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JEE-MAIN 2013

Test Booklet Code

P

PAPER - 1 : PHYSICS, CHEMISTRY & MATHEMATICS

Do not open this Test Booklet until you are asked to do so.

Read carefully the Instructions on the Back Cover of this Test Booklet.

Important Instructions :

1. Immediately fill in the particulars on this page of the Test Booklet with Blue/Black Ball Point Pen. *Use of pencil is strictly prohibited.*
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of **90** questions. The maximum marks are **360.**
5. There are ***three*** parts in the question paper A, B, C consisting of, **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
6. *Candidates will be awarded marks as stated above in instruction No. 5 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
7. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.
8. Use ***Blue/Black Ball Point Pen only*** for writting particulars/ marking responses on **Side−1** and **Side−2** of the Answer Sheet. ***Use of pencil is strictly prohibited***.
9. No candidates is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc., except the Admit Card inside the examination hall/room.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in **3** pages (Pages **21−23**) at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room / Hall. ***However, the candidates are allowed to take away this Test Booklet with them.***
12. The CODE for this Booklet is **P**. Make sure that the CODE printed on **Side−2** of the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the invigilator for replacement of both the Test Booklet and the Answer Sheet.
13. **Do not fold or make any stray marks on the Answer Sheet.**

Name of the Candidate (in Capital letters) : ____________________

Roll Number : in figures | | | | | | | | |

: in words ____________________

Examination Centre Number : | | | | | | |

Name of Examination Centre (in Capital letters) : ____________________

Candidate's Signature : ____________________ Invigilator's Signature : ____________________

READ THE FOLLOWING INSTRUCTIONS CAREFULLY :

1. The candidates should fill in the required particulars on the Test Booklet and Answer Sheet (***Side-1***) with ***Blue/Black Ball Point Pen.***

2. For writing/marking particulars on ***Side-2*** of the Answer Sheet, use ***Blue/Black Ball Point Pen only.***

3. The candidates should not write their Roll Numbers anywhere else (except in the specified space) on the Test Booklet/Answer Sheet.

4. Out of the four options given for each question, only one option is the correct answer.

5. For each ***incorrect response, one-fourth (¼)*** of the total marks allotted to the question would be deducted from the total score. ***No deduction*** from the total score, however, will be made ***if no response*** is indicated for an item in the Answer Sheet.

6. Handle the Test Booklet and Answer Sheet with care, *as under no circumstances (except for discrepancy in Test Booklet Code and Answer Sheet Code), will another set be provided.*

7. The candidates are not allowed to do any rough work or writing work on the Answer Sheet. All calculations/writing work are to be done in the space provided for this purpose in the Test Booklet itself, marked 'Space for Rough Work'. This space is given at the bottom of each page and in 3 pages (Pages 21 – 23) at the end of the booklet.

8. On completion of the test, the candidates must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. **However, the candidates are allowed to take away this Test Booklet with them.**

9. Each candidate must show on demand his/her Admit Card to the Invigilator.

10. No candidate, without special permission of the Superintendent or Invigilator, should leave his/her seat.

11. The candidates should not leave the Examination Hall without handing over their Answer Sheet to the Invigilator on duty and sign the Attendance Sheet again. Cases where a candidate has not signed the Attendance Sheet a second time will be deemed not to have handed over the Answer Sheet and dealt with as an unfair means case. **The candidates are also required to put their left hand THUMB impression in the space provided in the Attendance Sheet.**

12. Use of Electronic/Manual Calculator and any Electronic Item like mobile phone, pager etc. is prohibited.

13. The candidates are governed by all Rules and Regulations of the JAB/Board with regard to their conduct in the Examination Hall. All cases of unfair means will be dealt with as per Rules and Regulations of the JAB/Board.

