Code : N

Mathematics

1. If (2, 3, 5) is one end of a diameter of the sphere $x^2+y^2+z^2-6x-12y-2z+20=0,$ then the coordinates of the other end of the diameter are
(1) (4, 3, –3) (2) (4, 9, –3) (3) (4, –3, 3) (4) (4, 3, 5)

Sol. (2)
Centre of sphere is (3, 6, 1)
Let other end is (x_1, y_1, z_1)

$\therefore \frac{x_1+2}{2}=3 \Rightarrow x_1=4$

$\frac{y_1+3}{2}=6 \Rightarrow y_1=9$

$\frac{z_1+5}{2}=1 \Rightarrow z_1=-3$

$\therefore$ Other end of diameter is (4, 9, –3)

2. Let $\bar{a}=\hat{i}+\hat{j}+\hat{k}, \bar{b}=\hat{i}-\hat{j}+2\hat{k}$ and $\bar{c}=x\hat{i}+(x-2)\hat{j}-\hat{k},$. If the vector $\bar{c}$ lies in the plane of $\bar{a}$ and $\bar{b}$, then x equals
(1) –2 (2) 0 (3) 1 (4) –4

Sol. (1)

$\because \vec{c}, \vec{a}$ and $\vec{b}$ are coplanar

$\therefore \vec{c}=\lambda\vec{a}+\mu\vec{b}$

$x\hat{i}+(x-2)\hat{j}-\hat{k}=\lambda(\hat{i}+\hat{j}+\hat{k})+\mu(\hat{i}-\hat{j}+2\hat{k})$

$\Rightarrow x=\lambda+\mu$ ... (i)

$x-2=\lambda-\mu$ ... (ii)

$-1=\lambda+2\mu$ ... (iii)

From (i) and (ii)
$\lambda = x - 1, \mu = 1$
$\therefore$ From (iii) $-1 = x - 1 + 2$
$\Rightarrow x = -2$

Method II

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 1(1-2x+4)-1(-1-2x)+1(x-2+x)=0$$

$$\Rightarrow 2x = -4$$

$$\Rightarrow x = -2$$

3. Let A(h, k), B(1, 1) and C(2, 1) be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1, then the set of values which 'k' can take is given by

(1) {–3, –2} (2) {1, 3} (3) {0, 2} (4) {–1, 3}

Sol. (4)

$\Delta = 1$

$$\Rightarrow \frac{1}{2}\left\|\begin{matrix} h & k & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{matrix}\right\| = 1$$

$$\Rightarrow h(1-1)-k(1-2)+1(1-2)=\pm 2$$

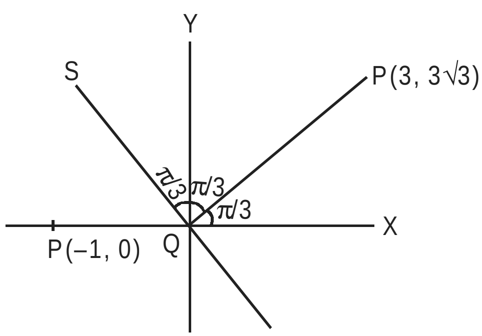
$\Rightarrow k - 1 = \pm 2$

$\Rightarrow k = 3$ or -1

4. Let P = (–1, 0), Q = (0, 0) and R = $(3, 3\sqrt{3})$ be three points. The equation of the bisector of the angle PQR is

(1) $x+\sqrt{3}y=0$ (2) $\sqrt{3}x+y=0$ (3) $x+\frac{\sqrt{3}}{2}y=0$ (4) $\frac{\sqrt{3}}{2}x+y=0$

Sol. (2)



Slope of QS, m = tan120° = $-\sqrt{3}$

$y = -\sqrt{3}x$

$y + \sqrt{3}x = 0$

5. If one of the lines of $my^2 + \left(1 - m^2\right)xy - mx^2 = 0$ is a bisector of the angle between the lines xy = 0, then m is

(1) 2 2. $-\dfrac{1}{2}$ (3) –2 (4) 1

Sol. (4)

Joint equation of bisector of the lines xy = 0 is $y^2 - x^2 = 0$

Since $my^2 + \left(1 - m^2\right)xy - mx^2 = 0$

$\Rightarrow (y - mx)(my + x) = 0$

$\Rightarrow$ One of the line is bisector of xy = 0

$\Rightarrow$ m = 1

6. Let F(x) = f(x) + $f\left(\dfrac{1}{x}\right)$, where $f(x) = \int_1^x \dfrac{\log t}{1+t}dt.$ Then F(e) equals

(1) 2 (2) $\dfrac{1}{2}$ (3) 0 (4) 1

Sol. (2)

