SOLVED PAPER 2006

Mathematics

1. When 2^{301} is divided by 5, the least positive remainder is :
   (a) 4 (b) 8
   (c) 2 (d) 6

2. If ω is a complex cube root of unity, then
   $$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$ is equal to :
   (a) -1 (b) 1
   (c) 0 (d) ω

3. The ends of the latus rectum of the conic $x^2 + 10x - 16y + 25 = 0$ are :
   (a) $(3, -4), (13, 4)$
   (b) $(-3, -4), (13, -4)$
   (c) $(3, 4), (-13, 4)$
   (d) $(5, -8), (-5, 8)$

4. The equation to the hyperbola having its eccentricity 2 and the distance between its foci is 8, is :
   (a) $\frac{x^2}{12} - \frac{y^2}{4} = 1$ (b) $\frac{x^2}{4} - \frac{y^2}{12} = 1$
   (c) $\frac{x^2}{8} - \frac{y^2}{2} = 1$ (d) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

5. The solution of $\sin^{-1} x - \sin^{-1} 2x = \pm \frac{\pi}{3}$ is :
   (a) $\pm \frac{1}{3}$ (b) $\pm \frac{1}{4}$
   (c) $\pm \frac{\sqrt{3}}{2}$ (d) $\pm \frac{1}{2}$

6. In a ΔABC if the sides are $a = 3, b = 5$ and $c = 4$, then $\sin \frac{B}{2} + \cos \frac{B}{2}$ is equal to :
   (a) $\sqrt{2}$ (b) $\frac{\sqrt{3}+1}{2}$
   (c) $\frac{\sqrt{3}-1}{2}$ (d) 1

7. The two circles $x^2 + y^2 - 2x + 22y + 5 = 0$ and $x^2 + y^2 + 14x + 6y + k = 0$ intersect orthogonally provided k is equal to :
   (a) 47 (b) -47
   (c) 49 (d) -49

8. The radius of the circle $x^2 + y^2 + 4x + 6y + 13 = 0$ is :
   (a) $\sqrt{26}$ (b) $\sqrt{13}$
   (c) $\sqrt{23}$ (d) 0

9. The centre of the circle $x = 2 + 3\cos\theta$, $y = 3\sin\theta - 1$ is :
   (a) $(3, 3)$ (b) $(2, -1)$
   (c) $(-2, 1)$ (d) $(-1, 2)$

10. The sum of the focal distances of any point on the conic $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is :
    (a) 10 (b) 9
    (c) 41 (d) 18

11. The solutions of the equation $\begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -1 & 2 & x \end{vmatrix} = 0$ are :
    (a) $3, -1$ (b) $-3, 1$
    (c) $3, 1$ (d) $-3, -1$

12. If $A = \begin{bmatrix} 3 & 5 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix}$, then $|AB|$ is equal to :
    (a) 80 (b) 100
    (c) -110 (d) 92

13. The inverse of the matrix $\begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}$ is :
    (a) $\frac{1}{11}\begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$
    (c) $\frac{1}{13}\begin{bmatrix} -2 & 5 \\ 1 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

14. The projection of the vector $2\hat{i} + \hat{j} - 3\hat{k}$ on the vector $\hat{i} - 2\hat{j} + \hat{k}$ is :

(a) $-\frac{3}{\sqrt{14}}$ (b) $\frac{3}{\sqrt{14}}$

(c) $-\sqrt{\frac{3}{2}}$ (d) $\frac{3}{\sqrt{2}}$

15. If $12 \cot^2 \theta - 31 \operatorname{cosec} \theta + 32 = 0$, then the value of $\sin \theta$ is :

(a) $\frac{3}{5}$ or 1 (b) $\frac{2}{3}$ or $-\frac{2}{3}$

(c) $\frac{4}{5}$ or $\frac{3}{4}$ (d) $\pm\frac{1}{2}$

16. The circumradius of the triangle whose sides are 13, 12 and 5, is :

(a) 15 (b) $\frac{13}{2}$

(c) $\frac{15}{2}$ (d) 6

17. The general solution of $\sin x - \cos x = \sqrt{2}$, for any integer n is :

(a) $n\pi$ (b) $2n\pi + \frac{3\pi}{4}$

(c) $2n\pi$ (d) $(2n+1)\pi$

18. The amplitude of $\frac{1 + i\sqrt{3}}{\sqrt{3} + i}$ is :

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{6}$

19. If ${}^nC_{12} = {}^nC_6$, then ${}^nC_2$ is equal to :

(a) 72 (b) 153

(c) 306 (d) 2556

20. The middle term in the expansion of $\left(x - \frac{1}{x}\right)^{18}$ is :

(a) ${}^{18}C_9$ (b) $-{}^{18}C_9$

(c) ${}^{18}C_{10}$ (d) $-{}^{18}C_{10}$

21. If α, β, γ are the roots of the equation $2x^3 - 3x^2 + 6x + 1 = 0$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

(a) $-\frac{15}{4}$ (b) $\frac{15}{4}$

(c) $\frac{9}{4}$ (d) 4

22. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular unit vectors, then $|\vec{a} + \vec{b} + \vec{c}|$ is equal to :

(a) 3 (b) $\sqrt{3}$

(c) $\frac{(\sqrt{a^2 + b^2 + c^2})}{3}$ (d) 1

23. $(0, -1)$ and $(0, 3)$ are two opposite vertices of a square. The other two vertices are :

(a) $(0, 1), (0, -3)$ (b) $(3, -1), (0, 0)$

(c) $(2, 1), (-2, 1)$ (d) $(2, 2), (1, 1)$

24. The equation to the line bisecting the join of $(3, -4)$ and $(5, 2)$ and having its intercepts on the x-axis and the y-axis in the ratio $2 : 1$ is :

(a) $x + y - 3 = 0$ (b) $2x - y = 9$

(c) $x + 2y = 2$ (d) $2x + y = 7$

25. The distance between the pair of parallel lines $x^2 + 2xy + y^2 - 8ax - 8ay - 9a^2 = 0$ is :

(a) $2\sqrt{5}\, a$ (b) $\sqrt{10}\, a$

(c) $10\, a$ (d) $5\sqrt{2}\, a$

26. The equation to the circle with centre (2 ... and touching the line $3x + 4y = 5$ is :

(a) $x^2 + y^2 - 4x - 2y + 5 = 0$

(b) $x^2 + y^2 - 4x - 2y - 5 = 0$

(c) $x^2 + y^2 - 4x - 2y + 4 = 0$

(d) $x^2 + y^2 - 4x - 2y - 4 = 0$

27. The condition for a line $y = 2x + c$ to touch the circle $x^2 + y^2 = 16$ is :

(a) $c = 10$ (b) $c^2 = 80$

(c) $c = 12$ (d) $c^2 = 64$

28. $\int \frac{\sin(2x)}{1 + \cos^2 x}\, dx$ is equal to :

(a) $-\frac{1}{2}\log(1 + \cos^2 x) + c$

(b) $2\log(1 + \cos^2 x) + c$

(c) $\frac{1}{2}\log(1 + \cos 2x) + c$

(d) $c - \log(1 + \cos^2 x)$

29. $\int \frac{e^x(1 + \sin x)}{1 + \cos x}\, dx$ is equal to :

(a) $e^x \tan\left(\frac{x}{2}\right) + c$

(b) $e^x \tan x + c$

(c) $e^x \left(\frac{1 + \sin x}{1 - \cos x}\right) + c$

(d) $c - e^x \cot\left(\frac{x}{2}\right)$

30. $\int_{\pi/4}^{\pi/2} \operatorname{cosec}^2 x\, dx$ is equal to :

(a) -1 (b) 1

(c) 0 (d) $\frac{1}{2}$

31. $\int_0^{\pi/4} \log(1 + \tan x)\,dx$ is equal to :

(a) $\frac{\pi}{8}\log_e 2$ (b) $\frac{\pi}{4}\log_2 e$

(c) $\frac{\pi}{4}\log_e 2$ (d) $\frac{\pi}{8}\log_e\left(\frac{1}{2}\right)$

32. The modulus and amplitude of $\frac{1+2i}{1-(1-i)^2}$ are :

(a) $\sqrt{2}$ and $\frac{\pi}{6}$ (b) 1 and 0

(c) 1 and $\frac{\pi}{3}$ (d) 1 and $\frac{\pi}{4}$

33. $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$ is equal to :

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) 0 (d) 1

34. If $f(x) = \begin{cases} \frac{\sin 5x}{x^2 + 2x}, & x \neq 0 \\ k + \frac{1}{2}, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is :

(a) 1 (b) -2

(c) 2 (d) $\frac{1}{2}$

35. The area bounded by the parabola $y^2 = 4ax$ and the line $x = a$ and $x = 4a$ is :

(a) $\frac{35a^2}{3}$ (b) $\frac{4a^2}{3}$

(c) $\frac{7a^2}{3}$ (d) $\frac{56a^2}{3}$

36. A population $p(t)$ of 1000 bacteria introduced into nutrient medium grows according to the relation $p(t) = 1000 + \frac{1000t}{100 + t^2}$. The maximum size of this bacterial population is:

