

IIT-JEE – 2010 (Code-8)

Answer Keys

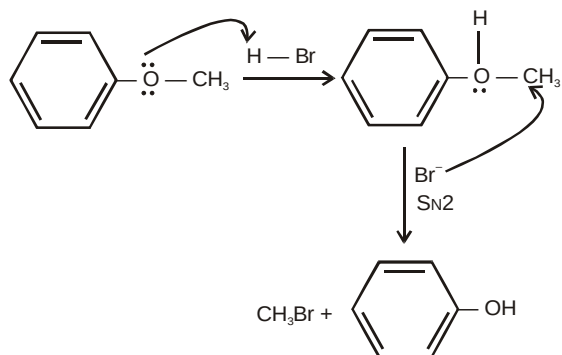
(Paper-I)

Chemistry	Mathematics	Physics
1. A	29. B	57. C
2. D	30. A	58. B
3. A	31. A	59. C
4. D	32. C	60. D
5. B	33. C	61. B
6. C	34. D	62. C
7. C	35. D	63. A
8. B	36. B	64. A
9. A,C	37. B	65. A,C
10. A,B	38. B,C	66. A,B,C
11. B,C,D	39. C,D	67. A,D
12. C,D	40. A	68. A,B
13. B,D	41. A,C,D	69. A,C
14. B	42. D	70. C
15. D	43. C	71. B
16. C	44. D	72. D
17. B	45. B	73. A
18. C	46. A	74. B
19. 1	47. 5	75. 4
20. 4	48. 2	76. 9
21. 3	49. 6	77. 5
22. 0	50. 4	78. 4
23. 3	51. 0	79. 6
24. 3	52. 1	80. 3
25. 0	53. 3	81. 8
26. 3	54. 9	82. 7
27. 2	55. 3	83. 6
28. 5	56. 2	84. 2

Chemistry

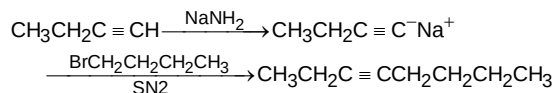
1. A $k = Ae^{\frac{-E_a}{RT}}$; k increases exponentially with temperature (T).

2. D

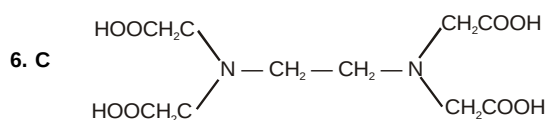


3. A

4. D



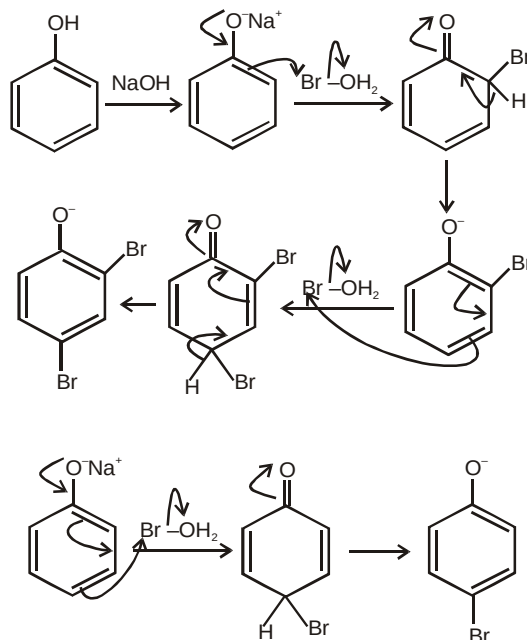
5. B $[\text{Cr}(\text{H}_2\text{O})_4\text{Cl}_2]\text{NO}_2$ is ionization isomer of $[\text{Cr}(\text{H}_2\text{O})_4\text{NO}_2\text{Cl}]\text{Cl}$.



7. C The C — C single bond dissociation energy is approximately 100 Kcal/mol.

8. B Standard molar enthalpy of formation of elements in their elemental state or naturally occurring state is said to be zero. Br_2 is liquid at room temperature, H_2O and CH_4 are compounds.

9. A,C $\text{H}_2\text{O} + \text{Br}_2 \rightarrow \text{H}_2\text{O}^+ - \text{Br}$



10. A,B

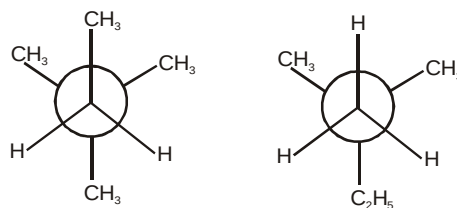
11. B,C,D

$\text{Ca}(\text{OH})_2$, Na_2CO_3 , NaOCl can be used to remove temporary hardness.

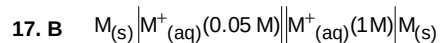
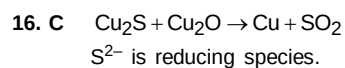
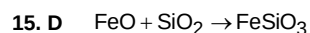
12. C,D

$\text{CH}_3\text{COONa} + \text{HNO}_3 + \text{H}_2\text{O} \rightarrow \text{CH}_3\text{COOH} + \text{NaNO}_3 + \text{HO}^-$
can act as buffer solution because weak acid (CH_3COOH) and its conjugate base (CH_3COO^-) present in solution.