$F(e) = f(e) + f\left(\dfrac{1}{e}\right)$

$= \int_1^e \dfrac{\log t}{1+t}dt + \int_1^{1/e} \dfrac{\log t}{1+t}dt$

$= I_1 + I_2$

For $I_2 = \int_1^{1/e} \dfrac{\log t}{1+t}dt$

Let $t = \dfrac{1}{z}$ $\Rightarrow dt = -\dfrac{1}{z^2}dz$

When t = 1 $\Rightarrow$ z = 1

$$t = \frac{1}{e} \Rightarrow z = e$$

$$\therefore I_2 = \int_1^e \frac{-\log z}{1+\frac{1}{z}}.\left(-\frac{1}{z^2}\right)dz$$

$$= \int_1^e \frac{\log z}{z(1+z)}dz = \int_1^e \frac{\log t}{t(1+t)}dt$$

$$\therefore F(e) = I_1 + I_2$$

$$= \int_1^e \left(\frac{\log t}{1+t} + \frac{\log t}{t(1+t)}\right)dt$$

$$= \int_1^e \frac{\log t}{t}dt = \int_0^1 s\,ds \qquad s = \log t;\ ds = \frac{1}{t}dt,\ \text{when } t = 1.\ s = 0 \text{ and } t = e.\ s = 1$$

$$= \left[\frac{s^2}{2}\right]_0^1 = \frac{1}{2}$$

7. Let $f: R \to IR$ be a function defined by f(x) = Min{x + 1, |x| + 1}. Then which of the following is true?

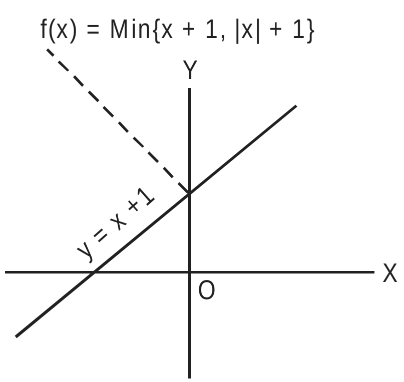
(1) f(x) is not differentiable at x = 0

(2) f(x) $\geq$ 1 for all $x \in R$

(3) f(x) is not differentiable at x = 1.

(4) f(x) is differentiable everywhere

Sol. (4)



$$f(x) = \begin{cases} x + 1 & x \geq 0 \\ x + 1 & x < 0 \end{cases}$$

$\Rightarrow$ f(x) is differentiable everywhere.

8. The function $f : R - \{0\} \to R$ given by

$$f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$$

can be made continuous at x = 0 by defining f(0) as

(1) 1 (2) 2 (3) –1 (4) 0

Sol. (1)

$$\lim_{x \to 0} \left(\frac{1}{x} - \frac{2}{e^{2x} - 1} \right)$$

$$= \lim_{x \to 0} \frac{e^{2x} - 1 - 2x}{x\left(e^{2x} - 1\right)} \qquad \left[\frac{0}{0}\right]$$

$$= \lim_{x \to 0} \frac{2e^{2x} - 2}{e^{2x} - 1 + x.2e^{2x}} \qquad \left[\frac{0}{0}\right]$$

$$= \lim_{x \to 0} \frac{4e^{2x}}{2e^{2x} + 2e^{2x} + 4xe^{2x}}$$

$$= \frac{4}{4} = 1$$

$\therefore$ f(0) = 1

9. The solution for x of the equation $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2 - 1}} = \frac{\pi}{2}$ is

(1) $2\sqrt{2}$ (2) 2 (3) π (4) $\frac{\sqrt{3}}{2}$

Sol. Wrong Question

10. $\int \frac{dx}{\cos x + \sqrt{3}\sin x}$ equals

(1) $\log \tan\left(\frac{x}{2} - \frac{\pi}{12}\right) + C$

(2) $\frac{1}{2}\log \tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$

(3) $\frac{1}{2}\log \tan\left(\frac{x}{2} - \frac{\pi}{12}\right) + C$

(4) $\log \tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$

Sol. (2)

$$I = \frac{1}{2}\int \frac{dx}{\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x}$$