(a) 1100 (b) 1250

(c) 1050 (d) 5250

37. The differential equation representing a family of circles touching the y-axis at the origin is :

(a) $x^2 + y^2 - 2xy\frac{dy}{dx} = 0$

(b) $x^2 + y^2 + 2xy\frac{dy}{dx} = 0$

(c) $x^2 - y^2 - 2xy\frac{dy}{dx} = 0$

(d) $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$

38. The general solution of the differential equation $(2x - y + 1)\,dx + (2y - x + 1)\,dy = 0$ is :

(a) $x^2 + y^2 + xy - x + y = c$

(b) $x^2 + y^2 - xy + x + y = c$

(c) $x^2 - y^2 + 2xy - x + y = c$

(d) $x^2 - y^2 - 2xy + x - y = c$

39. If $y = \tan^{-1}\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}$, then $\frac{dy}{dx}$ is equal to :

(a) $\frac{x^2}{\sqrt{1-x^4}}$ (b) $\frac{x^2}{\sqrt{1+x^4}}$

(c) $\frac{x}{\sqrt{1+x^4}}$ (d) $\frac{x}{\sqrt{1-x^4}}$

40. If $x = \sin t$, $y = \cos pt$, then :

(a) $(1 - x^2)y_2 + xy_1 + p^2y = 0$

(b) $(1 - x^2)y_2 + xy_1 - p^2y = 0$

(c) $(1 + x^2)y_2 - xy_1 + p^2y = 0$

(d) $(1 - x^2)y_2 - xy_1 + p^2y = 0$

41. If ST and SN are the lengths of the subtangent and the subnormal at the point $\theta = \frac{\pi}{2}$ on the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, $a \neq 1$, then :

(a) $ST = SN$ (b) $ST = 2SN$

(c) $ST^2 = a\,SN^3$ (d) $ST^3 = a\,SN$

42. If θ is the acute angle of intersection at a real point of intersection of the circle $x^2 + y^2 = 5$ and the parabola $y^2 = 4x$, then $\tan\theta$ is equal to :

(a) 1 (b) $\sqrt{3}$

(c) 3 (d) $\frac{1}{\sqrt{3}}$

43. Universal set,

$U = \{x \mid x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$

$A = \{x \mid x^2 - 5x + 6 = 0\}$

$B = \{x \mid x^2 - 3x + 2 = 0\}$

what is $(A \cap B)'$ equal to?

(a) $\{1, 3\}$ (b) $\{1, 2, 3\}$

(c) $\{0, 1, 3\}$ (d) $\{0, 1, 2, 3\}$

44. Which of the following statements is not correct for the relation R defined by aRb, if and only, if b lives within on kilometre from a?

(a) R is reflexive

(b) R is symmetric

(c) R is not anti-symmetric

(d) None of the above

45. What is the value of

$$\frac{(1001)_2^{(11)_2} - (101)_2^{(11)_2}}{(1001)_2^{(10)_2} + (1001)_2^{(01)_2}(101)_2^{(01)_2} + (101)_2^{(10)_2}}\ ?$$

(a) $(1001)_2$ (b) $(101)_2$
(c) $(110)_2$ (d) $(100)_2$

Physics

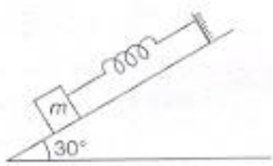
46. The angle turned by a body undergoing circular motion depends on time as $\theta = \theta_0 + \theta_1 t + \theta_2 t^2$. Then the angular acceleration of the body is :

(a) θ_1 (b) θ_2
(c) $2\theta_1$ (d) $2\theta_2$

47. The moment of inertia of a circular disc about an axis passing through the circumference perpendicular to the plane of the disc is :

(a) MR^2 (b) $\frac{3}{2}MR^2$
(c) $\frac{MR^2}{2}$ (d) $\frac{5}{4}MR^2$

48. A body of mass 5 kg is suspended by a spring balance on an inclined plane as shown in figure. The spring balance measure :



(a) 50 N (b) 25 N
(c) 500 N (d) 10 N

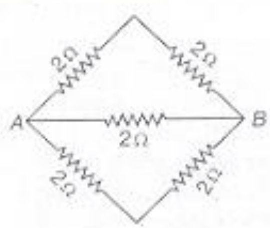
49. Under the action of a force $F = Cx$ the position of a body changes from 0 to x. The work done is :

(a) $\frac{1}{2}Cx^2$ (b) Cx^2
(c) Cx (d) $\frac{1}{2}Cx$

50. If $\vec{A}\cdot\vec{B} = \vec{A}\times\vec{B}$, then angle between $\vec{A}$ and $\vec{B}$ is :

(a) 45° (b) 30°
(c) 60° (d) 90°

51. Each resistance shown in figure is 2Ω. The equivalent resistance between A and B is :



(a) 2Ω (b) 4Ω
(c) 8Ω (d) 1Ω

52. A physical quantity is given by $X = [M^a L^b T$
The percentage error in measurement of M,
and T are α, β and γ respectively. Then, th
maximum % error in the quantity X is :

(a) $a\alpha + b\beta + c\gamma$ (b) $a\alpha + b\beta - c\gamma$
(c) $\frac{a}{\alpha} + \frac{b}{\beta} + \frac{c}{\gamma}$ (d) none of these

53. If emf induced in a coil is 2 V by changing th
current in it from 8 A to 6 A in 2×10^{-3} s, the
the coefficient of self induction is :

(a) 2×10^{-3} H (b) 10^{-3} H
(c) 0.5×10^{-3} H (d) 4×10^{-3} H

54. A hollow metallic sphere of radius R is give
charge Q. Then, the potential at the centre
(a) zero (b) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}$
(c) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{2Q}{R}$ (d) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2R}$

55. Susceptibility of ferromagnetic substance is

(a) > 1 (b) < 1
(c) zero (d) 1

56. What is the refractive index of a prism wi
angle $A = 60°$ and angle of minimum deviati
$d_m = 30°$?

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$
(c) 1 (d) $\frac{1}{\sqrt{3}}$

57. A satellite of mass m is placed at a distanc
from the centre of earth (mass M). T
mechanical energy of the satellite is :

(a) $-\frac{GMm}{r}$ (b) $\frac{GMm}{r}$
(c) $\frac{GMm}{2r}$ (d) $-\frac{GMm}{2r}$

58. A cell of constant emf first connected to
resistance R_1 and then connected to
resistance R_2. If power delivered in both ca

is same then the internal resistance of the cell is :

(a) $\sqrt{R_1R_2}$ (b) $\sqrt{\frac{R_1}{R_2}}$

(c) $\frac{R_1-R_2}{2}$ (d) $\frac{R_1+R_2}{2}$

59. Energy gas between valence band and conduction band of a semiconductor is :

(a) zero (b) infinite

(c) 1 eV (d) 10 eV

60. At what point of a projectile motion, acceleration and velocity are perpendicular to each other ?

(a) At the point of projection

(b) At the point of drop

(c) At the top most point

(d) Anywhere in between the point of projection and top most point

61. An object is placed at a distance 20 cm from the pole of a convex mirror of focal length 20 cm. The image is produced at :

(a) 13.3 cm (b) 20 cm

(c) 25 cm (d) 10 cm

62. Angular momentum is conserved :

(a) always

(b) never

(c) when external force is absent

(d) when external torque is absent

63. The plano-convex lens of focal length 20 cm and 30 cm are placed together to form a double convex lens. The final focal length will be :

(a) 12 cm (b) 60 cm

(c) 20 cm (d) 30 cm

64. Initially two gas samples 1 and 2 are at the same condition. The volume of the two are halved, one isothermally and the other adiabatically. What is the relation between the final pressure P_1 and P_2 ?

(a) $P_1 = P_2$

(b) $P_1 > P_2$

(c) $P_2 > P_1$

(d) Cannot be determined

65. A can is taken out from a refrigerator at 0°C. The atmospheric temperature is 25°C. If t_1 is the time taken to heat from 0°C to 5°C and t_2 is the time taken from 10°C to 15°C, then :

(a) $t_1 > t_2$ (b) $t_1 < t_2$

(c) $t_1 = t_2$ (d) there is no relation

66. A simple pendulum hanging from the ceiling of a stationary lift has time period t_1. When the lift moves downward with constant velocity, the time period is t_2, then :

(a) t_2 is infinity (b) $t_2 > t_1$

(c) $t_2 < t_1$ (d) $t_2 = t_1$

67. Two progressive waves having equation $x_1 = 3 \sin \omega t$ and $x_2 = 4 \sin (\omega t = 90°)$ are super imposed. The amplitude of the resultant wave is :

(a) 5 unit (b) 1 unit

(c) 3 unit (d) 4 unit

68. In a magnetic field of 0.05 T area of coil changes from 101 cm^2 to 100 cm^2 without changing the resistance which is 2Ω. The amount of charge that flow during this period is :

(a) 2.5×10^{-6} C (b) 2×10^{-6} C

(c) 10^{-6} C (d) 8×10^{-6} C

69. A dielectric is introduced in a charged and isolated parallel plate capacitor, which of the following remains unchanged ?

(a) Energy

(b) Charge

(c) Electric field

(d) Potential difference

70. If in a triode valve amplification factor is 20 and plate resistance is 10 kΩ, then its mutual conductance is :

(a) 2 milli mho

(b) 20 milli mho

(c) (1/2) milli mho

(d) 200 milli mho

71. Which of the following is a fusion reaction?

(a) ${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4$

(b) ${}_1H^2 + {}_1H^2 \rightarrow 2({}_1He^2)$

(c) ${}_1H^1 + {}_1H^1 \rightarrow {}_2He^4$

(d) ${}_1H^1 + {}_1H^2 \rightarrow {}_2He^4 + n$

72. The correct relation between α and β in a transistor is :

(a) $\beta = \frac{\alpha}{1-\alpha}$ (b) $\beta = \frac{\alpha}{1+\alpha}$

(c) $\beta = \frac{1+\alpha}{\alpha}$ (d) $\beta = 1 - \alpha$

73. Which of the following law states that "good absorbers of heat are good emitters"?

(a) Stefan's law (b) Kirchhoff's law

(c) Planck's law (d) Wien's law

74. Doping of intrinsic semiconductor is done :
(a) to neutralize charge carriers
(b) to increase the concentration of majority charge carriers
(c) to make it neutral before disposal
(d) to carry out further purification

75. If λ is the wavelength of hydrogen atom from the transition $n = 3$ to $n = 1$, then what is the wavelength for doubly ionised lithium ion for same transition?
(a) $\frac{\lambda}{3}$ (b) 3λ
(c) $\frac{\lambda}{9}$ (d) 9λ