13. B,D



14. B $\text{CuFeS}_2 + \text{O}_2 \xrightarrow{\text{Roasting}} \text{Cu}_2\text{S} + \text{Fe}_2\text{O}_3$
 $\text{Cu}_2\text{S} + \text{O}_2 \rightarrow \text{Cu}_2\text{O}$



$$E_{\text{cell}} = E_{\text{cell}}^0 - \frac{0.0591}{1} \log \frac{[M^+_{(aq)}]_{\text{LHS}}}{[M^+_{(aq)}]_{\text{RHS}}}$$

$$E_{\text{cell}} = -\frac{0.0591}{1} \log \frac{0.05}{1} = 70 \text{ mV}$$

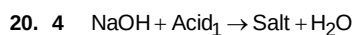
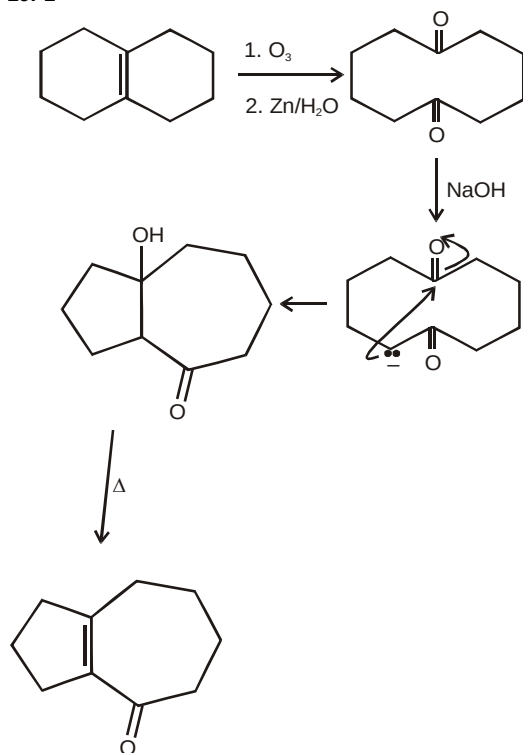
So, E_{cell} is positive

$$\Delta G = -nFE \Rightarrow \Delta G \text{ is +ve}$$

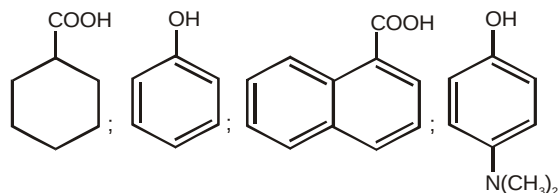
18. C $E_{\text{cell}} = -\frac{0.0591}{1} \log \frac{0.0025}{1}$

$$E_{\text{cell}} = 2 \left(-\frac{0.0591}{1} \log \frac{0.05}{1} \right) = 2 \times 70 = 140 \text{ mV}$$

19. 1

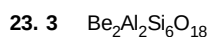
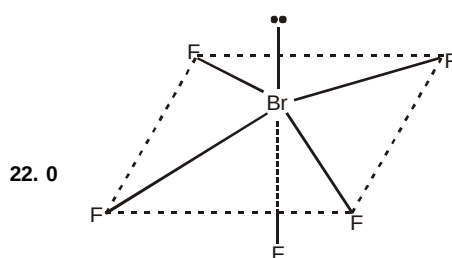


If acid in above reaction is stronger acid than H_2O



are stronger acid than H_2O .

21. 3 KCN , K_2CO_3 , LiCN on dissolving in water gives basic solution which turns red litmus into blue.



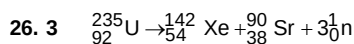
24. 3 $\frac{25.2 + 25.25 + 25.0}{3} = \frac{75.45}{3} = 25.15$

25. 0 $k = \frac{dx}{dt}$

$$k = \frac{0.25}{0.05} = 5$$

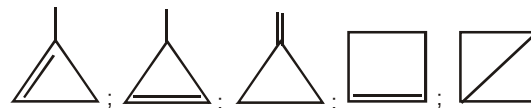
$$k = \frac{0.6}{0.12} = 5$$

$$k = \frac{0.9}{0.18} = 5$$



27. 2

28. 5



Mathematics

29. B $\lim_{x \rightarrow 0} \frac{1}{x^3} \int_0^x \frac{t \ln(1+t)}{t^4+4} dt$ $\frac{0}{0}$ form

Use L'Hospital rule $\lim_{x \rightarrow 0} \frac{\frac{x \ln(1+x)}{x^4+4} - 0}{3x^2} = \lim_{x \rightarrow 0} \frac{\ln(1+x)}{3x(x^4+4)}$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{1+x}}{3(x^4+4)+12x^3} = \frac{1}{12}$$

30. A Three planes cannot cut at two distinct point.

31. A $\overrightarrow{PQ} = 6\hat{i} + \hat{j}, \overrightarrow{QR} = -\hat{i} + 3\hat{j}, \overrightarrow{RS} = -6\hat{i} - \hat{j}$
 $\overrightarrow{SP} = -\hat{i} + 3\hat{j}$

Now $|\overrightarrow{PQ}| \neq |\overrightarrow{QR}|$ so it can not be a square or rhombus.

but $|\overrightarrow{PQ}| = |\overrightarrow{RS}|$ and $|\overrightarrow{QR}| = |\overrightarrow{SP}|$ so it can be a rectangle or a parallelogram.

But $\overrightarrow{PQ} \cdot \overrightarrow{QR} \neq 0$ so it a parallelogram.

32. C Number of events in sample space = $6 \times 6 \times 6 = 216$

for $\omega^1 + \omega^2 + \omega^3 = 0$

Number of possible triplets are eight

(1, 2, 3) (2, 3, 4) (3, 4, 5) (4, 5, 6)...

