(1) Find the co-ordinates of the focus, length of the latus-rectum and equation of the directrix of the parabola $x^2 = -8y$.

[Ans: $(0, -2)$, 8, $y = 2$]

(2) If the line $3x + 4y + k = 0$ is a tangent to the parabola $y^2 = 12x$, then find k and obtain the co-ordinates of the point of contact.

$$\left[\text{Ans: } k = 16, \left(\frac{16}{3}, -8\right)\right]$$

(3) Derive the equations of the tangents drawn from the point (1, 3) to the parabola $y^2 = 8x$. Obtain the co-ordinates of the point of contact.

$$\left[\text{Ans: } y = x + 2 \text{ at } (2, 4) \text{ and } y = 2x + 1 \text{ at } \left(\frac{1}{2}, 2\right)\right]$$

(4) Find the equation of the chord of the parabola joining the points $P(t_1)$ and $Q(t_2)$. If this chord passes through the focus, then prove that $t_1 t_2 = -1$.

[Ans: $(t_1 + t_2)y = 2(x + a t_1 t_2)$]

(5) If one end-point of a focal chord of the parabola $y^2 = 16x$ is (9, 12), then find its other end-point.

$$\left[\text{Ans: } \left(\frac{16}{9}, -\frac{16}{3}\right)\right]$$

(6) The points $P(t_1)$, $Q(t_2)$ and $R(t_3)$ are on the parabola $y^2 = 4ax$. Show that the area of triangle PQR is $a^2 \left|(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)\right|$.

(7) If the focus of the parabola $y^2 = 4ax$ divides a focal chord in the ratio 1 : 2, then find the equation of the line containing this focal chord.

[Ans: $y = \pm 2\sqrt{2}(x - a)$]

(8) If a focal chord of the parabola $y^2 = 4ax$ forms an angle of measure θ with the positive X-axis, then show that its length is $4\,|\,a\,|\,\text{cosec}^2\,\theta$.

(9) Show that the length of the focal chord of the parabola $y^2 = 4ax$ at the point $P(t)$ is $|\,a\,|\left(t + \dfrac{1}{t}\right)^2$

(10) Find the condition for the line $x\cos\alpha + y\sin\alpha = p$ to be a tangent to the parabola $y^2 = 4ax$ and obtain the co-ordinates of the point of contact.

[Ans: $p + a\sin\alpha\tan\alpha = 0$, $(a\tan^2\alpha,\ -2a\tan\alpha)$]

(11) Show that the equation of the common tangent to the parabolas $y^2 = 4ax$ and $x^2 = 4by$ is $a^{\frac{1}{3}}x + b^{\frac{1}{3}}y + (ab)^{\frac{2}{3}} = 0$.

(12) Find the equations of tangents to the parabola $y^2 = 12x$ from the point $(2, 5)$ and the co-ordinates of the point of contact.

$$\left[\text{Ans: } 3x - 2y + 4 = 0 \text{ at } \left(\frac{4}{3},\ 4\right) \text{ and } x - y + 3 = 0 \text{ at } (3, 6)\right]$$

(13) The line $\overleftrightarrow{PA}$ joining a point P on the parabola and the vertex of the parabola intersects the directrix in K. If M is the foot of the perpendicular to the directrix from P, then show that $\angle MSK$ is a right angle.

(14) If the tangent at point P of the parabola $y^2 = 4ax$ intersects the line $x = a$ in K and the directrix in U, then prove that SK = SU.

(15) $\overline{PQ}$ is a focal chord of the parabola $y^2 = 4ax$. The lengths of the perpendicular line segments from the vertex and the focus to the tangents at P and Q are p_1, p_2, p_3 and p_4 respectively. Show that $p_1 p_2 p_3 p_4 = a^4$.

(16) Prove that the orthocentre of the triangle formed by any three tangents to a parabola lies on the directrix.

(17) A tangent of a parabola has a line segment between the tangents at the points P and Q. Show that the mid-point of this line segment lies on the tangent parallel to $\overline{PQ}$.

(18) If a chord of the parabola $y^2 = 4ax$ subtends a right angle at the vertex, then show that the point of intersection of the tangents drawn at the end-points of this chord is on the line $x + 4a = 0$.