14. No part of the Test Booklet and Answer Sheet shall be detached under any circumstances.

15. **Candidates are not allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, electronic device or any other material except the Admit Card inside the examination hall/room.**

Questions and Solutions

PART- A : PHYSICS

1. A uniform cylinder of length L and mass M having cross - sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is :

(1) $\frac{Mg}{k}$ (2) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$ (3) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$ (4) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$

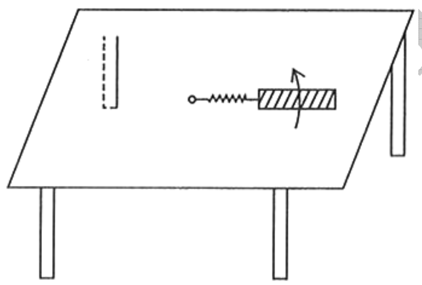
(Here, k is spring constant)

1. (3)

$$\left(Kx_0+\frac{LA}{2}\sigma g\right)=mg$$

$$\therefore\ x_0=\frac{Mg}{k}\left(1-\frac{LA\alpha}{2M}\right)$$

2. A metallic rod of length 'ℓ' is tied to a string of length 2ℓ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field 'B' in the region, the e.m.f. induced across the ends of the rod is :

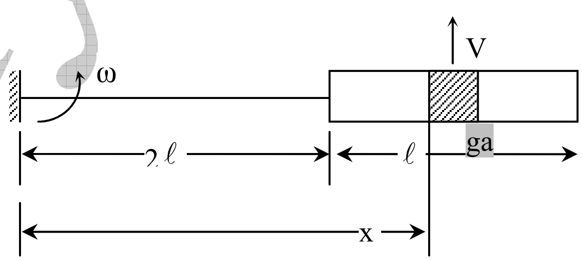


(1) $\frac{2B\omega\ell^2}{2}$ (2) $\frac{3B\omega\ell^2}{2}$ (3) $\frac{4B\omega\ell^2}{2}$ (4) $\frac{5B\omega\ell^2}{2}$

2. (4)

$$V=\int\left(\vec{v}\times\vec{B}\right)\cdot\vec{d\ell}=\int_{2\ell}^{3\ell}x\,\omega B\,dx$$

$$V=\frac{5B\omega\ell^2}{2}$$



3. This question has statement I and Statement II. Of the four choice given after the statements, choose the one that best describes the two statements.

Statement – I : A Point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as

$f\left(\frac{1}{2}mv^2\right)$ then $f=\left(\frac{m}{M+m}\right)$.

Statement – II : Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(1) Statement - I is true, Statement - II is true, statement - II is a **correct** explanation of Statement - I
(2) Statement - I is true, Statement - II is true, statement - II is **not** a correct explanation of Statement – I
(3) Statement - I is true, Statement - II is false
(4) Statement – I is false, Statement – II is true

3. (4)

Maximum energy loss when inelastic collision takes place

$mv = (m + M)\, v'$

$$v' = \frac{m}{m+M} v$$

$$k_i = \frac{1}{2} mv^2$$

$$k_f = \frac{1}{2}(m+M)v'^2 = \frac{1}{2}(m+M)\frac{m^2v^2}{(m+M)^2} = \frac{1}{2}mv^2\left(\frac{m}{M+m}\right)$$

Loss of energy $= k_i - k_f = \frac{1}{2}mv^2\left[1 - \frac{m}{M+m}\right] = \frac{M}{M+m} \times \frac{1}{2}mv^2$

4. Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = Time and A = electric current, then :

(1) $[\varepsilon_0] = [M^{-1} L^{-3} T^2 A]$ (2) $[\varepsilon_0] = [M^{-1} L^{-3} T^4 A^2]$
(3) $[\varepsilon_0] = [M^{-1} L^2 T^{-1} A^{-2}]$ (4) $[\varepsilon_0] = [M^{-1} L^2 T^{-1} A]$

4. (2)

$$[\varepsilon_0] = \left[\frac{C^2}{N - m_2}\right] = \frac{A^2T^2}{MLT^{-2}L^2}$$

$$[\varepsilon_0] = M^{-1}L^{-3}T^4A^2$$

5. A Projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s^2, the equation of its trajectory is :

(1) $y = x - 5x^2$ (2) $y = 2x - 5x^2$ (3) $4y = 2x - 5x^2$ (4) $4y = 2x - 25x^2$

