

MODEL PAPER (NEW)
PREPARATORY EXAMINATION
II PUC MATHEMATICS(35)

Time : 3 hours

Max. Marks : 100

PART - A

Answer ALL the questions.

10X1=10

1. Find the number of incongruent solutions of $12x \equiv 6 \pmod{3}$.
2. If a line makes angle 90° , 60° and 30° with the positive direction of x, y and z axis respectively, find its direction cosines.
3. Find x such that $\begin{bmatrix} 2 & 4x+5 \\ x+2 & 1 \end{bmatrix}$ is symmetric matrix..
4. Find the inverse of 3 in the group $\{1,3,7,9\}$ under multiplication modulo 10
5. Find the power of the point $(-2,1)$ w.r.t. the circle $x^2 + y^2 - 3x + 5y - 7 = 0$
6. Write the asymptotes of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
7. Find the value of $\cos(\tan^{-1} \frac{3}{4})$
8. If $(1+ni)^2$ is purely imaginary, find n
9. If $y = \log_6 x$, find $\frac{dy}{dx}$
10. Evaluate $\int_0^{\frac{\pi}{4}} \sec^2 x \, dx$

PART - B

Answer any TEN questions

10X2=20

11. If c and a are relatively prime and $c|ab$ prove that $c|b$
12. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = c\hat{i} + (x-2)\hat{j} - \hat{k}$ and \vec{c} lies in the plane of \vec{a} and \vec{b} then find x
13. If $A(\text{adj}A) = 5I$ where I is identity matrix of order 3, then find $|\text{adj}A|$
14. Define a subgroup of a group. Give an example.
15. Find the equation of circle with centre (3,2) touching y axis.
16. Find the focus of the parabola $x^2 + 2x - 6y - 11 = 0$.
17. If $\tan^{-1}x + 2 \cot^{-1}x = \frac{2\pi}{3}$ find x
18. Find the modulus and amplitude of $\frac{1+2i}{1-(1-i)^2}$
19. If $x = a \cos^3 t$, $y = a \sin^3 t$, then find $\frac{dy}{dx}$
20. With usual notation if $s = 4t^3 - 6t^2 + t - 7$ find the acceleration and velocity when $t = 2$ seconds
21. Integrate $\frac{1}{\sqrt{6-x-x^2}}$ w.r.t x
22. Form the differential equation of families of parabolas $y^2 = 4ax$, where a is an arbitrary constant.

PART - C

Answer any THREE questions

3X5=15

23. Find the GCD of 495 and 675 and find k and l such that $495k + 675l = 1$ and show that k and l are not unique. 5
24. a) if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ show that $\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$ 3
 b) Find the projection of $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ on $\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$ 2
25. If Q^+ is the set of all positive rational numbers, Prove $(Q^+, *)$ is an abelian group where * is defined by $a*b = \frac{2ab}{3}$, $a, b \in Q^+$ 5
26. Solve the equations $x - y - 2z = 3$ and $2x + y + z = 5$, $4x - y - 2z = 11$ using matrix method. 5

Answer any TWO questions

2X5=10

27. Find the equation of the circle, cutting the three circles $x^2 + y^2 + 4x + 2y + 1 = 0$, $2x^2 + 2y^2 + 8x + 6y - 3 = 0$ and $x^2 + y^2 + 6x - 2y - 3 = 0$ orthogonally 5
28. a) Find the eccentricity and equations to directrices of the ellipse $4x^2 + 9y^2 - 8x + 36y + 4 = 0$ 3
 b) Find k if $y = 3x + k$ touches the ellipse $4x^2 + 9y^2 = 36$ 2
29. a) Prove that $\sin^{-1}(4/5) + \sin^{-1}(5/13) + \sin^{-1}(16/65) = \frac{\pi}{2}$ 3
 b) Find the general solution of $\tan 5x = \cot 3x$ 2

Answer any THREE questions

3X5=15

30. a) Differentiate 5^x w.r.t x by first principle method. 3
 b) If $y = (\cos x)^{\log x}$ $\frac{dy}{dx}$ 2
31. a) If $y = \sin(a \sinh^{-1} x)$ Prove that $(1+x^2)y^2 + xy_1 + a^2y = 0$. 3
 b) Integrate $\cos \sqrt{x}$ w.r.t x 2
32. a) Integrate $\int \frac{2 \sin x + 3 \cos x}{4 \sin x + 5 \cos x} dx$ 3
 b) Prove that subtangent for any point on the curve $y = be^{\frac{x}{a}}$ is of constant length 2
33. a) If $\tan^{-1} \left(\frac{y}{x} \right) = \frac{1}{2} \log(x^2 + y^2)$ Show that $\frac{dy}{dx} = \frac{x+y}{x-y}$ 3
 b) Evaluate $\int \frac{x^2}{16+x^6} dx$ 2
34. Find the area of the region bounded by ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ by integration method 5

PART-D

Answer any TWO questions

35. a) Find all values of $\left(\frac{1}{2} + i \frac{\sqrt{3}}{2} \right)^{\frac{3}{4}}$ show that continued product of all the values is 1. 6
 b) In a triangle prove that $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ using vector method 4
36. a) Derive the equation of the hyperbola in the standard form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ 6
 b) Show that $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ 4
37. a) A metal cube expands on heating such that its side is increasing uniformly at 2 mm/sec. Find the rate at which its i) volume ii) Surface area and iii) diagonals are increasing when the side is 10 mm 6
 b) Find the general solution of the equation $(\sqrt{3}-1) \sin \theta + (\sqrt{3}+1) \cos \theta = 2$ 4
38. a) Prove that $\int_0^a f(x) dx = \int_0^a f(a-x) dx$ and hence Evaluate $\int_0^{4018} \frac{2^x}{2^x + 2^{4018-x}} dx$ 6
 b) Solve the Differential equation $\frac{dy}{dx} = \frac{1}{\cos(x+y)}$ 4

PART -E

Answer any ONE question

39. a) If \vec{a} is a unit vector and $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 8$ then find $|\vec{x}|$ 4
 b) Find the equation of chord of circle $x^2 + y^2 - 2x + 4y - 17 = 0$ bisected at (-1,2) 4
 c) Find the derivative of $\sin(3x)^0$ 2
40. a) Show that maximum rectangle that can be inscribed in a circle is a square 4
 b) Evaluate $\int \tan^4 x dx$ 4
 c) Find the order and degree of the differential equation $a^2 \frac{d^2 y}{dx^2} = \left[b + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{3}{4}}$ 2