Each triplet gives $|\underline{3}| = 6$ ways

so possible favourable ways = $6 \times 8 = 48$

Required probability = $\frac{48}{216} = \frac{2}{9}$

33. C Let the plane be $ax + by + cz = d$

as it contains $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$

$\Rightarrow d = 0$ and $2a + 3b + 4c = 0 \dots (i)$

Further the plane is perpendicular to plane containing

straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$

Hence the normal to desired plane will be perpendicular to both the straight lines

Hence normal will be at 90° to the vector formed by

$(3\hat{i} + 4\hat{j} + 2\hat{k}) \times (4\hat{i} + 2\hat{j} + 3\hat{k}) = 8\hat{i} - \hat{j} - 10\hat{k}$

$\therefore 8a - b - 10c = 0 \dots (ii)$

Solving (i) and (ii) we get

$a = c$ and $b = -2c$

$\therefore$ required plane will be $cx - 2cy + cz = 0$

or $x - 2y + z = 0$

34. D A, B, C are in A.P.

$\therefore A + C = 2B$

$\therefore A + B + C = 180^\circ$

$A + C = 120^\circ$ and $B = 60^\circ$

$\frac{a}{c} \sin 2c + \frac{c}{a} \sin 2A$

$= \frac{\sin A}{\sin C} \cdot 2 \sin C \cos C + \frac{\sin C}{\sin A} \cdot 2 \sin A \cos A$

(by sine rule)

$= 2 \sin(A + C) = 2 \sin 120^\circ$

$= \frac{2\sqrt{3}}{2} = \sqrt{3}$

35. D All $f(x)$, $g(x)$, $h(x)$ are monotonically increasing functions in $[0, 1]$

$\therefore a = f(1)$, $b = g(1)$, $c = h(1)$

36. B $\alpha + \beta = -p$ $\alpha^3 + \beta^3 = q$

$(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = q$

$-p^3 + 3p\alpha\beta = q$

$\alpha\beta = \frac{q + p^3}{3p}$

Desired equation

$x^2 - \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha} \right) x + 1 = 0$

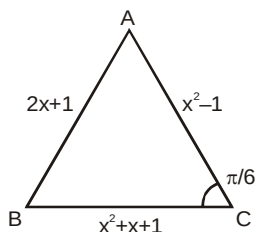
$x^2 - \left\{ \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} \right\} x + 1 = 0$

$\Rightarrow x^2 - \left(\frac{p^2 - 2(q + p^3)}{\frac{q + p^3}{3p}} \right) x + 1 = 0$

$\Rightarrow x^2 - \left(\frac{3p^3 - 2q - 2p^3}{q + p^3} \right) x + 1 = 0$

$(q + p^3)x^2 - (p^3 - 2q)x + q + p^3 = 0$

37. B



$$\cos \frac{\pi}{6} = \frac{(x^2+x+1)^2 + (x^2-1)^2 - (2x+1)^2}{2(x^2+x+1)(x^2-1)}$$

$$\frac{\sqrt{3}}{2} = \frac{x^4 + x^2 + 1 + 2x + 2x^2 + 2x^3 + x^4 + 1 - 2x^2 - 4x^2 - 1 - 4x}{2(x^2-1)(x^2+x+1)}$$

$$\frac{\sqrt{3}}{2} = \frac{2x^4 + 2x^3 - 3x^2 - 2x + 1}{2(x^2-1)(x^2+x+1)}$$

$$\sqrt{3} = \frac{2x^3(x-1) + 4x^2(x-1) + x(x-1) - 1(x-1)}{(x^2-1)(x^2+x+1)}$$

$$\sqrt{3} = \frac{2x^3 + 4x^2 + x - 1}{(x+1)(x^2+x+1)}$$

$$\sqrt{3} = \frac{2x^2(x+1) + 2x(x+1) - 1(x+1)}{(x+1)(x^2+x+1)}$$

$$\sqrt{3} = \frac{(2x^2 + 2x + 1)}{(x^2 + x + 1)}$$

$$x^2(2 - \sqrt{3}) + x(2 - \sqrt{3}) - (1 + \sqrt{3}) = 0$$

$$x^2 + x - \frac{(1 + \sqrt{3})}{2 - \sqrt{3}} = 0$$

$$x^2 + x - (1 + \sqrt{3})(2 + \sqrt{3}) = 0$$

$$x^2 + (2 + \sqrt{3}) - (1 + \sqrt{3})x - (1 + \sqrt{3})(2 + \sqrt{3}) = 0$$

$$x = -(2 + \sqrt{3}), 1 + \sqrt{3}$$

$$x \neq -(2 + \sqrt{3}) \text{ as } c \text{ will be negative}$$

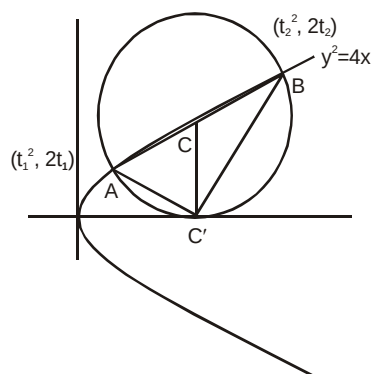
$$\therefore x = 1 + \sqrt{3}$$

38. B, C $f'(x) = \frac{1}{x} + \sqrt{1 + \sin x}$

$$f''(x) = -\frac{1}{x^2} + \frac{\cos x}{2\sqrt{1 + \sin x}}$$

f' exist for all x belonging to $(0, \infty)$, f'' does not exist for all $x \in (0, \infty)$