$$= \frac{1}{2}\int \frac{dx}{\sin x \cos\frac{\pi}{6} + \cos x \sin\frac{\pi}{6}}$$

$$= \frac{1}{2}\int \frac{dx}{\sin\left(x + \frac{\pi}{6}\right)}$$

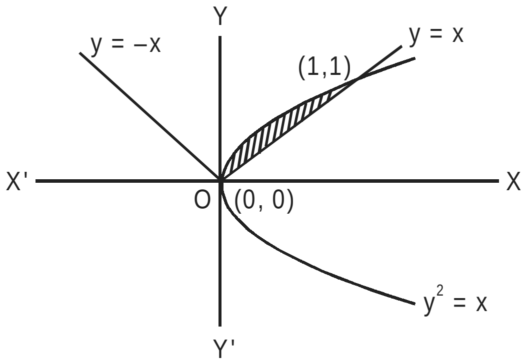
$$= \frac{1}{2}\int \operatorname{cosec}\left(x + \frac{\pi}{6}\right)dx$$

$$= \frac{1}{2}\log \tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$$

11. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$ (3) 1 (4) $\frac{1}{6}$

Sol. (4)



$$\text{Area} = \int_0^1 \left(\sqrt{x} - x\right) dx$$

$$= \left[\frac{x^{3/2}}{\frac{3}{2}} - \frac{x^2}{2}\right]_0^1$$

$$= \frac{2}{3} - \frac{1}{2} = \frac{1}{6} \text{ sq. units}$$

12. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is

(1) $(-\infty, -3)$ (2) $(-3, 3)$ (3) $(-3, \infty)$ (4) $(3, \infty)$

Sol.

$\because |\alpha - \beta| < \sqrt{5}$

Again $\alpha + \beta = -a,\ \alpha\beta = 1$

$\Rightarrow (\alpha - \beta)^2 < 5$

$$\Rightarrow (\alpha+\beta)^2 - 4\alpha\beta < 5$$

$$\Rightarrow a^2 - 4 < 5$$

$$\Rightarrow a^2 < 9 \quad \Rightarrow a \in (-3, 3) \qquad \text{... (i)}$$

Also $D \geq 0$

$a^2 - 4 \geq 0$

$$\Rightarrow a \in (-\infty, -2) \cup (2, \infty) \qquad \text{... (ii)}$$

From (i) and (ii)

$$a \in (-3, -2) \cup (2, 3)$$

13. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals

(1) $\frac{1}{2}(\sqrt{5}-1)$ (2) $\frac{1}{2}(1-\sqrt{5})$ (3) $\frac{1}{2}\sqrt{5}$ (4) $\sqrt{5}$

Sol. (1)

Let the GP be a, ar, ar^2, ar^3...

$\therefore a = ar + ar^2$

$\Rightarrow 1 = r + r^2$

$\Rightarrow r^2 + r - 1 = 0$

$$\therefore r = \frac{-1 \pm \sqrt{1+4}}{2}$$

$$\therefore r = \frac{\sqrt{5}-1}{2} \qquad (\because \text{ GP has positive terms})$$

14. If $\sin^{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$ then a value of x is

(1) 5 (2) 1 (3) 3 (4) 4

Sol. (3)

$$\sin^{-1}\left(\frac{x}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{2}$$

$$\Rightarrow \frac{x}{5} = \frac{3}{5} \Rightarrow x = 3$$

15. In the binomial expansion of $(a-b)^n, n \geq 5$, the sum of 5th and 6th term is zero, then $\frac{a}{b}$ equals

(1) $\frac{n-4}{5}$ (2) $\frac{5}{n-4}$ (3) $\frac{6}{n-5}$ (4) $\frac{n-5}{6}$

Sol. (1)

$T_5 + T_6 = 0$

${}^nC_4\, a^{n-4}.b^4 - {}^nC_5\, a^{n-5}\, b^5 = 0$

$$\Rightarrow \frac{a^{n-4}\, b^4}{a^{n-4}\, b^5} = \frac{{}^nC_5}{{}^nC_4}$$

$$\Rightarrow \frac{a}{b} = \frac{n!}{5!(n-5)!} \times \frac{4!(n-4)!}{n!} = \frac{n-4}{5}$$

16. The set S : = {1, 2, 3..., 12} is to be partitioned into three sets A, B and C of equal size. Thus, $A \cup B \cup C = S, A \cap C = B \cap C = A \cap C = \phi$. The numbers of ways to partition S is