76. A rocket of mass 1000 kg is exhaust gases at a rate of 4 kg/s with a velocity 3000 m/s. The thrust developed on the rocket is :
(a) 12000 N (b) 120 N
(c) 800 N (d) 200 N

77. Ampere-hour is the unit of :
(a) quantity of charge
(b) potential
(c) energy
(d) current

78. Water falls from a tap, down the streamline :
(a) area decreases
(b) area increases
(c) velocity remains same
(d) area remains same

79. Positively charged particles are projected into a magnetic field. If the direction of the magnetic field is along the direction of motion of the charge particles, the particles get :
(a) accelerated
(b) decelerated
(c) deflected
(d) no changed in velocity

80. In Young's double slit experiment a minim[um] observed when path difference between [t]he interfering beam is :
(a) λ (b) 1.5λ
(c) 2λ (d) 2.25λ

81. Calculate the energy released when th[ree] α-particles combined to from a ^{12}C nucleus, th[e] mass defect is :
(atomic mass of $_2He^4$ is 4.002603 u)
(a) 0.007809 u (b) 0.002603 u
(c) 4.002603 u (d) 0.5 u

82. In a step-up transformer, if ratio of turns [of] primary to secondary is 1 : 10 and primar[y] voltage is 230 V. If the load current is 2 A, the[n] the current in primary is :
(a) 20 A (b) 10 A
(c) 2 A (d) 1 A

83. If the equation of transverse wave [is] $Y = 2\sin(kx - 2t)$, then the maximum partic[le] velocity is :
(a) 4 unit (b) 2 unit
(c) zero (d) 6 unit

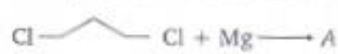
84. Fusion reaction takes place at high temperatu[re] because :
(a) KE is high enough to overcome repulsi[on] between nuclei
(b) nuclei are most stable at this temperatur[e]
(c) nuclei are unstable at this temeprature
(d) none of the above

85. An isotope decays to 1/16th of its mass in 1[...] What is the half-life period of the isotope?
(a) 15 min (b) 30 min
(c) 12 min (d) 10 min

Chemistry

86. The crystal field splitting energy for octahedral (Δ_o) and tetrahedral (Δ_t) complexes is related to :
(a) $\Delta_t = \frac{4}{9}\Delta_o$ (b) $\Delta_t = \frac{1}{2}\Delta_o$
(c) $\Delta_o = 2\Delta_t$ (d) $\Delta_o = \frac{4}{9}\Delta_t$

87. What is the product A in the following ?



(a) 

(b) 

(c) Both (a) and (b)
(d) None of the above

88. Which of the following species has a bond order other than 3 ?
(a) CO (b) CN^-
(c) NO^+ (d) O_2^+

89. The number of waves in an orbit are :
(a) n^2 (b) n
(c) $n-1$ (d) $n-2$

90. When glucose reacts with bromine water the main product is :
(a) gluconic acid (b) glyceraldehyde
(c) sorbitol (d) saccharic acid

91. The ion which exhibits green colour ?
(a) Cu^{2+} (b) Mn^{2+}
(c) Co^{2+} (d) Ni^{2+}

92. The probability of finding the electron in the orbital is :
(a) 100% (b) 90–95%
(c) 70–80% (d) 50–60%

93. Which destroy antigens ?
(a) Insulin (b) Antibodies
(c) Chromoprotein (d) Phosphoprotein

94. In $2HI \rightleftharpoons H_2 + I_2$, the forward reaction is not affected by change in :
(a) catalyst (b) pressure
(c) volume (d) temperature

95. Nylon-66 is an example of :
(a) poly propylene (b) polyester
(c) polyamide (d) polystyrene

96. 1 mole of $N_2O_4(g)$ at 300 K is kept in a closed container under one atmosphere. It is heated to 600 K when 20% by mass of $N_2O_4(g)$ decomposes to $NO_2(g)$. The resultant pressure is :
(a) 1.2 atm (b) 2.4 atm
(c) 2.0 atm (d) 1.0 atm

97. A hypothetical reaction $A \rightarrow 2B$, proceeds through following sequence of steps :
(i) $A \longrightarrow C; \Delta H = q$
(ii) $C \longrightarrow D; \Delta H = v$
(iii) $\frac{1}{2}D \longrightarrow B; \Delta H = x$

Then the heat of reaction is :
(a) $q - v + 2x$ (b) $q + v - 2x$
(c) $q + v + 2x$ (d) $q + 2v - 2x$

98. Following reaction is :

(a) S_N (b) S_E
(c) E1 (d) E1-CB

99. The cathodic reaction of a dry cell is represented by

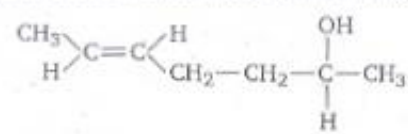
$$2MnO_2(s) + Zn^{2+} + 2e^- \longrightarrow ZnMn_2O_4(s)$$

If, there are 8 g of MnO_2 in the cathodic compartment then the time for which the dry cell will continue to give a current of 2 milliampere is :
(a) 25.675 day (b) 51.35 day
(c) 12.8 day (d) 6.423 day

100. On heating with oxalic acid at 110°C, glycerine gives :
(a) glyceryl trioxalate
(b) formic acid
(c) glyceryl dioxalate
(d) none of the above

101. Which of the following is not the example of pseudounimolecular reactions ?
(a) $CH_3COOC_2H_5 + H_2O \xrightarrow{H^+} CH_3COOH + C_2H_5OH$
(b) $C_{12}H_{22}O_{11} + H_2O \xrightarrow{H^+} \underset{\text{(Glucose)}}{C_6H_{12}O_6} + \underset{\text{(Fructose)}}{C_6H_{12}O_6}$
(c) $CH_3COCl + H_2O \longrightarrow CH_3COOH + HCl$
(d) $CH_3COOC_2H_5 + H_2O \xrightarrow{OH^-} CH_3COOH + C_2H_5OH$

102. The compound, whose stereo-chemical formula is written below, exhibits x geometrical isomers and y optical isomers.



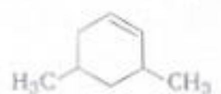
The values of x and y are :
(a) 4 and 4 (b) 2 and 2
(c) 2 and 4 (d) 4 and 2

103. When Na reacts with liquid NH_3 the following substance is formed :

(a) $[Na(NH_3)_x]^+$ (b) $[e(NH_3)_y]^-$

(c) $NaNH_2$ (d) Na_xNH_{3y}

104. IUPAC name of the following compound is :



(a) 3,5-dimethylcyclohexene

(b) 3,5-dimethyl-1-cyclohexene

(c) 1,5-dimethyl-5-cyclohexene

(d) 1,3-dimethyl-5-cyclohexene

105. The purine base present in RNA is :

(a) guanine (b) thymine

(c) cytosine (d) uracil

106. The molar volume of CO_2 is maximum at :

(a) NTP

(b) 0°C and 2.0 atm

(c) 127°C and 1 atm

(d) 273°C and 2 atm

107. SO_2 does not acts as :

(a) bleaching agent (b) oxidising agent

(c) reducing agent (d) dehydrating agent

108. The noble gas which is not found in atmosphere :

(a) Ne (b) Ar

(c) Rn (d) Kr

109. Which one of the following product is formed when calcium salt of adipic acid is heated ?

(a) $CH_2—CH_2$ / $CH_2—CH_2$ >O

(b) $CH_2—CH_2$ / $CH_2—CH_2$ >C=O

(c) CH_2CH_2CO / CH_2CH_2CO >C=O

(d) CH_2CH_2COOH / CH_2CH_2COOH

110. Which is not present in chlorophyll ?

(a) Carbon (b) Calcium

(c) Magnesium (d) Hydrogen

111. Glyptal polymer is obtained by the reaction of glycerol with :

(a) malonic acid (b) acetic acid

(c) phthalic acid (d) maleic acid

112. For the following two reactions

(i) $CH_4(g) + 2O_2(g) \longrightarrow CO_2(g) + 2H_2O$
$\Delta H = -890.4$ kJ

(ii) $2HgO(s) \longrightarrow 2Hg(l) + O_2(g) - 181.6$ k

Which one of the following statement correct ?

(a) Both of them are exothermic

(b) Both of them are endothermic

(c) (i) is exothermic and (ii) is endotherm

(d) (i) is endothermic and (ii) is exotherm

113. A compound contains X, Y and Z atoms. oxidation states of X, Y and Z are +2, +2 respectively. The possible formula of compound is :

(a) XYZ_2

(b) $Y_2(XZ_3)_2$

(c) $X_3(Y_4Z)_2$

(d) $X_3(YZ_4)_3$

114. Pinacol is :

(a) 3-methylbutan-2-ol

(b) 2,3-dimethyl-2,3-butanediol

(c) 2,3-dimethyl-2-propanone

(d) none of the above

115. If the H^+ concentration is decreased from 1 to 10^{-4} M at 25°C for the couple MnO_4^-/Mn^{2+} then the oxidising power of the MnO_4^-/Mn^{2+} couple decreases by :

(a) −0.18 V (b) 0.18 V

(c) 0.38 V (d) −0.38 V

116. For a first order reaction with rate constant and initial concentration 'a', the half-life period is given by :

(a) $\frac{\ln 2}{k}$

(b) $\frac{1}{ka}$

(c) $\frac{3}{2ka^2}$

(d) none of the above

117. Aldol condensation will not take place in :

(a) HCHO

(b) CH_3CH_2CHO

(c) CH_3CHO

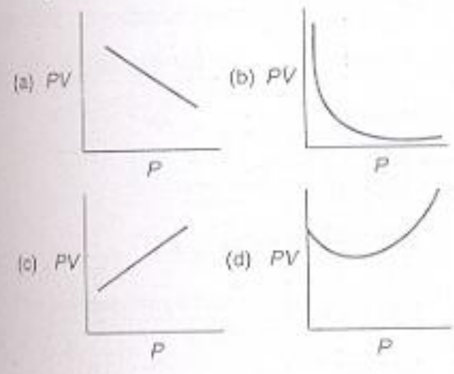
(d) CH_3COCH_3

118. Which of the following is called Berthelot salt ?

(a) $(NaPO_3)_6$ (b) NaOCl

(c) $KClO_3$ (d) KHF_2

119. Which of the following is a Boyle plot at very low pressure?



120. Calgon used as water softner is :
(a) $Na_2[Na_4(PO_3)_6]$ (b) $Na_4[Na_2(PO_3)_6]$
(c) $Na_3[Na_4(PO_4)_5]$ (d) none of these