(19) Find the equation of a tangent to the parabola $y^2 = 8x$ which cuts off equal intercepts along the two axes, and find the co-ordinates of the point of contact.

[Ans: $x + y + 2 = 0$, $(2, -4)$]

(20) Prove that the segment cut out on a tangent to a parabola by the point of contact and the directrix subtends a right angle at the focus.

(21) Prove that the foot of the perpendicular from the focus on any tangent to a parabola lies on the Y-axis.

(22) Show that the circle described on any focal chord of a parabola as a diameter touches the directrix.

(23) Prove that, if P is any point on the parabola $y^2 = 4ax$ whose focus is S, the circle described on $\overline{SP}$ as diameter touches the Y-axis.

(24) A quadrilateral ABCD is inscribed inside a parabola. If the sides $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CD}$ and $\overleftrightarrow{DA}$ of the quadrilateral make angles θ_1, θ_2, θ_3 and θ_4 respectively with the axis of the parabola, then prove that

$$\cot\theta_1 + \cot\theta_3 = \cot\theta_2 + \cot\theta_4 .$$

(25) Find the points on the parabola $y^2 = 16x$ which are at a distance of 13 units from the focus.

[Ans: (9, - 12), (9, 12)]

(26) Prove that the parabola $y^2 = 2x$ divides the line-segment joining (1, 1) and (2, 3) internally and externally in the same ratio numerically.

(27) Find the measure of the angle between the two tangents drawn from (1, 4) to the parabola $y^2 = 12x$.

$\left[\text{Ans: } \tan^{-1}\frac{1}{2} \right]$

(28) Prove that the measure of the angle between the two parabolas $x^2 = 27y$ and $y^2 = 8x$ is $\tan^{-1}\frac{9}{13}$.

(29) If the tangents at the points P and Q on the parabola meet at T, then prove that $ST^2 = SP \cdot PQ$.

(30) Find the point on the parabola $y^2 = 64x$ which is nearest to the line $4x + 3y + 64 = 0$.

[Ans: (9, - 24)]

(31) The tangents at the points P and Q to the parabola make complementary angles with the axis of the parabola. Prove that the line $\overleftrightarrow{PQ}$ passes through the point of intersection of the directrix and the axis of the parabola.

(32) The tangents at the points P and Q to the parabola with vertex A meet at the point T. If the lines $\overleftrightarrow{AP}$, $\overleftrightarrow{AT}$ and $\overleftrightarrow{AQ}$ intersect the directrix at the points P, T and Q respectively, then prove that PT = TQ.

(33) Prove that the area of the triangle inscribed in the parabola $y^2 = 4ax$ is $\frac{1}{8|a|}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$, where y_1, y_2 and y_3 are the Y-coordinates of the vertices.

(34) Prove that the area of the triangle formed by the tangents at the parametric points $P(t_1)$, $Q(t_2)$ and $R(t_3)$ to the parabola $y^2 = 4ax$ is $\frac{a^2}{2}|(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)|$.

(35) Find the equation of the common tangents to the parabolas $y^2 = 4x$ and $x^2 = 32y$.

[Ans: $x + 2y + 4 = 0$]

(36) If (h, k) is the point of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4by$ other than the origin, then prove that the equation of their common tangent is $4(kx + hy) + hk = 0$.

(37) Find the equation of the common tangent to the circle $x^2 + y^2 = 2a^2$ and the parabola $y^2 = 8ax$.

[Ans: $x \pm y + 2a = 0$]

(38) Find the equation of the line containing the chord of the parabola $y^2 = 4ax$ whose midpoint is (x_1, y_1).

[Ans: $y_1y - y_1^2 = 2a(x - x_1)$]

(39) The tangent at any point P on the parabola $y^2 = 4ax$ meets the X-axis at T and the Y-axis at R. A is the vertex of the parabola. If RATQ is a rectangle, prove that the locus of the point Q is $y^2 + ax = 0$.

(40) If the angle between two tangents from point P to the parabola $y^2 = 4ax$ is α, then prove that the locus of point P is $y^2 - 4ax = (x + a)^2 \tan^2 \alpha$.