(1) $\frac{12!}{(3!)^4}$ (2) $\frac{12!}{3!(4!)^3}$ (3) $\frac{12!}{3!(3!)^4}$ (4) $\frac{12!}{(4!)^3}$

Sol. (4)

Total number of ways = $\frac{12!}{(4!)^3 \times 3!} \times 3! = \frac{12!}{(4!)^3}$

17. The largest interval lying in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ for which the function

$$\left[f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)\right]$$

is defined, is

(1) $\left[0, \frac{\pi}{2}\right)$ (2) $[0, \pi]$ (3) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (4) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$

Sol. (1)

$$f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$$

For $4^{-x^2} \Rightarrow x \in R$ ... (i)

For $\cos^{-1}\left(\frac{x}{2}-1\right)$

$-1 \le \frac{x}{2} - 1 \le 1 \Rightarrow 0 \le \frac{x}{2} \le 2$

$\Rightarrow 0 \le x \le 4$ ... (ii)

For log(cosx)

$\cos x > 0 \Rightarrow \frac{-\pi}{2} < x < \frac{\pi}{2}$ ... (iii)

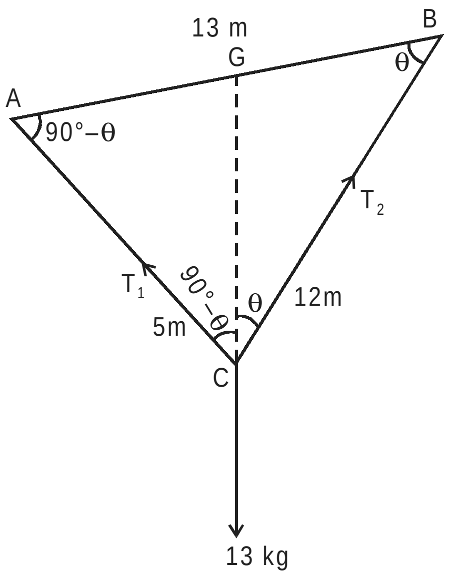
From (i), (ii) and (iii)

$x \in \left[0, \frac{\pi}{2}\right)$

18. A body weighing 13 kg is suspended by two strings 5 m and 12 m long, their other ends being fastened to the extremities of a rod 13 m long. If the rod be so held that the body hands immediately below the middle point. The tension in the strings are

(1) 5 kg and 13 kg (2) 12 kg and 13 kg (3) 5 kg and 5 kg (4) 5 kg and 12 kg

Sol. (4)



$13^2 = 5^2 + 12^2 \Rightarrow AB^2 = AC^2 + BC^2 \Rightarrow \angle ACB = 90°$

$\because$ G is mid-point of hypotenuse AB.

$\therefore$ GA = GB = GC $\Rightarrow$ GC = 6.5m

Let $\angle GBC = \theta$, then, $\angle GCB = \theta$

By Lami's theorem

$$\frac{T_1}{\sin(180° - \theta)} = \frac{T_2}{\sin(90° + \theta)} = \frac{13}{\sin 90°}$$

$$\Rightarrow \frac{T_1}{\sin\theta} = \frac{T_2}{\cos\theta} = \frac{13}{1}$$

$\Rightarrow T_1 = 13\sin\theta$ and $T_2 = 13\cos\theta$

$\Rightarrow T_1 = 13 \times \frac{5}{13}$ and $T_2 = 13 \times \frac{12}{13}$ as $\sin\theta = \frac{5}{13}, \cos\theta = \frac{12}{13}$

$\Rightarrow T_1 = 5$ kg and $T_2 = 12$ kg

19. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is

(1) $\frac{8}{243}$ (2) $\frac{1}{729}$ (3) $\frac{8}{9}$ (4) $\frac{8}{729}$

Sol. (1)

Probability of getting exactly 9 is $\frac{1}{9}$

and probability of not getting 9 is $1 - \frac{1}{9} = \frac{8}{9}$

$\therefore$ Required probability = ${}^3C_2 \left(\frac{1}{9}\right)^2 \times \frac{8}{9}$