121. How many asymmetric carbon atoms are present in :
(i) 1,2-dimethylcyclohexane,
(ii) 3-methylcyclopentene and
(iii) 3-methylcyclohexene ?
(a) two, one, one (b) one, one one
(c) two, none, two (d) two, none, one

122. For which order half-life period is independent of initial concentration ?
(a) Zero (b) First
(c) Second (d) Third

123. In the reaction

$$CH_3CN + 2H \xrightarrow[SnCl_2]{HCl} X \xrightarrow{\text{boiling } H_2O} Y,$$

The term Y is :
(a) acetone (b) ethanamine
(c) acetaldehyde (d) dimethyl amine

124. The speices that undergoes disproportionation in an alkaline medium is :
(a) MnO_4^{2-} (b) ClO_4^-
(c) NO_2 (d) all of these

125. On shaking H_2O_2 with acidified potassium dichromate and ether, etheral layer becomes :
(a) green (b) red
(c) blue (d) brown

English

Directions : Read the following passage carefully and answer the questions given below it.

PASSAGE

India is a country which has been subjected to foreign invasions since the dawn of Indian history. The fertile plains of India have been attracting avaricious tribals from all over the world. Long back the Aryans from Central Asia invaded India and settled down permanently in this beautiful land where food and fodder were available in plenty. After a chain of invasions from the bordering countries through land routes, the European nations, including the British, finally came to India to exploit her rich resources. The Englishmen came to India as traders but stealthily became her masters. India became the 'brightest Jewel' in the British diadem. They proclaimed to civilise her and started to exploit her. Neither the imperialist might, nor the treachery of some of her sons, nor the treachery of some of her sons, nor the nerve-racking exploitation could curb the indomitable urge for freedom of the people who bid defiance to time. They fought and fought heroically; they never submitted. Their struggle for independence is an inspiring and exhilarating story. It is a story not only of firm determination and will but also of sacrifice and suffering, a story of heroism and courage that happens in all revolutionaries.

126. Why did the Aryans settle permanently in India?
(a) Because here food and fodder were available in plenty
(b) Because they invaded India
(c) Because here food was available in plenty
(d) Because here fodder was available in plenty

127. British came to India:
(a) to exploit her rich resources
(b) to become her master
(c) to civilise her
(d) none of the above

128. The Indians indomitable urge for freedom could be curbed neither :

(a) by the nerve-racking exploitation of the British

(b) by the treachery of some of her sons

(c) by the imperialist might

(d) all of the above

Directions : *In the following questions, out of the four alternatives choose the one which is most opposite in meaning to the word given in capital letters.*

129. INGRATITUDE :

(a) Stimulation (b) Reward

(c) Sympathy (d) Thankfulness

130. GIGANTIC :

(a) Tiny (b) Narrow

(c) Vulgar (d) Attentive

131. COLOURLESS :

(a) Resolute (b) Steadfast

(c) Pleasant (d) Terrible

Directions : *Choose the word which is most nearly the same in meaning to the word given in capital letters.*

132. CONNOISSEUR :

(a) Lover of art (b) Interpreter

(c) Delinquent (d) Ignorant

133. LETHAL :

(a) Unlawful (b) Sluggish

(c) Deadly (d) Smooth

134. PICTURESQUE :

(a) Photogenic

(b) Ugly

(c) Simple

(d) Stimulating

Directions : *The following sentences have been divided into three parts (a), (b), (c). One of the parts may contain an error. Write down the part of the sentence that has an error. If there is No error., mark (d) as your answer.*

135. Whenever is the matter I shall (a) do this work because I have (b) to expose my working capacity at any co[st]. (c) No error (d)

136. She is so lazy as she cannot (a) do this work properly and (b) cannot cooperate us in your scheme. (c) No error (d)

Directions : *Each of the idioms or phra[ses is] followed by four meanings out of which only [one] is correct. Pick out the correct meaning.*

137. A hard nut to crack :

(a) Difficult things require extra effort

(b) A difficult problem to solve

(c) A difficult problem solved effortlessly

(d) Costly things need careful handling

138. To beat about the bush :

(a) Not to come to the point

(b) Vigorous search for the culprit

(c) Easily achieved success without m[uch] effort

(d) Working hard to achieve the goal

Directions : *Choose the suitable prepos[ition] from the given alternatives to fill in the bla[nks in] the following sentences.*

139. So many servants attended ...... him during illness.

(a) with (b) on

(c) for (d) to

140. At last he yielded ...... the temptation.

(a) on (b) off

(c) for (d) to

Reasoning

141. Kilogram is related to Quintal in the same way as Paisa is related to :

(a) Rupee

(b) Coin

(c) Wealth

(d) Money

142. In the following question four groups of let[ters] are given. Three of them are alike in a cer[tain] way while one is different. Select the one w[hich] is different.

(a) xXYA (b) iIMP

(c) hHIR (d) DBCE

Directions : *Find the missing character from among the given alternatives.*

148.

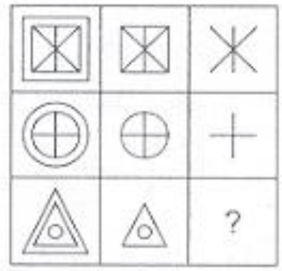
(a) 625 (b) 25
(c) 125 (d) 156

149. Find the wrong term in the letter-number series given below:

G4T, J10R, M20P, P43N, S90L

(a) G4T
(b) J10R
(c) M20P
(d) P43N

150. Select one alternative figure out of (a), (b) and (d), which completes the given matrix.



(a) 

(b) 

(c) 

(d) 

ANSWERS

MATHEMATICS

1.	(c)	2.	(c)	3.	(c)	4.	(b)	5.	(d)	6.	(a)	7.	(a)	8.	[illegible]
9.	(b)	10.	(a)	11.	(a)	12.	(b)	13.	(a)	14.	(c)	15.	(c)	16.	[illegible]
17.	(b)	18.	(d)	19.	(b)	20.	(b)	21.	(a)	22.	(b)	23.	(c)	24.	[illegible]
25.	(d)	26.	(c)	27.	(b)	28.	(d)	29.	(a)	30.	(b)	31.	(a)	32.	[illegible]
33.	(a)	34.	(c)	35.	(d)	36.	(c)	37.	(d)	38.	(b)	39.	(d)	40.	[illegible]
41.	(a)	42.	(c)	43.	(c)	44.	(c)	45.	(d)						

PHYSICS

46.	(d)	47.	(b)	48.	(b)	49.	(a)	50.	(a)	51.	(a)	52.	(a)	53.	[illegible]
54.	(b)	55.	(a)	56.	(a)	57.	(d)	58.	(a)	59.	(c)	60.	(c)	61.	[illegible]
62.	(d)	63.	(a)	64.	(c)	65.	(b)	66.	(d)	67.	(a)	68.	(a)	69.	[illegible]
70.	(a)	71.	(a)	72.	(b)	73.	(b)	74.	(b)	75.	(c)	76.	(a)	77.	[illegible]
78.	(b)	79.	(d)	80.	(b)	81.	(a)	82.	(a)	83.	(a)	84.	(a)	85.	[illegible]

CHEMISTRY

86.	(a)	87.	(a)	88.	(d)	89.	(b)	90.	(a)	91.	(d)	92.	(b)	93.	[illegible]
94.	(a)	95.	(c)	96.	(b)	97.	(c)	98.	(a)	99.	(b)	100.	(b)	101.	[illegible]
102.	(b)	103.	(b)	104.	(a)	105.	(d)	106.	(c)	107.	(d)	108.	(b)	109.	[illegible]
110.	(b)	111.	(c)	112.	(d)	113.	(a)	114.	(b)	115.	(c)	116.	(a)	117.	[illegible]
118.	(c)	119.	(d)	120.	(a)	121.	(a)	122.	(b)	123.	(c)	124.	(c)	125.	[illegible]

- ENGLISH

126.	(a)	127.	(a)	128.	(d)	129.	(d)	130.	(a)	131.	(c)	132.	(a)	133.	(c)
134.	(a)	135.	(a)	136.	(a)	137.	(b)	138.	(a)	139.	(d)	140.	(d)		

- REASONING

141.	(a)	142.	(d)	143.	(d)	144.	(b)	145.	(c)	146.	(a)	147.	(a)	148.	(a)
149.	(b)	150.	(d)												

HINTS & SOLUTIONS

Mathematics

1. Since, $2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16, 2^5 = 32$. It is clear that unit place is repeated after every four power.

Then, $2^{301} = (2^4)^{75} \cdot 2$
$= (16)^{75} \cdot 2$

$\therefore$ Digit at unit place in $(16)^{75}$ is 6.

$\therefore$ Digit at units place in 2^{301}
$=$ digit at units place in $(6) \cdot 2$
$= 2$

Hence, the remainder, when 2^{301} is divided by 5, is 2.

2. Let $A = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$

$$= \begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ 1+\omega+\omega^2 & \omega^2 & 1 \\ 1+\omega+\omega^2 & 1 & \omega \end{vmatrix} \quad C_1 \to C_1 + C_2 + C_3$$

$$= \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & \omega^2 & 1 \\ 0 & 1 & \omega \end{vmatrix} \quad [\because 1 + \omega + \omega^2 = 0]$$

$= 0$

3. Given that

$x^2 + 10x - 16y + 25 = 0$
$\Rightarrow (x+5)^2 = 16y$
$\Rightarrow X^2 = 4AY$

where $X = x + 5, A = 4, Y = y$.