5. (2)

$u = \sqrt{2^2 + 1^2} = \sqrt{5}$

$\theta = \tan^{-1} 2$

Equation : $y = x \tan\theta - \dfrac{gx^2}{2u^2 \cos^2\theta}$

$$y = 2x - \frac{10x^2}{2 \times 5 \times \frac{1}{5}}$$

$$y = 2x - 5x^2$$

6. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s it will decrease to α times its original magnitude, where α equals :

(1) 0.7 (2) 0.81 (3) 0.729 (4) 0.6

6. (3)

$A = A_o\, e^{-kt}$

$0.9A_o = A_o e^{-kt}$

$-kt = \ell n\ (0.9)$

$-5\,k = \ell n\,(0.9) \Rightarrow -15\,k = 3\,\ell n\ (0.9)$

$A = A_o e^{-15k} = A_o\, e^{-\ell n(0.9)^3}$

$= (0.9)^3 A_o = 0.729\, A_o$

7. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then :
(1) $5C_1 = 3C_2$ (2) $3C_1 = 5C_2$ (3) $3C_1 + 5C_2 = 0$ (4) $9C_1 = 4C_2$

7. (2)
Potential = 0 on connecting them together i.e. Q = 0
i.e. $C_1V_1 = C_2V_2$ [capacitance is positive but they are connected with opposite polarity]
$120\ C_1 = 200\ C_2$
$3C_1 = 5C_2$

8. A sonometer wire of length 1.5 m is made of steel. The tension in it produce an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg / m^3 and 2.2×10^{11} N/m^2 respectively ?
(1) 188.5 Hz (2) 178.2 Hz (3) 200.5 Hz (4) 770 Hz

8. (2)

$$\text{Stress} = Y \times \frac{1}{100} = 2.2 \times 10^9$$

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\frac{M}{L}}} = \sqrt{\frac{\frac{T}{L^2}}{\frac{M}{L^3}}} = \sqrt{\frac{\text{stress}}{\text{density}}} = \sqrt{\frac{2.2\times10^9}{7.7\times10^3}} = \sqrt{\frac{2}{7}} \times 10^3$$

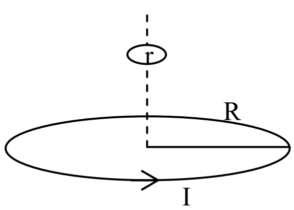
$$V = \frac{V}{2\ell} = \sqrt{\frac{2\times10^3}{72\times1.5}} = 178.2 \text{ Hz}$$

9. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is:
(1) 9.1×10^{-11} weber (2) 6×10^{-11} weber
(3) 3.3×10^{-11} weber (4) 6.6×10^{-9} weber

9. (1)
First we find the mutual inductance of the assembly.
Let current I flow through the larger loop.

The strength of induction at the smaller loop = $\dfrac{\mu_0 I R^2}{2\left(R^2+d^2\right)^{3/2}}$

$\therefore$ Flux through smaller loop = $\dfrac{\mu_0 \pi I R^2 r^2}{2\left(R^2+d^2\right)^{3/2}}$



$\therefore$ Mutual inductance = $\dfrac{\mu_0 R^2 r^2}{2\left(R^2+d^2\right)^{3/2}}$

$\therefore$ Flux linked through coil = $\dfrac{\mu_0 \pi R^2 r^2}{2\left(R^2+d^2\right)^{3/2}} \cdot i$

$$= \frac{4\pi\times10^{-7}\times\pi\times4\times10^{-2}\times9\times10^{-6}\times2}{2\left(4\times10^{-2}+2.25\times10^{-2}\right)^{3/2}}$$