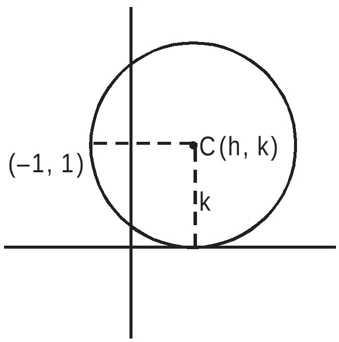
$$= \frac{3!}{2!} \times \frac{1}{81} \times \frac{8}{9}$$

$$= \frac{6 \times 8}{2 \times 81 \times 9} = \frac{8}{243}$$

20. Consider a family of circles which are passing through the point (–1, 1) and are tangent to x-axis. If (h, k) are the coordinates of the centre of the circles, then the set of values of k is given by the interval

(1) $k \le \frac{1}{2}$ (2) $0 < k < \frac{1}{2}$ (3) $k \ge \frac{1}{2}$ (4) $-\frac{1}{2} \le k \le \frac{1}{2}$

Sol. (3)



$$(h+1)^2 + (k-1)^2 = k^2$$

$$\Rightarrow h^2 + 2h + 1 + k^2 - 2k + 1 = k^2$$

$$\Rightarrow h^2 + 2h + (2 - 2k) = 0$$

$\therefore D \ge 0$
$\Rightarrow 4 - 4(2 - 2k) \ge 0$
$\Rightarrow 4 - 8 + 8k \ge 0$
$\Rightarrow 8k \ge 4$

$k \ge \frac{1}{2}$

21. Let L be the line of intersection of the planes 2x + 3y + z = 1 and x + 3y + 2z = 2. If L makes an angle α with the positive x-axis, then cos α equals.

(a) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{\sqrt{3}}$ (3) $\frac{1}{2}$ (4) 1

Sol. (2)

Given planes are

$2x + 3y + z - 1 = 0$ ... (i)

$x + 3y + 2x - 2 = 0$ ... (ii)

Let l, m, n be the direction cosines of line of intersection of plane (i) and (ii).

$2l + 3m + n = 0$ ... (iii)

$l + 3m + 2n = 0$ ... (iv)

Solving (iii) and (iv), we get
m = –l, n = l

$l^2 + m^2 + n^2 = 1$

$\Rightarrow 3l^2 = 1 \quad \Rightarrow l = \pm\frac{1}{\sqrt{3}}$

22. The differential equation of all circles passing through the origin and having their centres on the x-axis is

(1) $y^2 = x^2 - 2xy\frac{dy}{dx}$

(2) $x^2 = y^2 + xy\frac{dy}{dx}$

(3) $x^2 = y^2 + 3xy\frac{dy}{dx}$

(4) $y^2 = x^2 + 2xy\frac{dy}{dx}$

Sol. (4)

General equation of circle

$x^2 + y^2 + 2gx + 2fy + c = 0$

As centre is on x-axis, f = 0
As circle is passing through origin, c = 0
Equation of required circle will be

$x^2 + y^2 + 2gx = 0$ ... (i)

Differentiating w.r.t. x, we get

$2x + 2y\frac{dy}{dx} + 2g = 0$ ... (ii)

Eliminating g from (i) and (ii)

$$x^2 + y^2 + x\left(-2x - 2y\frac{dy}{dx}\right) = 0$$

$$y^2 = x^2 + 2xy\frac{dy}{dx}$$

23. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of (p + q) is

(1) $\sqrt{2}$ (2) 2 (3) $\frac{1}{2}$ (4) $\frac{1}{\sqrt{2}}$

Sol. (1)

Given $p^2 + q^2 = 1$ ... (i)

From (i), we can say $0 \le p \le 1$ and $0 \le q \le 1$

$\therefore$ Put $p = \sin\theta$ $q = \cos\theta$

$\therefore$ $p + q = \sin\theta + \cos\theta$

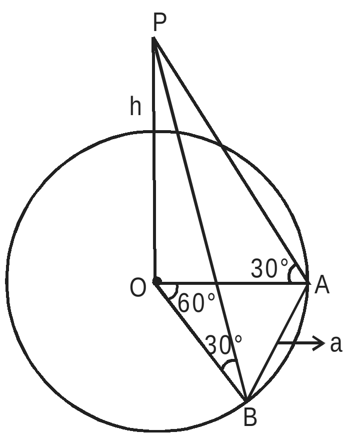
Maximum value of $\sin\theta + \cos\theta = \sqrt{2}$

$\therefore$ Maximum value of $p + q = \sqrt{2}$

24. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that AB (= a) subtends an angle of 60° at the foot of the tower, and the angle of elevation of the top of the tower from A or B is 30°. The height of the tower is