The ends of the latus rectum are
$(2A, A)$ and $(-2A, A)$
$\Rightarrow x + 5 = 2(4)$
$\Rightarrow x = -8 - 5 = 3, y = 4$
and $x + 5 = -2(4)$
$\Rightarrow x = -8 - 5 = -13, y = 4$
$\Rightarrow (3, 4)$ and $(-13, 4)$.

4. Let the equation of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Given, $e = 2, 2ae = 8$
$\Rightarrow ae = 4 \Rightarrow a = 2$
Now, $b^2 = a^2(e^2 - 1)$
$\Rightarrow b^2 = 4(4-1)$
$\Rightarrow b^2 = 12$

$\therefore$ Equation of hyperbola is

$$\frac{x^2}{4} - \frac{y^2}{12} = 1$$

5. $\sin^{-1} x - \sin^{-1} 2x = \pm \frac{\pi}{3}$

$$\Rightarrow \sin^{-1} x - \sin^{-1}\left(\pm \frac{\sqrt{3}}{2}\right) = \sin^{-1} 2x$$

$$\Rightarrow \sin^{-1}\left[x\sqrt{1-\frac{3}{4}} - \left(\pm\frac{\sqrt{3}}{2}\sqrt{1-x^2}\right)\right] = \sin^{-1} 2x$$

$$\Rightarrow \frac{x}{2} - \left(\pm\frac{\sqrt{3}}{2}\sqrt{1-x^2}\right) = 2x$$

$$\Rightarrow -(\pm\sqrt{3}\sqrt{1-x^2}) = 3x$$

On squaring, both sides we get,

$$3(1-x^2)=9x^2$$

$$\Rightarrow \quad 1-x^2=3x^2$$

$$\Rightarrow \quad 4x^2=1$$

$$\Rightarrow \quad x=\pm\frac{1}{2}$$

6. We know,

$$\cos B=\frac{a^2+c^2-b^2}{2ac}$$

$$\therefore \quad \cos B=\frac{3^2+4^2-5^2}{2(3)(4)}=\frac{9+16-25}{2(3)(4)}=0$$

$$\Rightarrow \quad B=90^\circ$$

$$\therefore \quad \sin\frac{B}{2}+\cos\frac{B}{2}=\sin 45^\circ+\cos 45^\circ$$

$$=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\sqrt{2}$$

7. By using the condition that, if two circles are intersect orthogonally, then

$$2(g_1g_2+f_1f_2)=c_1+c_2$$

where $g_1=-1,\ f_1=11,\ c_1=5$

and $g_2=7,\ f_2=3,\ c_2=k$

$$\Rightarrow \quad 2(-1\cdot 7+11\cdot 3)=5+k$$

$$\Rightarrow \quad 2(26)=5+k$$

$$\Rightarrow \quad k=47$$

8. Given equation is

$$x^2+y^2+4x+6y+13=0$$

or $(x^2+4x+4)+(y^2+6y+9)+13=4+9$

or $\quad (x+2)^2\ (y+3)^2=0$

$\therefore$ Radius of circle $=0$.

9. Given parametric equations are

$$x=2+3\cos\theta,\ y=3\sin\theta-1$$

or $\quad \cos\theta=\dfrac{x-2}{3},\ \sin\theta=\dfrac{y+1}{3}$

Since, $\quad \sin^2\theta+\cos^2\theta=1$

$$\Rightarrow \quad \left(\frac{x-2}{3}\right)^2+\left(\frac{y+1}{3}\right)^2=1$$

$$\Rightarrow \quad (x-2)^2+(y+1)^2=3^2$$

$\therefore$ Centre of circle is $(2,-1)$.

10. We know, if P is any point on the curve, then Sum of focal distances = length of major axis

i.e., $SP+S'P=2a$

$$=2(5) \qquad [\because a^2=5^2]$$

$$=10$$

11. Since, $\quad \begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -1 & 2 & x \end{vmatrix}=0$

$$\Rightarrow \quad \begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -3 & -3 & 0 \end{vmatrix}=0 \qquad R_3\to R_3-$$

$$\Rightarrow \quad -1(-6+15)-x[-3x+6]=0$$

$$\Rightarrow \quad -9+3x^2-6x=0$$

$$\Rightarrow \quad x^2-2x-3=0$$

$$\Rightarrow \quad (x-3)(x+1)=0$$

$$\Rightarrow \quad x=-1,3$$

12. $A=\begin{bmatrix} 3 & 5 \\ 2 & 0 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix}$

$$\therefore \quad AB=\begin{bmatrix} 3 & 5 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix}$$

$$=\begin{bmatrix} 3+0 & 51-50 \\ 2+0 & 34-0 \end{bmatrix}=\begin{bmatrix} 3 & 1 \\ 2 & 34 \end{bmatrix}$$

$$\Rightarrow \quad |AB|=\begin{vmatrix} 1 & 1 \\ 2 & 34 \end{vmatrix}$$

$$=102-2$$

$$=100$$

13. Let $A=\begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}$

$$|A|=5+6=11$$

and $\quad \text{adj } A=\begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$

$$A^{-1}=\frac{1}{|A|}(\text{adj } A)$$

$$=\frac{1}{11}\begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$$

14. Let $\vec{\mathbf{a}}=2\hat{\mathbf{i}}+\hat{\mathbf{j}}-3\hat{\mathbf{k}}$ and $\vec{\mathbf{b}}=\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}}$

$$\text{Projection of } \vec{\mathbf{a}} \text{ on } \vec{\mathbf{b}}=\frac{\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}}{|\vec{\mathbf{b}}|}$$

$$=\frac{(2\hat{\mathbf{i}}+\hat{\mathbf{j}}-3\hat{\mathbf{k}})\cdot(\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}})}{\sqrt{1^2+(-2)^2+1^2}}$$

$$=\frac{2-2-3}{\sqrt{6}}=-\frac{3}{\sqrt{6}}$$

$$=-\sqrt{\frac{3}{2}}$$

15. $\quad 12\cot^2\theta-31\operatorname{cosec}\theta+32=0$

$$\Rightarrow \quad 12\cos^2\theta-31\sin\theta+32\sin^2\theta=0$$

$$\Rightarrow \quad 12(1-\sin^2\theta)-31\sin\theta+32\sin^2\theta=0$$

$$\Rightarrow \quad 20\sin^2\theta-31\sin\theta+12=0$$

This is a quadratic equation in $\sin\theta$.

$$\therefore \sin\theta = \frac{31 \pm \sqrt{31^2 - 4\cdot 20\cdot 12}}{2\cdot 20}$$

$$= \frac{31 \pm \sqrt{961 - 960}}{40} = \frac{31 \pm 1}{40}$$

$$\Rightarrow \quad \sin\theta = \frac{4}{5}, \frac{3}{4}$$

16. Let sides are $a = 13$, $b = 12$, $c = 5$

Now, $a^2 = b^2 + c^2$

$\Rightarrow \quad (13)^2 = (12)^2 + 5^2$

$\Rightarrow \quad 169 = 169$

$\Rightarrow \quad \angle A = 90^\circ$

We know, $R = \dfrac{a}{2\sin A}$

$$R = \frac{13}{2\cdot \sin 90^\circ} = \frac{13}{2}$$

17. Given that, $\sin x - \cos x = \sqrt{2}$

$\Rightarrow \quad \dfrac{1}{\sqrt{2}}\sin x - \dfrac{1}{\sqrt{2}}\cos x = 1$

$\Rightarrow \quad \sin 45^\circ \sin x - \cos 45^\circ \cos x = 1$

$\Rightarrow \quad \cos\left(x + \dfrac{\pi}{4}\right) = -1$

$\Rightarrow \quad \cos\left(x + \dfrac{\pi}{4}\right) = \cos(\pi)$

$\Rightarrow \quad x + \dfrac{\pi}{4} = 2n\pi + \pi$

$\Rightarrow \quad x = 2n\pi + \dfrac{3\pi}{4}$

18. Let $z = \dfrac{1 + i\sqrt{3}}{\sqrt{3} + i}$

$$= \frac{1 + i\sqrt{3}}{(\sqrt{3} + i)} \times \frac{(\sqrt{3} - i)}{(\sqrt{3} - i)}$$

$$= \frac{\sqrt{3} - i + 3i + \sqrt{3}}{3 + 1} = \frac{\sqrt{3} + i}{2}$$

$$\text{amp}(z) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{\pi}{6}$$

19. Given that $^nC_{12} = {}^nC_6$

or $^nC_{n-12} = {}^nC_6$

$\Rightarrow \quad n - 12 = 6$

$\Rightarrow \quad n = 18$

$\therefore \quad {}^nC_2 = {}^{18}C_2 = \dfrac{18 \times 17}{2 \times 1}$

$= 153$

20. The general term in the expansion $\left(x - \dfrac{1}{x}\right)^{18}$ is given by

$$T_{r+1} = {}^{18}C_r (x)^{18-r}\left(-\frac{1}{x}\right)^r$$

Here, $n = 18$

$\therefore$ The middle term is T_{9+1}, where $r = 9$

$$\therefore \quad T_{9+1} = {}^{18}C_9 (-1)^9 x^{18-2r}$$

$$= -{}^{18}C_9\, x^{18-18} = -{}^{18}C_9$$

21. Since, α, β, γ are the roots of the equation

$$2x^3 - 3x^2 + 6x + 1 = 0,$$

then $\alpha + \beta + \gamma = \dfrac{3}{2}$ ...(i)

$\alpha\beta + \beta\gamma + \gamma\alpha = 3$ ...(ii)

$\alpha\beta\gamma = -\dfrac{1}{2}$ ...(iii)