39. C, D



Point C $\Rightarrow \left(\frac{t_1^2 + t_2^2}{2}, t_1 + t_2 \right)$

$$CC' = r = t_1 + t_2$$

$$m_{AB} = \frac{2}{t_1 + t_2} = \frac{2}{r}$$

Similarly a circle exist for lower lobe

$$m_{AB} = \frac{-2}{r}$$

40. A $I = \int_0^1 f(x) dx$ where $f(x) = \frac{x^4(1-x)^4}{1+x^2}$

$$f(x) = \frac{x^4(1-2x+x^2)^2}{1+x^2}$$

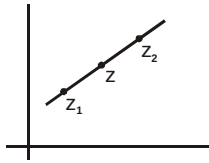
$$f(x) = \frac{x^4[(1+x^2)^2 - 4x(1+x^2) + 4x^2]}{1+x^2}$$

$$\Rightarrow f(x) = x^6 - 4x^5 + 5x^4 - 4x^2 + 4 - \frac{4}{1+x^2}$$

$$\therefore I = \frac{1}{7} - \frac{2}{3} + 1 - \frac{4}{3} + 4 - \pi$$

$$\therefore I = \frac{22}{7} - \pi$$

41. A, C, D



A is true as $|z - z_1| + |z - z_2| = |z_1 - z_2|$
 C is true as this is nothing the equation of line
 D is true as $\text{Arg}(z - z_1) = (z_2 - z_1)$
 B is not true as $\text{Arg}(z - z_1) = -\text{Arg}(z - z_2)$

42. D

43. C

44. D

45. B $(x - 4)^2 + y^2 = 4^2$
 $y = m(x - 4) \pm \sqrt{1 + m^2} \cdot 4, y = mx \pm \sqrt{9m^2 - 4}$
 $-4m \pm 4\sqrt{1 + m^2} = \pm \sqrt{9m^2 - 4}$
 $m = \frac{2}{\sqrt{5}}, -\frac{8}{\sqrt{5}} \pm 4\sqrt{1 + \frac{4}{5}} = \pm \sqrt{9 \times \frac{4}{5} - 4}$
 $-8 \pm 4.3 = \pm 4$
 $y = \frac{2}{\sqrt{5}}x + \sqrt{9 \cdot \frac{4}{5} - 4}$
 $\sqrt{5}y = 2x + 4$

46. A $y^2 = 8x - x^2$
 $\frac{x^2}{9} - \frac{8x - x^2}{4} = 1$
 $4x^2 - 72x + 9x^2 = 36$
 $13x^2 - 72x - 36 = 0$
 $x = \frac{72 \pm \sqrt{(72)^2 + 4 \cdot 13 \cdot 36}}{26}$
 $x = \frac{72 \pm 84}{26} = 6, -\frac{6}{13}$
 For $x = 6, y^2 = 12$
 $\Rightarrow y = \pm 2\sqrt{3}$
 Equation of circle will be
 $(x - 6)(x - 6) + (y - 2\sqrt{3})(y + 2\sqrt{3}) = 0$
 $x^2 - 12x + y^2 + 24 = 0$

47. 5

$$\vec{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}} \quad \vec{b} = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$$

since $|\vec{a}| = |\vec{b}| = 1$ and $\vec{a} \cdot \vec{b} = 0$

then let $\vec{a} \times \vec{b} = \vec{c}$

Such that $\vec{a}, \vec{b}$ and $\vec{c}$ form a right hand system of unit vectors.

$$\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a}, \vec{c} \times \vec{a} = \vec{b}$$

$$(2\vec{a} + \vec{b}) \cdot \{(\vec{a} \times \vec{b}) \times (\vec{a} - 2\vec{b})\}$$

$$= (2\vec{a} + \vec{b}) \cdot (\vec{c} \times \vec{a} - 2\vec{c} \times \vec{b})$$

$$= (2\vec{a} + \vec{b}) \cdot (\vec{b} + 2\vec{a})$$

$$= 4|\vec{a}|^2 + |\vec{b}|^2 = 4 + 1 = 5$$

48. 2

Line $y = -2x + 1$ is tangent to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$m = -2$$

$$c = 1$$

from condition of tangency $c^2 = a^2m^2 - b^2$

$$1 = 4a^2 - b^2$$

$$\boxed{b^2 = 4a^2 - 1}$$

Now line pass through $\left(\frac{a}{e}, 0\right)$

$$\Rightarrow 0 = -\frac{2a}{e} + 1 \therefore e = 2a$$

$$\text{Since } b^2 = a^2(e^2 - 1)$$

$$4a^2 - 1 = a^2(e^2 - 1)$$

$$e^2 - 1 = \frac{e^4}{4} - e^2 \Rightarrow e^4 - 5e^2 + 4 = 0$$

$$e = \pm 2 \text{ or } e = \pm 1$$

for hyperbola $e = 2$.

49. 6

Equation of plane containing lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$

will be

$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{c} = 3\hat{i} + 4\hat{j} + 5\hat{k} \quad (\vec{r} - \vec{a}) \cdot (\vec{b} \times \vec{c}) = 0$$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = -1 + 4 - 3 = 0$$

$$\vec{r} \cdot (\vec{b} \times \vec{c}) = \vec{a} \cdot (\vec{b} \times \vec{c})$$

$$\vec{r} \cdot (-\hat{i} + 2\hat{j} - \hat{k}) = 0$$

$$\text{put } \vec{r} = x\hat{i} + 4\hat{j} + z\hat{k}$$

$$\Rightarrow -x + 2y - z = 0, x - 2y + z = 0$$

Distance between planes will be

$$\left| \frac{d}{\sqrt{1+4+1}} \right| = \sqrt{6}$$

$$\Rightarrow |d| = 6$$

50. 4

$$f(x) = \begin{cases} \{x\} & \text{if } [x] \text{ is odd} \\ 1 - \{x\} & \text{if } [x] \text{ is even} \end{cases}$$