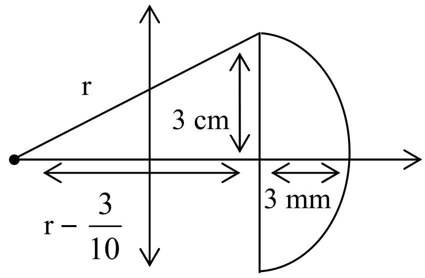
$$= \frac{16}{2}\times\frac{10\times18}{15.625}\times\left(10^{+3-15}\right) = 9.1 \times 10^{-11} \text{ Weber}$$

10. Diameter of a plano - convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is :

(1) 15 cm (2) 20 cm (3) 30 cm (4) 10 cm

10. (3)

$$\left(r - \frac{3}{10}\right)^2 + 3^2 = r^2$$



$$r^2 - \frac{6r}{10} + \frac{9}{10} + 9 = r^2$$

$$-\frac{6r}{10} + \frac{909}{100} = 0$$

r = 15.15 m

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$R_{plane} = \infty$

R_{convex} = 15 cm (approx.)

$$\mu = \frac{3\times10^8}{2\times10^8} = \frac{3}{2}$$

$$\frac{1}{f} = \frac{1}{2} \times \frac{1}{15}$$

f = 30 cm

11. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R ?

(1) $\frac{5GmM}{6R}$ (2) $\frac{2GmM}{3R}$ (3) $\frac{GmM}{2R}$ (4) $\frac{GmM}{3R}$

11. (1)

The kinetic energy at altitude 2R = $\frac{GMm}{6R}$

The gravitational potential energy at altitude 2R = $-\frac{GMm}{3R}$

$\therefore$ Total energy = $k\varepsilon + PE = -\frac{GMm}{6R}$

Potential energy at the surface is $-\frac{GMm}{R}$

$\therefore$ Req. kinetic energy = $\frac{GMm}{R} - \frac{GMm}{6R} = \frac{5GMm}{6R}$

12. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 pico farad in parallel with a load resistance 100 kilo ohm. Find the maximum modulated frequency which could be detected by it.

(1) 10.62 MHz (2) 10.62 kHz (3) 5.31 MHz (4) 5.31 kHz

12. (4)

Cut−off frequency is given by

$$f_c = \frac{1}{2\pi RC}$$

$$= \frac{1}{2\times3.14\times10^5\times250\times10^{-12}}$$

$$= 6.37 \text{ KHZ.}$$

Modulation factor = $\dfrac{\sqrt{1-m^2}}{m}$ (n = 0.6)

$= \dfrac{4}{3}$

So, maximum frequency that can be detected should be less than modulation factor $\times f_c$

i.e.,Less than $6.37\times\dfrac{4}{3} = 8.5\,\text{KH}.$

$\therefore$ Option (4) is correct.

13. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is :

(1) I_0 (2) $I_0/2$ (3) $I_0/4$ (4) $I_0/8$

13. (3)

Intensity of light is halved upon passage through first polaroid.

Using Malus' Law : $I = \dfrac{I_0}{2}\cos^2\theta$

θ = 45° (The angle between the polarization axes of the polaroids)

$\therefore\ I_0 = \dfrac{I_0}{2}\left(\dfrac{1}{\sqrt{2}}\right)^2 = \dfrac{I_0}{4}$

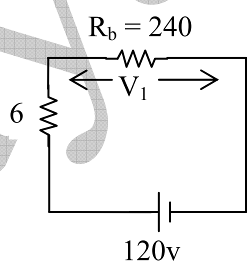
14. The supply voltage to a room is 120 V. The resistance of the lead wires is 6Ω. A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb ?