On squaring Eq. (i), we get

$$\alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha) = \frac{9}{4}$$

$$\alpha^2 + \beta^2 + \gamma^2 = \frac{9}{4} - 2(3) \quad \text{[from (ii)]}$$

$$= \frac{9}{4} - 6 = -\frac{15}{4}$$

22. Since, $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular to each other, then

$$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c})\cdot(\vec{a} + \vec{b} + \vec{c})$$

$$[\because |\vec{a}|^2 = \vec{a}\cdot\vec{a}]$$

$$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$$

$$= 1 + 1 + 1 + 2(0 + 0 + 0)$$

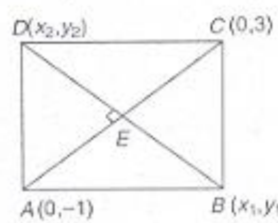
$$[\because \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0]$$

$\Rightarrow \quad |\vec{a} + \vec{b} + \vec{c}|^2 = 3$

$\therefore \quad |\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}$

23. Let the points be $B(x_1, y_1)$ and $D(x_2, y_2)$ mid point of

$$BD = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



and mid point of $AC = (0, 1)$

We know, mid point of both the diagonal lie on the same point E.

$$\Rightarrow \quad \frac{x_1 + x_2}{2} = 0 \text{ and } \frac{y_1 + y_2}{2} = 1$$

$$\Rightarrow \quad x_1 + x_2 = 0 \quad \ldots(i)$$

$$\text{and} \quad y_1 + y_2 = 2 \quad \ldots(ii)$$

Slope of BD × slope of $AC = -1$

$$\frac{(y_1 - y_2)}{(x_1 - x_2)} \times \frac{(3+1)}{(0-0)} = -1$$

$$\Rightarrow \quad y_1 - y_2 = 0 \quad \ldots(iii)$$

Solving Eqs. (ii) and (iii), we get

$$y_1 = 1, y_2 = 1$$

Now, slope of AB × slope of $BC = -1$

$$\Rightarrow \quad \frac{(y_1 + 1)}{(x_1 - 0)} \times \frac{(y_1 - 3)}{(x_1 - 0)} = -1$$

$$\Rightarrow \quad (y_1 + 1)(y_1 - 3) = -x_1^2$$

$$\Rightarrow \quad 2(-2) = -x_1^2 \quad [\because y_1 = 1]$$

$$\Rightarrow \quad x_1 = \pm 2$$

∴ The required points are (2, 1) and (− 2, 1).

24. Let the points be $A(3, -4)$ and $B(5, 2)$ and mid point of $AB = (4, -1)$.

It is given that the bisecting line intercept the co-ordinate axes in the ratio 2 : 1.

∴ Point of co-ordinate axes are $(2k, 0)$ and $(0, k)$. The equation of line passing through the above point is

$$y - 0 = \frac{k - 0}{0 - 2k}(x - 2k)$$

$$\text{or} \quad y = -\frac{1}{2}(x - 2k) \quad \ldots(i)$$

Since, it is passing through the mid point of AB *i.e.*, (4, − 1)

$$\Rightarrow \quad -1 = -\frac{1}{2}(4 - 2k)$$

$$\Rightarrow \quad 2 = 4 - 2k$$

$$\Rightarrow \quad k = 1$$

Putting the value of k in Eq. (i), we get

$$y = -\frac{1}{2}(x - 2)$$

$$\Rightarrow \quad x + 2y = 2$$

25. Given equation is

$$x^2 + y^2 + 2xy - 8ax - 8ay - 9a^2 = 0$$

$$\text{or} \quad x^2 + y^2 + (-4a)^2 + 2xy - 8ax - 8ay - 25a^2 = 0$$

$$\text{or} \quad (x + y - 4a)^2 - (5a)^2 = 0$$

$$\text{or} \quad (x + y - 9a)(x + y + a) = 0$$

$$\Rightarrow \quad x + y - 9a = 0$$

$$\text{or} \quad x + y + a = 0$$

These lines are parallel. Now, we find the distance from origin to the line.

Let, $p_1 = \dfrac{0 + 0 - 9a}{\sqrt{1^2 + 1^2}}, \; p_2 = \dfrac{0 + 0 + a}{\sqrt{1^2 + 1^2}}$

$$p_1 = -\frac{9a}{\sqrt{2}}, \; p_2 = \frac{a}{\sqrt{2}}$$

The distance between two lines is

$$|p_2 - p_1| = \left|\frac{a}{\sqrt{2}} + \frac{9a}{\sqrt{2}}\right| = \frac{10a}{\sqrt{2}}$$

$$= 5\sqrt{2}a$$

26. Distance from centre (2, 1) to the line $3x + 4y - 5$ = radius of circle

$$\Rightarrow \quad \frac{|3(2) + 4(1) - 5|}{\sqrt{3^2 + 4^2}} = r$$

$$\Rightarrow \quad \frac{5}{5} = r$$

$$\Rightarrow \quad r = 1$$

∴ Equation of circle is

$$(x - 2)^2 + (y - 1)^2 = 1^2$$

$$\Rightarrow \quad x^2 + y^2 - 4x - 2y + 4 + 1 = 1$$

$$\Rightarrow \quad x^2 + y^2 - 4x - 2y + 4 = 0$$

27. If $y = mx + c$ touches the circle $x^2 + y^2 = a^2$, then $c^2 = a^2(1 + m^2)$

Now, the line $y = 2x + c$ touches the circle $x^2 + y^2 = 16$, if

$$\therefore \quad c^2 = 16(1 + 4) = 16 \times 5$$

$$\text{or} \quad c^2 = 80$$

28. Let $I = \displaystyle\int \frac{2\sin x \cos x}{1 + \cos^2 x}\,dx$

$$\text{Put} \quad 1 + \cos^2 x = t$$

$$\Rightarrow \quad -2\cos x \sin x\,dx = dt$$

$$\therefore \quad I = \int -\frac{dt}{t} = -\log t + c$$

$$= -\log(1 + \cos^2 x) + c$$

29. Let $I = \displaystyle\int e^x \left(\frac{1 + \sin x}{1 + \cos x}\right) dx$

$$= \int e^x \frac{\left(1 + 2\sin\frac{x}{2}\cos\frac{x}{2}\right)}{2\cos^2\frac{x}{2}}\,dx$$

$$= \int \frac{1}{2} e^x \sec^2\frac{x}{2}\,dx + \int e^x \tan\frac{x}{2}\,dx$$

$$= \frac{1}{2}\left[2e^x \tan\frac{x}{2} - \int 2e^x \tan\frac{x}{2}\,dx + \int e^x \tan\frac{x}{2}\,dx\right.$$

$= e^x \tan\frac{x}{2} - \int e^x \tan\frac{x}{2}\,dx + \int e^x \tan\frac{x}{2}\,dx + c$

$= e^x \tan\frac{x}{2} + c$

30. $\int_{\pi/4}^{\pi/2} \operatorname{cosec}^2 x\,dx = [-\cot x]_{\pi/4}^{\pi/2}$

$$= \left(-\cot\frac{\pi}{2} + \cot\frac{\pi}{4}\right)$$

$$= -[0 - (1)] = 1$$

31. Let $I = \int_0^{\pi/4} \log(1 + \tan x)\,dx$ ...(i)

$\Rightarrow\ I = \int_0^{\pi/4} \log\left[1 + \tan\left(\frac{\pi}{4} - x\right)\right] dx$

$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$

$= \int_0^{\pi/4} \log\left[1 + \frac{1 - \tan x}{1 + \tan x}\right] dx$

$= \int_0^{\pi/4} \log\left[\frac{2}{1 + \tan x}\right] dx$

$= \int_0^{\pi/4} \log 2\,dx - \int_0^{\pi/4} \log(1 + \tan x)\,dx$

$\Rightarrow\ I = \log 2 [x]_0^{\pi/4} - I$ [from Eq. (i)]

$\Rightarrow\ 2I = \frac{\pi}{4}\log_e 2$

$\Rightarrow\ I = \frac{\pi}{8}\log_e 2$

32. Let $z = \frac{1 + 2i}{1 - (1 - i)^2}$

$= \frac{1 + 2i}{1 - (1^2 + i^2 - 2i)} = \frac{1 + 2i}{1 + 2i}$

$= 1$

$\therefore\ |z| = 1$ and $\text{amp}(z) = \tan^{-1}\left(\frac{0}{1}\right) = 0$

33. $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3} \left(\frac{0}{0} \text{ form}\right)$

(using L'Hospital's rule)

$= \lim_{x \to 0} \frac{\sec^2 x - \cos x}{3x^2} \quad \left(\frac{0}{0} \text{ form}\right)$

(using L'Hospital's rule)

$= \lim_{x \to 0} \frac{2\sec^2 x \tan x + \sin x}{6x} \quad \left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x \to 0} \frac{\left[\begin{array}{l} 2(\sec^2 x \sec^2 x + 2\sec x \\ \quad \times \sec x \tan x \tan x) + \cos x \end{array}\right]}{6}$

(using L'Hospital's rule)

$= \frac{2[1 \cdot 1 + 2(0)] + 1}{6}$

$= \frac{3}{6} = \frac{1}{2}$

34. $f(x) = \begin{cases} \frac{\sin 5x}{x^2 + 2x}, & x \neq 0 \\ k + \frac{1}{2}, & x = 0 \end{cases}$

LHL $f(0^-) = \lim_{h \to 0} f(0 - h)$

$= \lim_{h \to 0} \frac{\sin 5(0 - h)}{(0 - h)^2 + 2(0 - h)}$

$= \lim_{h \to 0} \frac{\sin(-5h)}{h^2 - 2h}$

$= -\lim_{h \to 0} \frac{\frac{\sin 5h}{5h}}{\frac{1}{5}(h - 2)} = -\frac{1}{\frac{1}{5}(-2)}$

