PART – A

Answer *all* the *ten* questions : $10 \times 1 = 10$

1. Find the least positive remainder when 7^{30} is divided by 5.

2. If $\begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is a symmetric matrix, find x.

3. Define a subgroup.

4. Find the direction cosine of the vector $2\hat{i} - 3\hat{j} + 2\hat{k}$.

5. If the radius of the circle $x^2 + y^2 + 4x - 2y - k = 0$ is 4 units, then find k.

6. Find the equation of the parabola if its focus is (2, 3) and vertex is (4, 3).

7. Find the value of $\sin\left[\frac{1}{2}\cos^{-1}(-1)\right]$.

8. If $1, \omega, \omega^2$ are the cube roots of unity, find the value of $\left(1 - \omega + \omega^2\right)^6$.

9. Differentiate $3^x \sinh x$ w.r.t. x.

10. Integrate $\sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$ w.r.t. x.

PART – B

Answer any *ten* questions : $10 \times 2 = 20$

11. If $a \equiv b$ (mod m) and n is a positive divisor of m, prove that

$a \equiv b \pmod{n}$.

12. Without actual expansion show that $\begin{vmatrix} 43 & 1 & 6 \\ 35 & 7 & 4 \\ 17 & 3 & 2 \end{vmatrix} = 0$.

13. Is $G = \{0, 1, 2, 3\}$, under $\otimes$ modulo 4 a group ? Give reason.

14. Find the equation of two circles whose diameters are $x + y = 6$ and $x + 2y = 4$ and whose radius is 10 units.

15. Find the area of the parallelogram whose diagonals are given by the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $3\hat{i} + 4\hat{j} - \hat{k}$.

Find the eccentricity of the ellipse ($a > b$). if the distance between the directrices is 5 and distance between the foci is 4.

Solve $\cot^{-1} x + 2 \tan^{-1} x = \frac{5\pi}{6}$.

Find the least positive integer n for which $\left(\frac{1+i}{1-i} \right)^n = 1$.

If $y = \left(x + \sqrt{1 + x^2} \right)^m$, prove that $\left(\sqrt{1 + x^2} \right) \frac{dy}{dx} - my = 0$.

). Show that for the curve $y = be^{\frac{x}{a}}$ the subnormal varies as the square of the ordinate y.

1. Evaluate $\int_1^e \log_e x \, dx$.

2. Find the order and degree of the differential equation

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}.$$

PART - C

$3 \times 5 = 15$

I. Answer any *three* questions :

23. Find the G.C.D. of $a = 495$ and $b = 675$ using Euclid Algorithm. Express it in the form $495(x) + 675(y)$. Also show that x and y are not unique where $x, y \in z$. 5

24. Solve the linear equations by matrix method :

$$3x + y + 2z = 3$$

$$2x - 3y - z = -3$$

$$x + 2y + z = 4$$

25. a) On the set of rational numbers, binary operation $*$ is defined t $a * b = \sqrt{a^2 + b^2}$, $a, b \in R$, show that $*$ is commutative an associative. Also find the identity element.

b) If a is an element of the group (G, *), then prove that

$$(a^{-1})^{-1} = a.$$

26. a) Find the sine of the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ an

$2\hat{i} + \hat{j} + \hat{k}$.

b) Show that the vectors $\hat{j} + 2\hat{k}$, $\hat{i} - 3\hat{j} - 2\hat{k}$ and $-\hat{i} + 2\hat{j}$ forn

the vertices of the vectors of an isosceles triangle. 2

II. Answer any *two* questions : $2 \times 5 = 10$

27. a) Derive the condition for the two circles

$x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and

$x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ to cut orthogonally. 3

b) Show that the radical axis of the two circles

$2x^2 + 2y^2 + 2x - 3y + 1 = 0$ and

$x^2 + y^2 - 3x + y + 2 = 0$ is perpendicular to the line joining the

centres of the circles. 2

28. a) Find the ends of latus rectum and directrix of the parabola

$y^2 - 4y - 10x + 14 = 0$. 3

b) Find the value of k such that the line $x - 2y + k = 0$ be a tangent to the ellipse $x^2 + 2y^2 = 12$. 2

29. a) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, show that

$x + y + z - xyz = 0$. 3

b) Find the general solution of $\tan 4\theta = \cot 2\theta$. 2

III. Answer any *three* of the following questions : $3 \times 5 = 15$

30. a) Differentiate $\tan x$ w.r.t. x from the first principle. 3

b) If $y = \tan^{-1}\left[\frac{2 + 3x^2}{3 - 2x^2}\right]$, prove that $\frac{dy}{dx} = \frac{2x}{1 + x^4}$. 2

31. a) If $y = \cos\left(p \sin^{-1} x\right)$, prove that

$\left(1 - x^2\right) y_2 - xy_1 + p^2 y = 0$. 3

b) Find the equation of the normal to the curve $y = x^2 + 7x - 2$ at the point where it crosses y-axis. 2

32. a) Integrate $e^{3x}\left(\frac{3 + \tan x}{\cos x}\right)$ w.r.t. x. 3

b) Find the angle between the curves $4y = x^3$ and $y = 6 - x^2$ at (2, 2). 2

33. a) If $x^m y^n = (x+y)^{m+n}$, prove that $x\frac{dy}{dx} = y$. 3

b) Integrate $\frac{1}{7-6x-x^2}$ w.r.t. x. 2

34. Find the area between the curves $y^2 = 6x$ and $x^2 = 6y$. 5

PART – D

Answer any *two* of the following questions : $2 \times 10 = 20$

35. a) Define hyperbola as a locus and hence derive the equation of the hyperbola in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. 6

b) Show that $\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$. 4

36. a) If $\cos\alpha + 2\cos\beta + 3\cos\gamma = 0$, $\sin\alpha + 2\sin\beta + 3\sin\gamma = 0$,

show that i) $\cos 3\alpha + 8\cos 3\beta + 27\cos 3\gamma = 18\cos(\alpha+\beta+\gamma)$

ii) $\sin 3\alpha + 8\sin 3\beta + 27\sin 3\gamma = 18\sin(\alpha+\beta+\gamma)$.

6

b) Prove that $[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}]$. 4

. a) The volume of a sphere is increasing at the rate of 4π c.c./sec. Find the rate of increase of the radius and its surface area when the volume of the sphere is 288π c.c. 6

b) Find the general solution of $\sqrt{3} \tan x = \sqrt{2} \sec x - 1$. 4

a) Show that $\int_0^{\pi/4} \log(1 + \tan x)\, dx = \frac{\pi}{8} \log 2$. 6

b) Solve the differential equation

$$\tan y \frac{dy}{dx} = \sin(x + y) + \sin(x - y).$$ 4

PART - E

Answer any *one* of the following questions : $1 \times 10 = 10$

9. a) Find the cube roots of $3 - i\sqrt{3}$ and find their continued product. 4

b) Show that $(\vec{a} \times \vec{b})^2 = \vec{a}^2 \vec{b}^2 - (\vec{a} \cdot \vec{b})^2$. 4

c) Find the length of the chord of the circle $x^2 + y^2 - 6x - 2y + 5 = 0$ intercepted by the line $x - y + 1 = 0$. 2

40. a) Evaluate $\int_0^3 \frac{\sqrt{x+2}}{\sqrt{x+2} + \sqrt{5-x}}\, dx$.

b) Show that among all the rectangles of a given perimeter, the square has maximum area. 4

c) Differentiate $\sec(5x)^0$ w.r.t. x. 2