(1) $a\sqrt{3}$ (2) $\frac{2a}{\sqrt{3}}$ (3) $2a\sqrt{3}$ (4) $\frac{a}{\sqrt{3}}$

Sol. (4)



OA = OB = radii

In ΔOAB, OA = OB = AB = a

In ΔPOB

$\tan 30° = \frac{h}{a}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{a}$$

$$\Rightarrow h = \frac{a}{\sqrt{3}}$$

25. The sum of the series
${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots - \ldots + {}^{20}C_{10}$ is

(1) ${}^{20}C_{10}$ (2) $-\,{}^{20}C_{10}$ (3) $\frac{1}{2}\,{}^{20}C_{10}$ (4) 0

Sol. (3)

Given series is

$x = {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots - \ldots + {}^{20}C_{10}$

$\Rightarrow 2x = 2\,{}^{20}C_0 - 2\,{}^{20}C_1 + 2\,{}^{20}C_2 + \ldots - \ldots + 2\,{}^{20}C_{10}$

$$= \left({}^{20}C_0 + {}^{20}C_{20}\right) - \left({}^{20}C_1 + {}^{20}C_{19}\right) + \left({}^{20}C_2 + {}^{20}C_{18}\right) + \ldots + \left({}^{20}C_{10} + {}^{20}C_{10}\right)$$

$$\Rightarrow 2x = \left({}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 + \ldots + {}^{20}C_{10} + \ldots + {}^{20}C_{18} - {}^{20}C_{19} + {}^{20}C_{20}\right) + {}^{20}C_{10}$$

As ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 + \ldots + {}^{20}C_{20} = 0$

$$\therefore 2x = {}^{20}C_{10} \quad \Rightarrow x = \frac{1}{2}\,{}^{20}C_{10}$$

26. The normal to a curve at P(x, y) meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a

(1) hyperbola (2) ellipse (3) parabola (4) circle

Sol. (1, 2)

Let y = f(x) be a curve

$$\therefore \frac{dy}{dx} = \text{slope of tangent}$$

$$\Rightarrow -\frac{dx}{dy} = \text{slope of normal}$$

Equation of normal

$$Y - y = -\frac{dx}{dx}(X - x)$$

$$\therefore G \equiv \left(x + y\frac{dy}{dx}, 0\right)$$

Given $\left|x+y\frac{dy}{dx}\right|=|2x|$

$\Rightarrow y\frac{dy}{dx}=x \Rightarrow ydy=xdx$ or $y=\frac{dy}{dx}=-3x$

$\Rightarrow \frac{y^2}{2}=\frac{x^2}{2}+K$ or $ydy=-3xdx$

$\Rightarrow y^2-x^2=K_1$ or $\frac{y^2}{2}=-\frac{-3x^2}{2}+K$ or $\frac{x^2}{2/3}+\frac{y^2}{2}=K$

$\therefore$ Curve is hyperbola or ellipse.

27. If $|z+4| \le 3$, then the maximum value of $|z+1|$ is

(1) 0 (2) 4 (3) 10 (4) 6

Sol. (4)

Given $|z+4| \le 3$

$|z+1|=|z+4+(-3)| \le |z+4|+|-3|$

$\Rightarrow |z+1| \le |z+4|+3$

$\Rightarrow |z+1| \le 3+3$

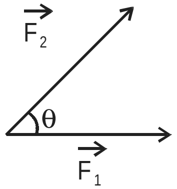
$\Rightarrow |z+1| \le 6$

Maximum value of $|z+1|$ is 6.

28. The resultant of two forces P N and 3 N is a force of 7 N. If the direction of the 3 N force were reversed, the resultant would be $\sqrt{19}$ N. The value of P is

(1) 4 N (2) 5 N (3) 6 N (4) 3 N

Sol. (2)

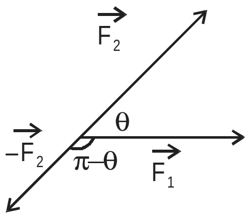


$|\vec{F_1}|=PN$

$\left|\vec{F_2}\right| = 3N$

$R_1 = \sqrt{3^2 + P^2 + 6P\cos\theta} = 7$

$\cos\theta = \dfrac{40 - P^2}{6P}$ ... (i)



$R_2 = \sqrt{9 + P^2 + 6P\cos(\pi - \theta)} = \sqrt{19}$

$P^2 - 6P\cos\theta = 10$

$P^2 - \left(40 - P^2\right) = 10$ [(from i)]

$2P^2 = 50$

$\therefore$ P = 5N

29. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is

(1) 0.7 (2) 0.06 (3) 0.14 (4) 0.2

Sol. (3)

Let A is the event of the plane I hit the target correctly.
B is the event of the plane II hit the target correctly.