$= \frac{5}{2}$

Since, it is continuous at $x = 0$

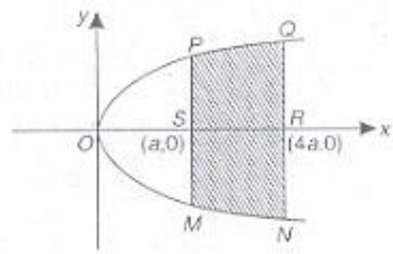
$\therefore$ LHL $= f(0)$

$\Rightarrow\ \frac{5}{2} = k + \frac{1}{2}$

$\Rightarrow\ k = 2$

35. Required area

= area of curve PSMNRQP



= 2 area of curve PSRQP

$= 2\int_a^{4a} \sqrt{4ax}\,dx$

$= 4\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_a^{4a} = \frac{8}{3}\sqrt{a}\,[(4a)^{3/2} - a^{3/2}]$

$= \frac{8}{3}\sqrt{a}\,(8a^{3/2} - a^{3/2}) = \frac{56}{3}a^2$

36. $p(t) = 1000 + \frac{1000t}{100 + t^2}$ ...(i)

On differentiating both side w.r.t. t,

$p'(t) = 0 + \frac{(100 + t^2)(1000) - 1000t\,(2t)}{(100 + t^2)^2}$

$= 1000\,\frac{(100 - t^2)}{(100 + t^2)^2}$ ...(ii)

Put $p'(t) = 0$ for maxima or minima

$$\Rightarrow \quad 100 - t^2 = 0$$

$$\Rightarrow \quad t = \pm 10$$

Now, again differentiating Eq. (ii), w.r.t. x

$$p''(t) = 1000 \times \left[\frac{(100+t^2)^2(-2t) - (100-t^2)2(100+t^2)2t}{(100+t^2)^4}\right]$$

$$= 1000t \frac{[(100+t^2)(-2) - (100-t^2)(4)]}{(100+t^2)^3}$$

$$= -1000t \frac{[600 - 2t^2]}{(100+t^2)^3}$$

At $\quad t = 10$

$$p''(t) < 0$$

$\therefore$ The maximum value is

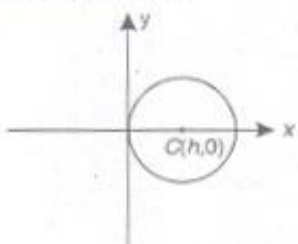
$$p(10) = 1000 + \frac{10000}{100+100}$$

$$= 1000 + \frac{10000}{200} = 1000 + 50$$

$$= 1050$$

37. Since, the circle touches the y-axis, therefore the centre lies on the x-axis. Let the centre be $(h, 0)$.

$\Rightarrow$ Radius of circle $= h$.



$\therefore$ The equation of circle is given by

$$(x-h)^2 + (y-0)^2 = h^2$$

$$\Rightarrow \quad x^2 + y^2 - 2hx = 0 \qquad \ldots(i)$$

On differentiating both sides w.r.t. x, we get

$$2x + 2y\frac{dy}{dx} - 2h = 0$$

$$\Rightarrow \quad h = x + y\frac{dy}{dx}$$

Putting the value of h in Eq. (i)

$$x^2 + y^2 - 2x\left(x + y\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \quad -x^2 + y^2 - 2xy\frac{dy}{dx} = 0$$

$$\Rightarrow \quad x^2 - y^2 + 2xy\frac{dy}{dx} = 0$$

This is the required differential equation.

38. Given differential equation is

$$(2x - y + 1)\,dx + (2y - x + 1)\,dy = 0$$

$$\Rightarrow 2x\,dx + 2y\,dy - (y\,dx + x\,dy) + dx + dy = 0$$

$$\Rightarrow (2x\,dx + 2y\,dy) - d(xy) + dx + dy = 0$$

On integrating both sides, we get

$$x^2 + y^2 - xy + x + y = c$$

39. $y = \tan^{-1}\dfrac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}$

Put $x^2 = \cos 2\theta$

$$\therefore \quad y = \tan^{-1}\frac{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}$$

$$= \tan^{-1}\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

$$= \tan^{-1}\tan\left(\frac{\pi}{4} - \theta\right)$$

$$\Rightarrow \quad y = \frac{\pi}{4} - \theta = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x^2$$

On differentiating both sides, we get

$$\frac{dy}{dx} = 0 - \frac{1}{2}\left(-\frac{(2x)}{\sqrt{1-x^4}}\right)$$

$$= \frac{x}{\sqrt{1-x^4}}$$

40. Given that

$$x = \sin t,\ y = \cos pt$$

$$\frac{dx}{dt} = \cot t,\ \frac{dy}{dt} = -p\sin pt$$

$$\frac{dy}{dx} = -\frac{p\sin pt}{\cos t}$$

$$\Rightarrow \quad y_1 = \frac{-p\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad y_1\sqrt{1-x^2} = -p\sqrt{1-y^2}$$

On squaring both sides, we get

$$y_1^2(1-x^2) = p^2(1-y^2)$$

Again differentiating

$$2y_1y_2(1-x^2) - 2xy_1^2 = -2yy_1p^2$$

or $\quad (1-x^2)y_2 - xy_1 + p^2y = 0$

41. Given that

$x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$

$\Rightarrow \dfrac{dx}{d\theta} = a(1+\cos\theta)$ and $\dfrac{dy}{d\theta} = a\sin\theta$

$$\therefore \quad \frac{dy}{dx} = \frac{a\sin\theta}{a(1+\cos\theta)}$$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$$

$$= \tan\frac{\theta}{2}$$

Now, length of subtangent $= \left|\frac{y}{dy/dx}\right|$

$$\therefore \quad ST = \frac{a(1-\cos\theta)}{\tan\frac{\theta}{2}}$$

$$= a\frac{2\sin^2\frac{\theta}{2}}{\sin\frac{\theta}{2}}\cos\frac{\theta}{2} = a\sin\theta$$

$\Rightarrow$ Length of subtangent at $\theta = \frac{\pi}{2}$,

$$ST = a\sin\frac{\pi}{2} = a$$

And length of subnormal $= \left|y\frac{dy}{dx}\right|$

$$\Rightarrow \quad SN = a(1-\cos\theta)\tan\frac{\theta}{2}$$

$$= a2\sin^2\frac{\theta}{2}\tan\frac{\theta}{2}$$

$\Rightarrow$ Length of subnormal at

$$\theta = \frac{\pi}{2}, SN = a\cdot 2\cdot\frac{1}{2} = a$$

Hence, $SN = ST$

42. Given equations are

$$x^2 + y^2 = 5 \quad \ldots\text{(i)}$$

and $$y^2 = 4x \quad \ldots\text{(ii)}$$

On solving Eqs. (i) and (ii), we get

$$x = -5, 1$$

at $x = -5$, $y^2 = -20$ (imaginary value)

$\therefore$ at $x = 1$, $y^2 = 4$

$$\Rightarrow \quad y = \pm 2$$

Hence, point of intersection are (1, 2) and (1, −2).

On differentiating Eq. (i) w.r.t. x, we get

$$2x + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{x}{y}$$

$$\therefore \quad m_1 = \left(\frac{dy}{dx}\right)_{(1,2)} = -\frac{1}{2}$$

And on differentiating Eq. (ii) w.r.t. x, we get

$$2y\frac{dy}{dx} = 4$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2}{y}, \; m_2 = \left(\frac{dy}{dx}\right)_{(1,2)} = \frac{2}{2} = 1$$

Now, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$= \left|\frac{-\frac{1}{2} - 1}{1 - \frac{1}{2}}\right| = \left|\frac{-\frac{3}{2}}{\frac{1}{2}}\right| = 3$$

43. $U = \{x : x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$

$= \{0, 1, 2, 3\}$

$A = \{x : x^2 - 5x + 6 = 0\}$

$= \{2, 3\}$

and $B = \{x : x^2 - 3x + 2 = 0\}$

$= \{2, 1\}$

$A \cap B = \{2\}$

$\therefore (A \cap B)' = U - (A \cap B)$

$= \{0, 1, 2, 3\} - \{2\} = \{0, 1, 3\}$

44. R is not anti-symmetric.

45. $\because (1001)_2 = 1 \times 2^3 + 2^0 = 8 + 1 = 9$

$(11)_2 = 2^1 + 2^0 = 2 + 1 = 3$

$(101)_2 = 2^2 + 2^0 = 4 + 1 = 5$

$(10)_2 = 2^1 = 2$

and $(01)_2 = 1$

$$\therefore \frac{(1001)_2^{(11)_2} - (101)_2^{(11)_2}}{(1001)_2^{(10)_2} + (1001)_2^{(01)_2}(101)_2^{(01)_2} + (101)_2^{(10)_2}}$$

$$= \frac{9^3 - 5^3}{9^2 + 9\times 5 + 5^2}$$

$$= \frac{(9-5)(9^2 + 9\times 5 + 5^2)}{(9^2 + 9\times 5 + 5^2)}$$

$$= 9 - 5 = 4 = (100)_2$$

Physics

46. Angle turned by the body

$$\theta = \theta_0 + \theta_1 t + \theta_2 t^2$$

Angular velocity

$$\omega = \frac{d\theta}{dt}$$

$$= \frac{d}{dt}(\theta_0 + \theta_1 t + \theta_2 t^2)$$

$$= \theta_1 + 2\theta_2 t$$

Angular acceleration

$$\alpha = \frac{d\omega}{dt}$$
$$= \frac{d}{dt}(\theta_1 + 2\theta_2 t)$$
$$= 2\theta_2$$

48. Acceleration of the body down the rough inclined plane = $g \sin \theta$

$\therefore$ Force applied on spring balance

$$= m g \sin \theta$$
$$= 5 \times 10 \times \sin 30^\circ$$
$$= 5 \times 10 \times \frac{1}{2} = 25 \text{ N}$$