$$\text{Now } \int_{-9}^{+10} f(x) \cos \pi x \, dx = \int_{-10}^{+10} \{x\} \cos \pi x \, dx$$

$$= - \int_{-9}^{-10} (1 - \{x\}) \cos \pi x \, dx$$

$$\Rightarrow f(x) \cos \pi x$$

$$= \int_{-10}^{-9} (1 - \{x\}) \cos \pi x \, dx$$

$$= + \int_{-10}^{-9} f(x) \cos \pi x \, dx$$

$$\therefore \int_{-10}^{+10} f(x) \cos \pi x \, dx$$

$$= 2 \int_0^{+10} f(x) \cos \pi x \, dx$$

$$I = \int_0^{+10} f(x) \cos \pi x \, dx$$

$$I_1 = \int_0^1 f(x) \cos \pi x \, dx = \int_0^1 (1 - x) \cos \pi x \, dx$$

$$= \int_0^1 \cos \pi x \, dx - \int_0^1 x \cos \pi x \, dx$$

$$= \left[\frac{\sin \pi x}{\pi} \right]_0^1 - \int_0^1 \frac{x \sin \pi x}{\pi} \, dx$$

$$- \frac{1}{\pi} \int \sin \pi x \, dx$$

$$= + \frac{1}{\pi} \left[-\frac{\cos \pi x}{\pi} \right]_0^1$$

$$= \frac{-1}{\pi^2} [\cos \pi x]_0^1 = + \frac{2}{\pi^2}$$

$$I = \int_1^2 f(x) \cos \pi x \, dx$$

$$= \int_1^2 x \cos \pi x \, dx$$

$$= \left[x \frac{\sin \pi x}{\pi} - \frac{1}{\pi} \int \sin \pi x \, dx \right]_1^2$$

$$= \frac{1}{\pi^2} [\cos \pi x]_1^2 = \frac{2}{\pi^2}$$

$$\Rightarrow I = \frac{20}{\pi^2}$$

$$\therefore \frac{\pi^2}{10} \int_{-10}^{+10} f(x) \cos \pi x \, dx = \frac{\pi^2}{10} \frac{40}{\pi^2} = 4$$

51.

0

Performing $C_1 \rightarrow C_1 + C_2 + C_3$

$$z \begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & z + \omega^2 & 1 \\ 1 & 1 & z + \omega \end{vmatrix}$$

Performing $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$

$$= z \begin{vmatrix} 1 & \omega & \omega^2 \\ 0 & z + \omega^2 - \omega & 1 - \omega^2 \\ 0 & 1 - \omega & z + \omega - \omega^2 \end{vmatrix}$$

$$= z \left[(z + \omega^2 - \omega)(z + \omega - \omega^2) - (1 + \omega^3 - \omega - \omega^2) \right]$$

$$= z \left[z^2 + z\omega - \omega^2 + z\omega^2 + \omega^3 - \omega - z\omega - \omega^2 + \omega^3 \right]$$

$$= z(z^2 + 2 - \omega - \omega^2 - 3)$$

$$= z^3$$

52.

1

$$S_k = \frac{(k-1)}{k!} = \frac{(k-1)k}{(k-1)k!} = \frac{k}{k!}$$

$$\therefore \sum_{k=1}^{100} (k^2 - 3k + 1) S_k = \left| \frac{k(k^2 - 3k + 1)}{k!} \right| = \left| \frac{k^3 - 3k^2 + k}{k!} \right|$$

$$= \left| \frac{k^2 - 3k + 1}{(k-1)!} \right| = \left| \frac{(k-1)^2}{(k-1)!} - \frac{k}{(k-1)!} \right|$$

$$= \left| \frac{k-1}{(k-2)!} - \frac{k}{(k-1)!} \right|$$

$$\begin{aligned}\therefore \left| (k^2 - 3k + 1)S_k \right| &= \frac{1}{0!} - \frac{2}{1!} & k = 2 \\ &= \frac{2}{1!} - \frac{3}{2!} & k = 3 \\ &= \frac{3}{2!} - \frac{4}{3!} & k = 4 \\ &= \frac{4}{3!} - \frac{5}{4!} & k = 5 \\ &= \frac{5}{4!} - \frac{6}{5!} & k = 6 \\ &= \frac{99}{98!} - \frac{100}{(99)!} & k = 100\end{aligned}$$

$$\sum_{k=1}^{100} (x^2 - 3k + 1)S_k = 1 - \frac{100}{99!}$$

$$\therefore \frac{100^2}{100!} + 1 - \frac{100^2}{(100)!} = 1$$

53. 3

$$(y + z)\cos 3\theta = xyz \sin 3\theta \quad \dots (1)$$

$$2z \cos 3\theta + 2y \sin 3\theta = xyz \sin 3\theta \quad \dots (2)$$

$$(y + 2z)\cos 3\theta + y \sin 3\theta = (xyz)\sin 3\theta \quad \dots (3)$$

$$\text{from (2) and (3)} \quad \frac{(xyz)\sin 3\theta}{2} + 2z \cos 3\theta = xyz \sin 3\theta$$

$$4z \cos 3\theta = xyz \sin 3\theta$$

$$= (y + z)\cos 3\theta$$

$$\text{from (1)} \quad 4z \cos 3\theta = x3z^2 \sin 3\theta \quad \boxed{y = 3z}$$