P(A) = .3 $P\left(A^c\right) = .7$

P(B) = .2 $P\left(B^c\right) = .8$

Probability that the target is hit by the second plane = $P\left(A^c\right).P(B) = .7 \times .2 = .14$

Assume second plane hit the target only one time.

30. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0,\ y \neq 0,$ then D is

(1) divisible by y but not x
(2) divisible by neither x nor y
(3) divisible by both x and y
(4) divisible by x but not y

Sol. (3)

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$$

$$C_1 \to C_1 - C_2, C_2 \to C_2 - C_3$$

$$D = \begin{vmatrix} 0 & 0 & 1 \\ -x & x & 1 \\ 0 & -y & 1+y \end{vmatrix} = xy$$

$\therefore$ D is divisible by both x and y.

31. For the hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant when α varies?

(1) Abscissae of foci
(2) Eccentricity
(3) Directrix
(4) Abscissae of vertices

Sol. (1)

$$\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$$

$$\sin^2\alpha = \cos^2\alpha\left(e^2 - 1\right)$$

$$\therefore \cos^2\alpha e^2 = 1 \ \Rightarrow e\cos\theta = \pm 1$$

Here, a = $\cos\alpha$, b = $\sin\alpha$
Abscissae of foci = $\pm ae = \pm e\cos\alpha = \pm 1$

$e = \frac{1}{\cos\alpha}$ (depends on α)

$$x = \pm\frac{a}{e} = \pm\cos^2\alpha$$

Abscissae of vertices = $\pm a = \pm\cos\alpha$

32. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of each of x-axis and y-axis, then the angle that the line makes with the positive direction of z-axis is

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

Sol. (1)

$$l = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$m = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\therefore l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} + n^2 = 1$$

$$\Rightarrow n^2 = 0 \Rightarrow n = 0$$

$$\therefore \cos\gamma = 0 \Rightarrow \gamma = \frac{\pi}{2}$$

33. A value of C for which the conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval [1, 3] is

(1) $\log_e 3$ (2) $2 \log_3 e$ (3) $\frac{1}{2}\log_e 3$ (4) $\log_3 e$

Sol. (2)

Given function $f(x) = \log_e x$

Mean Value Theorem for [1, 3]

$$f'(c) = \frac{f(3) - f(1)}{3 - 1}$$

$$\frac{1}{c} = \frac{\log_e 3 - \log_e 1}{2} = \frac{1}{2}\log_e 3$$

$$c = \frac{2}{\log_e 3} = 2\log_3 e$$

34. The function $f(x)=\tan^{-1}(\sin x+\cos x)$ is an increasing function in

(1) $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ (2) $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ (3) $\left(\frac{-\pi}{2},\frac{\pi}{4}\right)$ (4) $\left(0,\frac{\pi}{2}\right)$

Sol. (3)

$f(x)=\tan^{-1}(\sin x+\cos x)$

Let Z = sinx + cosx

$f(x)=\tan^{-1}(Z)$

f(x) is increasing only when Z increases

$$Z=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$$

From options Z increases only when $-\frac{\pi}{2}<x<\frac{\pi}{4}$

35. Let $A=\begin{bmatrix}5 & 5\alpha & \alpha\\ 0 & \alpha & 5\alpha\\ 0 & 0 & 5\end{bmatrix}$

If $\left|A^2\right|=25,$ then $|\alpha|$ equals

(1) 5 (2) 5^2 (3) 1 (4) $\frac{1}{5}$

Sol. (4)

$$A=\begin{bmatrix}5 & 5\alpha & \alpha\\ 0 & \alpha & 5\alpha\\ 0 & 0 & 5\end{bmatrix}$$

$$\left|A^2\right|=25 \Rightarrow |A|^2=25$$

$$\therefore |A|=\pm 5$$

$$A=\begin{bmatrix}5 & 5\alpha & \alpha\\ 0 & \alpha & 5\alpha\\ 0 & 0 & 5\end{bmatrix} = 5(5\alpha-0)=25\alpha$$