49. Work done $W = \int_0^x F \cdot dx$

$$= \int_0^x Cx \, dx = C\left(\frac{x^2}{2}\right)_0^x$$
$$= \frac{1}{2} Cx^2$$

50. $\vec{A} \cdot \vec{B} = \vec{A} \times \vec{B}$

$$AB \cos \theta = A B \sin \theta$$
$$\tan \theta = 1$$
$$\tan \theta = \tan 45^\circ$$
$$\theta = 45^\circ$$

51. Given circuit is a balanced Wheatstone bridge. So, diagonal resistance of 2Ω will be ineffective.

Equivalent resistance of upper arms

$$= 2 + 2 = 4\,\Omega$$

Equivalent resistance of lower arms

$$= 2 + 2 = 4\,\Omega$$

$$\therefore \quad R_{AB} = \frac{4 \times 4}{4 + 4}$$
$$= 2\Omega$$

52. $X = [M^a L^b T^c]$

Maximum % error in X

$$= a\alpha + b\beta + c\gamma$$

53. Induced emf $e = 2$ V

$$i_1 = 8 \text{ A}, i_2 = 6 \text{ A}$$
$$\Delta t = 2 \times 10^{-3} \text{ s}$$

Coefficient of self induction

$$L = \frac{e}{\Delta i / \Delta t} = \frac{-2}{(6-8)/2 \times 10^{-3}}$$
$$= \frac{-2 \times 2 \times 10^{-3}}{-2}$$
$$= 2 \times 10^{-3} \text{ H}$$

54. Potential at the centre of a hollow metal sphere

$$V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R}$$

56. Refractive index of prism

$$\mu = \frac{\sin \frac{A + \delta_m}{2}}{\sin \frac{A}{2}}$$
$$= \frac{\sin \frac{60^\circ + 30^\circ}{2}}{\sin \frac{60^\circ}{2}}$$
$$= \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1/\sqrt{2}}{1/2}$$
$$= \frac{1}{\sqrt{2}} \times \frac{2}{1} = \sqrt{2}$$

58. Current given by cell

$$I = \frac{E}{R + r}$$

Power delivered in first case

$$P_1 = I^2 R_1$$
$$= \left(\frac{E}{R_1 + r}\right)^2 R_1$$

Power delivered in second case

$$P_2 = I^2 R_2$$
$$= \left(\frac{E}{R_2 + r}\right)^2 R_2$$

Power delivered is same in the both the case

$$\left(\frac{E}{R_1 + r}\right)^2 R_1 = \left(\frac{E}{R_2 + r}\right)^2 R_2$$
$$\frac{R_1}{(R_1 + r)^2} = \frac{R_2}{(R_2 + r)^2}$$
$$R_1 (R_2^2 + r^2 + 2R_2 r) = R_2 (R_1^2 + r^2 + 2R_1 r)$$
$$R_1 R_2^2 + R_1 r^2 + 2R_1 R_2 r = R_2 R_1^2 + R_2 r^2 + 2R_1 R_2 r$$
$$R_1 R_2^2 - R_2 R_1^2 = R_2 r^2 - R_1 r^2$$
$$R_1 R_2 (R_2 - R_1) = r^2 (R_2 - R_1)$$
$$r = \sqrt{R_1 R_2}$$

60. At the top most point of the projectile there is only horizontal component of velocity and acceleration due to gravity is vertically downward, so velocity and acceleration are perpendicular to each other.

61. $u = -20$ cm, $f = 20$ cm

From mirror formula,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{20} = \frac{1}{v} + \frac{1}{-20}$$

$$\frac{1}{v} = \frac{1}{20} + \frac{1}{20}$$

$$\frac{1}{v} = \frac{2}{20} \Rightarrow v = 10 \text{ cm}$$

62. According to law of conseration of angular momentum, if there is no torque on the system, then the angular momentum remains constant.

63. Equivalent focal length

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$= \frac{1}{20} + \frac{1}{30}$$

$$F = \frac{20 \times 30}{20 + 30}$$

$$= \frac{600}{50} = 12 \text{ cm}$$

64. For isothermal process

$$PV = K \qquad \text{(constant)}$$

$$P = \frac{K}{V} \qquad \ldots(1)$$

$$= \frac{K}{V/2} = 2K$$

For adiabatic process

$$PV^{\gamma} = K \qquad \text{(constant)}$$

$$P = \frac{K}{V^{\gamma}} \qquad \ldots(2)$$

$$= \frac{K}{(V/2)^{\gamma}} = K(2^{\gamma})$$

From Eqs. (1) and (2), we have

$$P_2 > P_1$$

65. According to Newton's law of cooling, $t_1 < t_2$.

66. The lift is moving with constant velocity so, there will be no change in the acceleration hence time period will remain same.

67. $$x_1 = 2 \sin \omega t$$

$$x_2 = 4 \sin (\omega t + 90^\circ)$$

The phase difference between the two waves is 90°.

So, resultant amplitude

$$a = \sqrt{(3)^2 + (4)^2}$$

$$= \sqrt{9 + 16} = \sqrt{25}$$

$$= 5 \text{ unit}$$

68. $B = 0.5$ T

$A_1 = 101 \text{ cm}^2 = 101 \times 10^{-4} \text{ m}^2$

$A_2 = 100 \text{ cm}^2 = 100 \times 10^{-4} \text{ m}^2$

$R = 2\Omega$

Amount of charge

$$q = \frac{B\Delta A}{R}$$

$$= \frac{0.50 \times (101 \times 10^{-4} - 100 \times 10^{-4})}{2}$$

$$= \frac{0.05 \times 1 \times 10^{-4}}{2}$$

$$= 2.5 \times 10^{-6} \text{ C}$$

70. Amplification factor $\mu = 20$

Plate resistance $R_p = 10 \, k\Omega$

$= 10 \times 10^3 \, \Omega$

$\therefore$ Mutual conductance

$$g_m = \frac{\mu}{R_p}$$

$$= \frac{20}{10 \times 10^3} = 2 \times 10^{-3} \text{ mho}$$

$$= 2 \text{ milli mho}$$

75. For wavelength

$$\frac{1}{\lambda} = R Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Here, transition is same

So, $$\lambda \propto \frac{1}{Z^2}$$

$$\frac{\lambda_H}{\lambda_{Li}} = \frac{(Z_{Li})^2}{(Z_H)^2} = \frac{(3)^2}{(1)^2} = 9$$

$$\lambda_{Li} = \frac{\lambda_H}{9} = \frac{\lambda}{9}$$

76. $m = 1000$ kg

$$\frac{\Delta m}{\Delta t} = 4 \text{ kg/s}, \; v = 3000 \text{ m/s}$$

Thrust on the rocket

$$F = -v \frac{\Delta m}{\Delta t}$$

$$= -3000 \times 4$$

$$= -12000 \text{ N}$$

(Negative sign indicates that thrust applied in a direction opposite to the direction of escaping gas)

79. The direction of magnetic field is along the direction of motion of the charge particles, so angle will be 0°.

$\therefore$ Force $F = qvB \sin \theta$

$= qvB \sin 0$

$= 0 \qquad (\because \sin 0 = 0)$

So, there will be no change in the velocity.

81. Mass defect

Δm = Total mass of α-particles − mass of ^{12}C nucleus

$= 3 \times 4.002603 - 12$

$= 12.007809 - 12$

$= 0.007809$ unit

82. $\frac{N_p}{N_s} = \frac{1}{10}$

$V_p = 230$ V, $I_s = 2$ A,

$$\frac{I_p}{I_s} = \frac{N_s}{N_p}$$

$$\frac{I_p}{2} = \frac{10}{1}$$

$$\Rightarrow \quad I_p = 20 \text{ A}$$

83. Equation of wave

$$y = 2 \sin (kx - 2t)$$

Comparing with standard equation

$$y = A \sin (kx - \omega t)$$

$$A = 2, \omega = 2$$

$\therefore$ Maximum particle velocity

$$v_{max} = A\omega = 2 \times 2 = 4 \text{ unit}$$

85. $N = N_0 \left(\frac{1}{2}\right)^n$

$$\frac{N_0}{16} = N_0 \left(\frac{1}{2}\right)^2$$

$$\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n$$

$$\therefore \quad n = 4$$

$$\frac{T}{T_{1/2}} = 4$$

$$\Rightarrow \quad \frac{1}{T_{1/2}} = 4$$

$$\Rightarrow \quad T_{1/2} = \frac{1}{4} \text{ h} = 15 \text{ min}$$

Reasoning

141. Second is a bigger unit than the first, though both are used to measure the same quantity.

142. Except (d) all other groups contain only small letter.

143. Clearly. Fig. (d) will complete the pattern when placed in the blank space of fig. (x) as shown below.

In the following question, a statement group of statements is given followed by same conclusion choose the conclusion which logically follows from the given statement.

145. In fig. X, the upper triangular half of the paper has been folded over the lower half.

In fig. Y, the paper is refolded to a quarter triangle.

In fig. Z, a square has been punched in the folded paper.

Clearly, the square will appear in each of the trianglular quarters of the paper.

Thus, when the paper is unfolded, four squ[ares] will appear symmetrically over it and it [w]... resemble fig. (C).

148. Clearly, $(3 + 2)^2 = 25$;

$$(15 + 6)^2 = (21)^2 = 441;$$

$$(10 + 7)^2 = (17)^2 = 289.$$

So, missing number $= (12 + 13)^2 = (25)^2 = \ldots$

149. The first letter of each term is moved th[ree] steps forward and the last letter is moved ... steps backward to obtain the correspond[ing] letters of the next term. The numbers foll[ow] the sequence. $\times 2 + 1$, $\times 2 + 2$, $\times 2 + 3$, $\times 2 + 4$ ... So 10 is wrong and must be replaced ... $(4 \times 2 + 1)$ *i.e.*, 9.

150. Clearly, in the first and second rows, the sec[ond] figure is the inner part of the first figure and t[he] third figure is the inner part of the sec[ond] figure.

Thus, the missing figure should be the in[ner] part of the second figure in third row, *i.e.* ... small circle.