$$\Rightarrow xz \sin 3\theta = \frac{4}{3} \cos 3\theta$$

$$2z \cos 3\theta + 6z \sin 3\theta = xyz \cdot 3z^2 \sin 3\theta$$

$$2 \cos 3\theta + 6 \sin 3\theta = 4 \cos 3\theta$$

$$\Rightarrow 3 \sin 3\theta = \cos 3\theta$$

$$\tan 3\theta = \frac{1}{3} \quad \text{further } 0 < \theta < \pi$$

Hence three values of θ

54. 9

$$(Y - y) = \frac{dy}{dx}(X - x)$$

$$X = 0$$

$$Y = y - \frac{dy}{dx}x$$

$$y = -\frac{dy}{dx}x = x^3$$

$$\frac{dy}{dx} - \frac{y}{x} = -x^2$$

$$\text{I.F.} = \int_e -\frac{1}{x} dx = \frac{1}{x}$$

$$y \cdot \frac{1}{x} = -\int x^2 \frac{1}{x} dx + c$$

$$\frac{y}{x} = -\frac{x^2}{2} + c$$

$$x = y = 1 \quad c = \frac{3}{2}$$

$$x = -3$$

$$y = -3 \left(-\frac{(3)^2}{2} + \frac{3}{2} \right)$$

$$= -3 \left(\frac{-9+3}{2} \right)$$

$$= -3 \left(\frac{-6}{2} \right) = 9$$

55. 3

$$\tan \theta = \cot 5\theta$$

$$\tan \theta = \tan \left(\frac{\pi}{2} - 5\theta \right)$$

$$\theta = n\pi + \frac{\pi}{2} - 5\theta$$

$$\theta = \left(n + \frac{1}{2} \right) \frac{\pi}{6}$$

$$\text{for } n = 0, \theta = \frac{\pi}{12} \quad \text{for } n = 1, \theta = \frac{3\pi}{12}$$

$$n = 2, \theta = \frac{5\pi}{12} \quad n = -1, \theta = -\frac{\pi}{12}$$

$$n = -2, \theta = -\frac{3\pi}{12}$$

$$\sin 2\theta = \cos 4\theta$$

$$\cos 4\theta = \cos\left(\frac{\pi}{4} - 2\theta\right)$$

$$4\theta = 2n\pi \pm \left(\frac{\pi}{2}\right) \text{ or } \theta = \left(2n - \frac{1}{2}\right) \frac{\pi}{6}$$

$$\text{for } n = 0 \quad \theta = \frac{\pi}{4} \text{ or } -\frac{\pi}{12}$$

$$\text{for } n = 1 \quad \theta = \frac{3\pi}{4} \text{ or } \frac{3\pi}{12}$$

$$\text{for } n = -1 \quad \theta = -\frac{3\pi}{4} \text{ or } -\frac{5\pi}{12}$$

$$\text{for } n = 2, \quad \theta = \frac{5\pi}{12} \text{ or } \frac{7\pi}{12}$$

$$\text{for } n = -2, \quad \theta = -\frac{7\pi}{12} \text{ or } -\frac{9\pi}{12}$$

Now common solution are $-\frac{\pi}{12}, \frac{3\pi}{12}, \frac{5\pi}{12}$

56.

2

For maximum value

$\sin^2 \theta + 3 \sin \theta \cos \theta + 5 \cos^2 \theta$ should be minimum

$$= \sin^2 \theta + \cos^2 \theta + 4 \cos^2 \theta + \frac{3}{2} \sin 2\theta$$

$$= 1 + 4 \left(\frac{1 + \cos 2\theta}{2} \right) + \frac{3}{2} \sin 2\theta = 3 + 2 \cos 2\theta + \frac{3}{2} \sin 2\theta$$

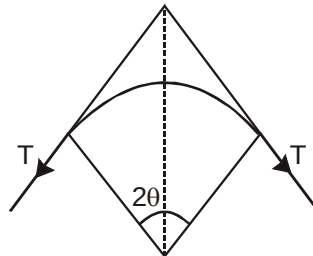
$$\text{Minimum value of } \frac{3}{2} \sin 2\theta + 2 \cos 2\theta = -\sqrt{\frac{9}{4} + 4} = -\frac{5}{2}$$

$$\therefore \text{ minimum value of denominator} = 2 - \frac{5}{2} = \frac{1}{2}$$

$$\therefore \text{ maximum value of expression} = \frac{1}{\frac{1}{2}} = 2$$

Physics

57. C



Consider a small segment of the circular wire

$$2T \sin \theta = B i (2KQ)$$

$$T = B i R \quad [\text{as } \theta \text{ is very small}]$$

$$T = \frac{B i L}{2\pi} \left[R = \frac{L}{2\pi} \right]$$

58. B

$$X_C = \frac{1}{\omega C} \quad \text{and} \quad z = \sqrt{R^2 + \frac{1}{(\omega C)^2}}$$

as ω increases

z decreases

Hence potential drop across the capacitor decreases and that across R increases.

Therefore the bulb glows brighter.

59. C

In the experiment one of the galvanometers has to serve as a voltmeter connected in parallel to R_T with a high resistance in series with itself. Moreover, the other galvanometer serve as an ammeter in series with R_T with a low resistance in parallel to galvanometer.

60. D

$$\text{For a bulb } P_{\text{rated}} = \frac{V_{\text{rated}}^2}{R}$$

This implies that the bulb with highest power rating should have least resistance

$$\text{Therefore } R_{40} > R_{60} > R_{100}$$

$$\frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$$

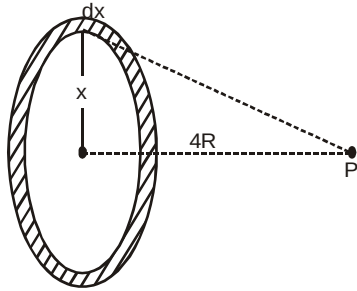
61. B

A real gas exhibits close to ideal behaviour at high temperature and low pressure.

