\ 1049807/11 \text{ Even} = 9 \text{ Odd} = 20 = 11 \end{array}$$

12 if test for divisibility by 3 & 4 work

15 if test for divisibility by 3 & 5 work

Others by using tests above in different multiplication combinations

### SQUARE ROOTS

### Examples

Separate the number into groups of 2 digits or less beginning from the right

$$(66049)^{1/2}$$

$$6 \ 60 \ 49$$

What number can be squared and be less than 6 = 2 with a remainder of 2

Bring down the second group of digits next to the remainder to give 260

**Double** the first part of the answer to get 4, divide into 26 of the 260 to get 6 as a trial number

Use 4 & 6 to get 46 and multiply by 6 to equal 276 which is larger than 260, therefore try 5

Use 4 & 5 to get 45,  $45 \times 5 = 225$ ,  $260 - 225 = 35$ , bring down the 49 to get 3549

**Double** the 25 to get 50, divide 50 into 354 to get 7 as a trial second part of divisor

Use 50 & 7 to get 507 and multiply  $507 \times 7$  to get 3549 with no remainder.

See complete calculations below:

$$\begin{array}{r} 6 \ 60 \ 49 \ (\mathbf{257} = \text{Answer}) \\ \quad 4 \\ 45)260 \\ \quad \underline{225} \\ 507)3549 \\ \quad \underline{3549} \\ \quad \quad 0 \end{array}$$

### CUBE ROOTS

Memorize the following:

Cube of 1 = 1, 2 = 8, 3 = 27, 4 = 64, 5 = 125, 6 = 216, 7 = 343, 8 = 512, 9 = 729

**Note:** no result ends in the same digit

$$(300763)^{1/3} \quad \text{Divide in to groups of 3 from right} = 300 \ 763$$

**Note:** the number ends in a 3. Last digit of cube will be 7 if this is a cube *without a remainder*

Since 7 cubed = 343 and 6 cubed = 216, the left most group of 300 is between them and we must use the smaller, or 6.

The answer is **67** This method works up to 1,000,000 for true cubes

## Mental Math with Tricks and Shortcuts (continued)

### Cube Roots the Long Way

$$(636056)^{1/3}$$

What number cubed is less than 636 = 8. Put 8 down as first part of answer

Square 8 for 64 and multiply by 300 = 19200. Divide into 114056 = 6, add the 8 and 6 = 14

Multiply 14 x 30 = 420, add 420 to 19200 = 19620, square the 6 and add to 19620 = 19656

If 6 x 19656 is less than 124056, then it is not necessary to use lower number.

$$\begin{array}{r} 636\ 056 \\ \underline{512} \\ 19656)124056 \\ \underline{117936} \\ 7120 \end{array} = 86^3$$

### USE ESTIMATES

Use estimates to check your answers. Get in the habit of doing this for all calculations.

***NOTE: Considerable care has been taken to eliminate errors in this document, but the author does not guarantee that the document is error free by implication or in fact.***