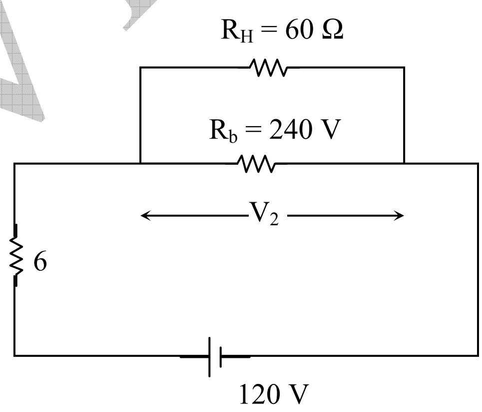
(1) zero Volt (2) 2.9 Volt (3) 13.3 Volt (4) 10.04 Volt

14. (no option matches)

$R_b = \dfrac{(120)^2}{60} = 240\ \Omega,\ R_H = \dfrac{240}{4} = 60\ \Omega$

$V_1 = 120\ \dfrac{240}{246} = 120.\dfrac{40}{41}\,V$





$$V_2 = 120\ \frac{(R_b \parallel R_H)}{(R_b \parallel R_H)+6} = 120\ \frac{48}{54} = 120\ \frac{8}{9}\ \text{V}$$

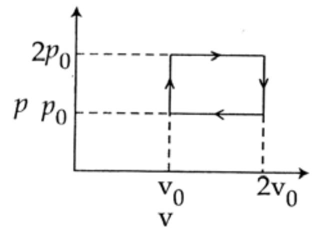
Loss in potential $= V_1 - V_2$

$= 120\left(\frac{40}{41} - \frac{8}{9}\right)$

$= 10.40$ V.

(no option matches)

15. The above p-v diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat, extracted from the source in a single cycle is :



(1) p_0v_0 (2) $\left(\frac{13}{2}\right)p_0v_0$

(3) $\left(\frac{11}{2}\right)p_0v_0$ (4) $4p_0v_0$

15. (2)

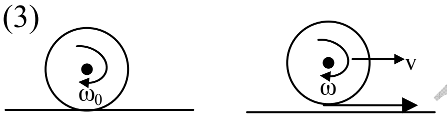
Heat extracted from the source

$$= \frac{C_V}{R}(P_2V_2 - P_1V_1) + \frac{C_p}{R}(P_3V_3 - P_2V_2) = \frac{3P_0V_0}{2} + 5P_0V_0 = \frac{13}{2}P_0V_0$$

16. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip ?

(1) $\frac{r\omega_0}{4}$ (2) $\frac{r\omega_0}{3}$ (3) $\frac{r\omega_0}{2}$ (4) $r\omega_0$

16. (3)



COAM about the contact point

$I\omega_0 = I\omega + mvr$

$m r^2 \omega_0 = mr^2\omega + mvr = 2mvr$

$\therefore\ v = \frac{r\omega_0}{2}$

17. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency :

(1) $\frac{1}{2\pi}\frac{A_\gamma P_0}{V_0 M}$ (2) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$ (3) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A_\gamma P_0}}$

17. (3)

As adiabatic process

$PV^\gamma = \text{Constant}$

$$\frac{dp}{p} + \gamma\frac{dv}{v} = 0$$

$$dp = -\frac{P\gamma}{V}dv = -\frac{P\gamma}{V}(A)\,x \qquad \text{(x is small displacement)}$$

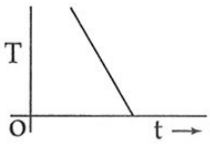
$$F = (dp)\,A = -\frac{P_0\gamma A^2}{V_0}x$$

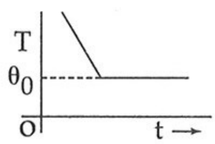
$$a = -\frac{P_0\gamma A^2}{mv_0}x$$

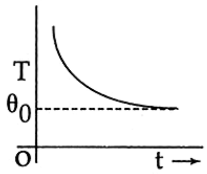
$$\omega = \sqrt{\frac{P_0\gamma A^2}{mv_0}}$$

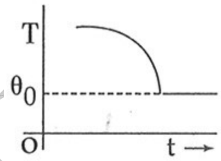
$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{P_0\gamma A^2}{mv_0}}$$

18. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0, the graph between the temperature T of the metal and time t will be closed to :

(1) 

(2) 

(3) 

(4) 

18. (3)

Since, by Newton's law of cooling, the rate of change of temperature is proportional to the difference in temperature between the object and surrounding, as the temperature of the piece of metal approaches θ_0, the rate of change of temperature will approach zero.