$$\therefore 25\alpha=\pm 5 \Rightarrow \alpha=\pm\frac{1}{5}$$

$$\therefore |\alpha|=\frac{1}{5}$$

36. The sum of the series $\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\dots$ up to infinity is

(1) $e^{+\frac{1}{2}}$ (2) e^{-2} (3) e^{-1} (4) $e^{-\frac{1}{2}}$

Sol. (3)

$$\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\dots\infty$$

$$e^x = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots\infty$$

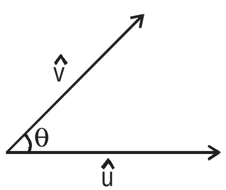
$$e^{-1} = 1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\dots\infty$$

$$\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}+\dots\infty = e^{-1}$$

37. If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them, then $2\hat{u}\times 3\hat{v}$ is a unit vector for

(1) Exactly one value of θ (2) Exactly two values of θ
(3) More than two values of θ (4) No value of θ

Sol. (1)



$2\hat{u}\times 3\hat{v}$

$= 6|\hat{u}||\hat{v}|\sin\theta\hat{n}$

Where $\hat{n}$ unit vector perpendicular to $\hat{u}$ and $\hat{v}$

$= 2\hat{u}\times 3\hat{v} = 6\sin\theta\hat{n}$

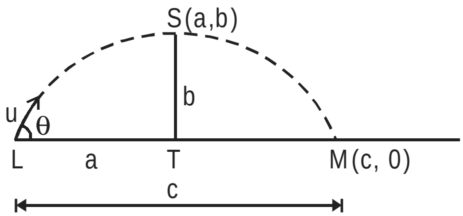
$6\sin\theta = 1 \Rightarrow \sin\theta = \frac{1}{6}$

$\therefore$ Here is one and only one value of θ between 0° and 90° for which $\sin\theta = \frac{1}{6}$

38. A particle just clears a wall of height b at a distance a and strikes the ground at a distance c from the point projection. The angle of projection is

(1) $\tan^{-1}\frac{bc}{a}$ (2) $\tan^{-1}\frac{b}{ac}$ (3) 45° (4) $\tan^{-1}\frac{bc}{a(c-a)}$

Sol. (4)



Let u = velocity of projection
θ = angle of projection

$$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2$$

As M is on the trajectory.

$$0 = c\tan\theta - \frac{g}{2u^2\cos^2\theta}c^2$$

$$\Rightarrow \tan\theta = \frac{g}{2u^2\cos^2\theta}c \quad \text{... (i)}$$

$$b = a\tan\theta - \frac{g}{2u^2\cos^2\theta}a^2 \quad \text{... (ii)}$$

From (i) and (ii), we get

$$\tan\theta = \frac{bc}{a(c-a)}$$

$$\therefore \theta = \tan^{-1}\frac{bc}{a(c-a)}$$

39. The average marks of boys in a class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is
(1) 60 (2) 40 (3) 20 (4) 80

Sol. (4)
Let x = Number of boys in the class
y = Number of girls in the class
Sum of marks of all boys = 52x

Sum of marks of all girls = 42y
Average of boys and girls combined marks = 50

$$50 = \frac{52x + 42y}{x + y}$$

$\Rightarrow$ x = 4y

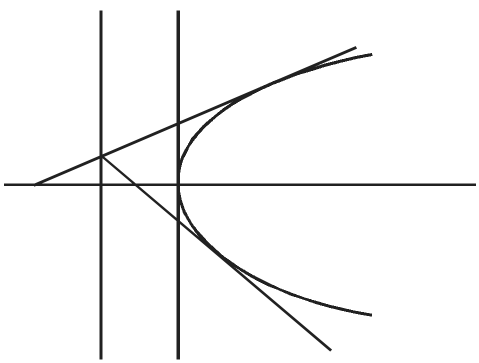
Percentage of boys in the class = $\frac{x}{x+y} \times 100 = \frac{4y}{5y} \times 100 = 80\%$

40. The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is

(1) (–2, 0) (2) (–1, 1) (3) (0, 2) (4) (2, 4)

Sol. (1)

Given parabola is $y^2 = 8x$



Given tangent y = x + 2 ...(i)

As second tangent is perpendicular to (i), so that pair is on the directrix as directrix is the director circle.

Equation of direcrix x = –2 ...(ii)

Solving (i) and (ii), we get (–2, 0)