62. C as $R = \rho \frac{l}{A}$

$$R = \frac{\rho(L)}{(Lt)} = \frac{\rho}{t}$$

63. A



Potential due to the differential ring at P

$$dV = \frac{-G(2\pi x dx)4r\sigma}{\sqrt{16R^2 + x^2}} \left[\sigma = \frac{M}{7\pi R^2} \right]$$

Substituting $16R^2 + x^2 = t$

$$= -G\pi r \frac{dt}{\sqrt{t}}$$

$$\therefore V_p = -2G\pi\sigma \left[\sqrt{t} \right]_{25R^2}^{32R^2}$$

$$= -\frac{2GM}{7R} [4\sqrt{2} - 5]$$

Now $V_\infty = 0$

hence work required $= [V_\infty - V_p] \times 1$

$$= \frac{2GM}{7R} (4\sqrt{2} - 5)$$

64. A The friction will vary as :

$$f = mg \sin \theta - P \text{ [till } P < mg \sin \theta]$$

$$f = 0 \text{ [when } P = mg \sin \theta]$$

$$f = -[mg \sin \theta - P] \text{ [when } P > mg \sin \theta]$$

overall, f follows the same straight line with negative slope.

65. A,C As the least count of the stopwatch is 1s, the error in measuring total time (40s) for 20 oscillations is 1s.

$$\therefore \text{percentage error} = \frac{1}{40} \times 100 = 2.5\%$$

$$\text{Now time period} = \frac{40}{20} = 2s$$

hence error in the measurement of T

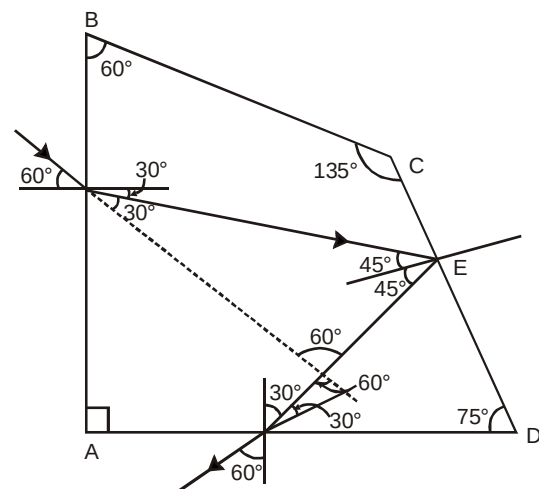
$$\Delta T = 2 \times \frac{2.5}{100} = 0.05s$$

$$\text{Now } \frac{\Delta T}{T} = \frac{1}{2} \left[\frac{\Delta l}{l} \pm \frac{\Delta g}{g} \right]$$

$$\therefore \frac{\Delta g}{g} = 2 \frac{\Delta T}{T} \left[\frac{\Delta l}{l} = 0 \right]$$

Hence percentage error in determination of g is 5%.

66. A,B,C



$$\text{At E } \sin i_c = \frac{1}{\mu} = \frac{1}{\sqrt{3}} = 0.57$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}} = 0.707$$

$$\sin i > \sin i_c \Rightarrow i > i_c$$

TIR occurs at E, (A) is correct

$$\sin 30^\circ = \frac{1}{2} < \sin i_c, \text{ ray cannot get totally internally}$$

reflected (B) is correct.

Angle between the incident and the emergent ray is 90° , as shown above by geometry.

67. A,D Electric field density is greater near Q_1 than Q_2 ,

$$\text{so } |Q_1| > |Q_2|$$

Charge Q_1 is positive and Q_2 is negative

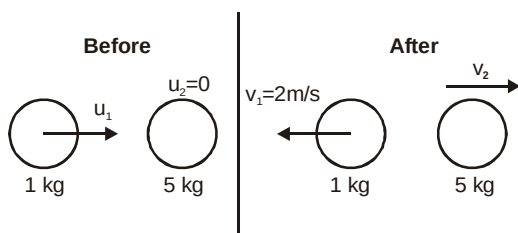
Electric field due to Q_1 and Q_2 will cancel out at a point to the right of Q_2

68. A,B As temperature is same at A and B, so the interval energies are same at A and B.

$$W_{AB} = nRT_0 \ln \frac{4V_0}{V_0} = P_0 V_0 \ln 4 \quad [\text{at A } P_0 V_0 = nRT_0]$$

As the graph of the process BC is not given to pass through the origin, so one cannot surely say any thing about the pressure and temperature at C.

69. A,C



Conserving the linear momentum

$$1 \times u_1 + 5 \times 0 = 1 \times (-2) + 5 \times v_2$$

$$u_1 = 5v_2 - 2 \quad \dots (1)$$

$$\text{coefficient of restitution } e = \frac{v_2 - (-2)}{u_1 - 0} = 1$$

$$u_1 = v_2 + 2 \quad \dots (2)$$

Solving (1) and (2)

$$u_1 = 3 \text{ m/s}, v_2 = 1 \text{ m/s}$$

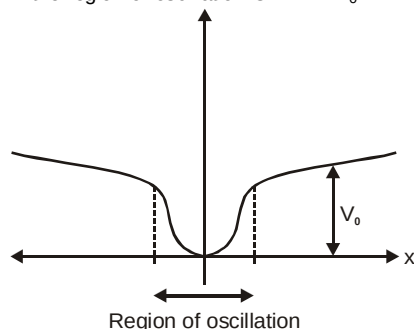
Momentum of the SOP = $3 \times 1 + 5 \times 0 = 3 \text{ kg-m/s}$