19. This question has statement I and Statement II. Of the four choices given after the Statements, choose the one that best describes the two statements.

Statement – I : Higher the range, greater is the resistance of ammeter.

Statement – II : To increase the range of ammeter, additional shunt needs to be used across it.

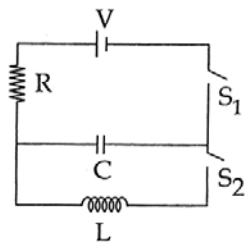
(1) Statement - I is true, Statements - II is true, Statement - II is the **correct** explanation of Statements - I
(2) Statement - I is true, Statement - II is true, Statement - II is **not** the correct explanation of Statement - I.
(3) Statement - I is true, Statement - II is false.
(4) Statement - I is false, Statement - II is true.

19. (4)

$i_1 = \left(\frac{r}{R_2 + r}\right)i$; i_1 = Maximum allowed current through ammeter coil.

r = resistance of the shunt. To increase the range i, r has to be reduced so effective resistance of ammeter decreases.

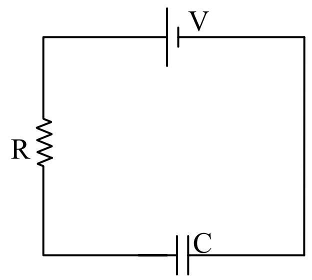
20. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and τ = RC is Capacitive time constant). Which of the following statement is correct?



(1) Work done by the battery is half of the energy dissipated in the resistor
(2) At $t = \tau$, $q = CV / 2$
(3) At $t = 2\tau$, $q = CV(1 - e^{-2})$
(4) At $t = \frac{\tau}{2}$, $q = CV(1 - e^{-1})$

20. (3)

Case – 1



Its normal RC circuit

$W_{bat} = CV^2$

$U = \frac{1}{2}CV^2$

$H = W_{bat} - U = \frac{1}{2}CV^2$

$\Rightarrow (W_{ba}) = 2(H)$

So (1) is wrong.

$q = CV\left(1-e^{-t/\alpha}\right) \Rightarrow$ At $t = \alpha$,

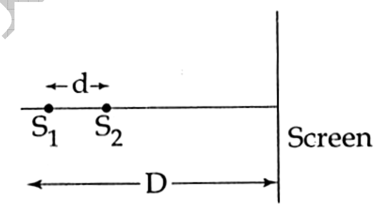
at $t = 2\alpha$, $q = cv(1 - e^2)$ 3 is correct

$q = CV\left(1-e^{-1}\right)$ 2 is wrong.

At $t = \frac{\alpha}{2}$

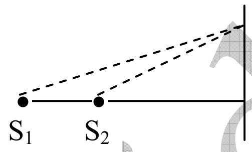
$q = CV\left(1-e^{-1/2}\right) \Rightarrow$ (4) is wrong.

21. Two coherent point sources S_1 and S_2 are separated by a small distance 'd' as shown. The fringes obtained on the screen will be :



(1) points
(2) straight lines
(3) semi–circles
(4) concentric circles

21. (4)



So, on a circle, path difference will be same locus of points having same path difference will be a circle, so fringes will make concentric circles.

22. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is :

(1) $\frac{3V}{m}$ (2) $\frac{6V}{m}$ (3) $\frac{9V}{m}$ (4) $\frac{12V}{m}$

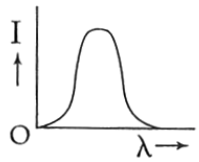
22. (2)

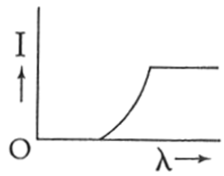
$\frac{E_o}{B_o} = c \quad \Rightarrow E_o = cB_o$

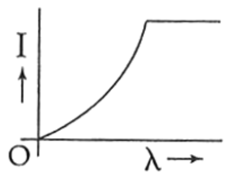
$= (20 \times 10^{-9}\ T)\ 3 \times 10^8$

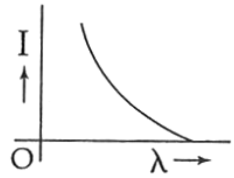
$= 6$ V/m.

23. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows :

(1) 

(2) 

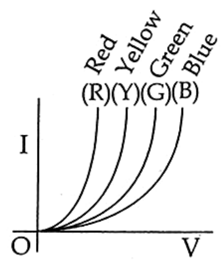
(3) 

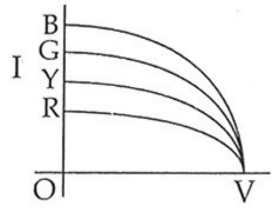
(4) 

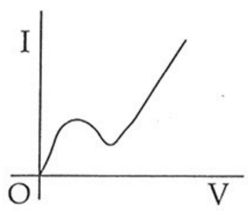
23. (4)

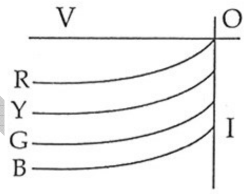
As λ will increase
k_{max} will decrease so current should decrease and finally fall to zero when λ_0 is achieved.

24. The I − V characteristic of an LED is :

(1) 

(2) 

(3) 

(4) 

24. (1)

25. Assume that a drop of liquid evaporates by decrease in its surface energy. So that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, density of liquid is ρ and L is its latent heat of vaporization.

(1) $\frac{\rho L}{T}$ (2) $\sqrt{\frac{T}{\rho L}}$ (3) $\frac{T}{\rho L}$ (4) $\frac{2T}{\rho L}$

25. (4)

Surface energy of the drop $= \left(4\pi R^2\right)T$

Let (δm) mass evaporate.

$(\delta m)L = 8\pi R.\ \delta RT$

$$m = \rho\frac{4}{3}\pi R^3 \quad \Rightarrow \frac{\delta m}{\delta R} = 4\pi R^2\rho$$

$$\therefore \quad \rho 4\pi R^2 L = 8\pi R.T$$

$$\Rightarrow R = \frac{2T}{L\rho}$$

26. In a hydrogen like atom electron makes transition from an energy level with quantum number n to another with quantum number (n − 1). If n >> 1, the frequency of radiation emitted is proportional to :

(1) $\frac{1}{n}$ (2) $\frac{1}{n^2}$ (3) $\frac{1}{n^{3/2}}$ (4) $\frac{1}{n^3}$

26. (4)

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)}$$

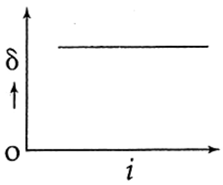
$$\frac{C}{\lambda} = RC\left(\frac{n^2-(n-1)^2}{(n-1)n^2}\right)$$

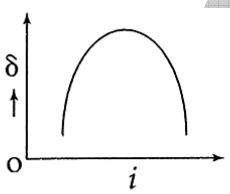
$$\nu = RC\left(\frac{2n-1}{(n-1)^2 n^2}\right) \approx RC\left(\frac{2n}{n^4}\right)$$

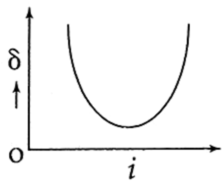
$$\boxed{\nu = \frac{2RC}{n^3}}$$

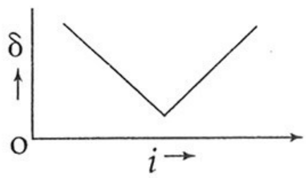
$$\boxed{\nu \propto \frac{1}{n^3}}$$

27. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by :

(1) 

(2) 

(3) 

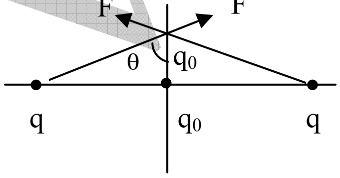
(4) 