$$V_{cm} = \frac{3 \times 1 + 5 \times 0}{6} = \frac{1}{2} \text{ m/s}$$

$$\text{KE of CM} = \frac{1}{2} \times (1+5) \times \left(\frac{1}{2}\right)^2 = 0.75 \text{ J}$$

$$\text{KE of SOP} = \frac{1}{2} \times 1 \times 3^2 = 4.5 \text{ J}$$

70. C In the region of oscillation $0 < E < V_0$



71. B $V(x) = \alpha x^4 = (\alpha x^2)x^2 = Kx^2$ {for small amplitude}

$$K = \alpha x^2$$

$$T = 2\pi \sqrt{\frac{m}{K}}$$

$$T = 2\pi \sqrt{\frac{m}{\alpha x^2}}$$

$$\text{putting } x = A, T = \frac{2\pi}{A} \sqrt{\frac{m}{\alpha}}$$

$$T \propto \frac{1}{A} \sqrt{\frac{m}{\alpha}}$$

72. D For $|x| > X_0$
 $V(x) = V_0 = \text{constant}$

$$F = \frac{-dV(x)}{dx} = 0$$

$$\therefore \text{acceleration} = 0$$

73. A Since $B_2 > B_1 \Rightarrow T_C(B_2) < T_C(B_1)$ and beyond T_C , B doesn't affect R.

74. B As $0 < 5 \text{ Tesla} < 7.5 \text{ Tesla} \Rightarrow 75 \text{ K} < T_C(5) < 100 \text{ K}$ certainly.

75. 4

$$\text{for series case, } i_1 = \frac{2E}{2+R}, J_1 = \left(\frac{2E}{2+R}\right)^2 R$$

$$\text{for parallel case, } i_2 = \frac{E}{\frac{1}{2}+R}, J_2 = \left(\frac{E}{\frac{1}{2}+R}\right)^2 R$$

$$\text{given } J_1 = 2.25 J_2$$

$$\text{solving } R = 4\Omega$$

76. 9

$$\lambda_1 T_1 = \lambda_2 T_2$$

$$\frac{E_1}{E_2} = \frac{A_1 T_1^4}{A_2 T_2^4} = \frac{r_1^2 T_1^4}{r_2^2 T_2^4}$$

$$\text{solving we get } \frac{E_1}{E_2} = 9$$

77. 5

$$R = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi}$$

$$\phi = 90^\circ$$

$$R = 5$$

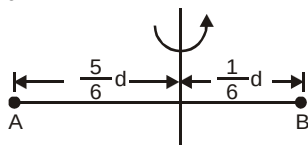
78. 4

Effective force constant of the wire $K_w = \frac{YA}{l}$

$$W = \sqrt{\frac{K_w}{m}}$$

solving $n = 4$

79. 6



$$L_{TE} = (2.2M_s) \left(\frac{5}{6}d \right)^2 \omega + 11M_s \left(\frac{1}{6}d \right)^2 \omega$$

$$L_B = (11M_s) \left(\frac{5}{6}d \right)^2 \omega$$

$$\frac{L_{TE}}{L_B} = 6$$

80. 3

$$g_p = \frac{\sqrt{6}}{11} g_e$$

$$c_p = \frac{2}{3} p_e$$

$$v_e = \sqrt{2gR} = \sqrt{\frac{2GM}{R}}$$

$$C = \frac{M}{\frac{4}{3}\pi R^3}$$

$$R = \left(\frac{3M}{4\pi e} \right)^{1/3}$$

$$\text{Also } g = \frac{GM}{R^2}$$

Using above we get $v_{ep} = 3\text{KM/sec}$

81. 8

Let m = mass of ice (in gm)

heat required to raise the temperature of ice from

$$-5^\circ\text{C to } 0^\circ\text{C is } \frac{m}{1000} \times 2100 \times 5 = H_1$$

heat required to melt 1 gm of ice = 3.36×10^5

$$\times \frac{1}{1000} = \text{Hz}$$

given $H_1 + H_2 = 420$

solving $m = 8 \text{ gm}$

82. 7

For reflected frequency from car

$$f = \left(\frac{V + V_c}{V - V_c} \right) f_0 \quad (V_c : \text{velocity of the car})$$

by differentiating w.r.t. V_c

$$\frac{df}{dV_c} = \frac{2V}{(V - V_c)^2} f_0$$

$$\text{Thus } dV_c = df \frac{(V - V_c)^2}{2V f_0}$$

Given $V \gg V_c$

$$\text{so } dV_c = \frac{V}{2} \frac{df}{f_0} \approx 7$$

83. 6

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

When $u_1 = 25 \text{ cm}$, $v_1 = 100 \text{ cm}$, $m_1 = 4$

when $u_2 = 50 \text{ cm}$, $v_2 = \frac{100}{3} \text{ cm}$, $m_2 = \frac{2}{3}$

$$\frac{m_1}{m_2} = 6$$

84. 2

$$KE_p = qv$$

$$KE_\alpha = 2qv$$

$$\Rightarrow \frac{\lambda_p}{\lambda_\alpha} = \frac{h/\sqrt{2K_p m_p}}{h/\sqrt{2K_\alpha m_\alpha}} = \sqrt{\frac{K_\alpha m_\alpha}{K_p m_p}} = \sqrt{\frac{2qv \cdot 4m_p}{qv m_p}} = 2\sqrt{2} \approx 3$$