27. (3)
Factual

28. Two charges, each equal to q, are kept at x = − a and x = a on the x−axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement (y << a) along the y−axis, the net force acting on the particle is proportional to :

(1) y (2) − y (3) $\frac{1}{y}$ (4) $-\frac{1}{y}$

28. (1)



$$F_{net} = \frac{1}{4\pi\varepsilon_0} \times \frac{2qq_0}{y^2+a^2}\cos\theta$$

$$= \frac{2qq_0\,y}{4\pi\varepsilon_0\left(y^2+a^2\right)^{3/2}} = \frac{2qq_0}{4\pi\varepsilon_0} \times \frac{y}{a^3} \text{ as } (y << a)$$

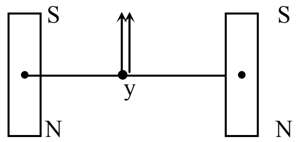
$\therefore F \propto y$

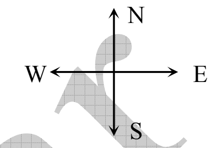
29. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2 respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid - point O of the line joining their centre is close to
(Horizontal component of earth′s magnetic induction is 3.6×10^{-5} Wb / m^2)

(1) 3.6×10^{-5} Wb/m^2 (2) 2.56×10^{-4} Wb/m^2
(3) 3.50×10^{-4} Wb/m^2 (4) 5.80×10^{-4} Wb/m^2

29. (2)

$$B = \frac{\mu_0}{4\pi}\frac{\mu}{\left[\left(\frac{L}{2}\right)^2 + y^2\right]^{3/2}}$$





$$B_1 = 10^{-7} \times \frac{1.2}{\left((0.1)^2 + (0.005)^2\right)^{3/2}}$$

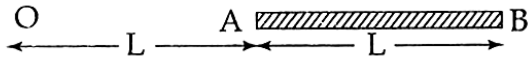
$$= 10^{-7}\frac{(1.2)}{10^{-3}} = 1.2 \times 10^{-4}\ \text{Wb / m}^2 \ \text{(South to North)}$$

$B_2 = 1 \times 10^{-4}$ Wb / m^2 (south to North)

$B_H = 3.6 \times 10^{-5}$ Wb / m^2 (South to North)

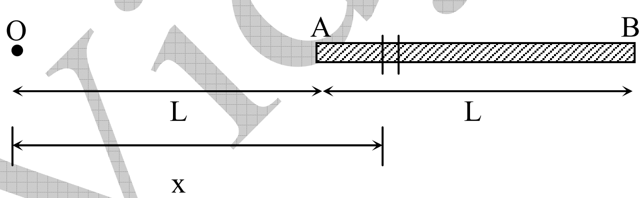
$B_{net} = (0.36 + 1 + 1.2) \times 10^{-4} = 2.56 \times 10^{-4}$ Wb/m^2

30. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at a distance L from the end A is :



(1) $\frac{Q}{8\pi\varepsilon_0 L}$ (2) $\frac{3Q}{4\pi\varepsilon_0 L}$ (3) $\frac{Q}{4\pi\varepsilon_0 L\,\ell n2}$ (4) $\frac{Q\,\ell n2}{4\pi\varepsilon_0 L}$

30. (4)



$$V = \int \frac{K\,dQ}{x}$$

$$= K\int\left(\frac{Q}{L}\right)\frac{1}{x}dx = \frac{KQ}{L}\int_L^{2L}\frac{1}{x}dx$$

$$= \frac{KQ}{L}(\ell n\,x)_L^{2L} = \frac{KQ}{L}\ell n\,2$$

$$= \frac{Q\,\ell n\,2}{4\pi\varepsilon_0 L}$$

PART- B : CHEMISTRY

31. Which of the following complex species is not expected to exhibit optical isomerism ?
(1) $[Co(en)_3]^{3+}$ (2) $[Co(en)_2Cl_2]^{+}$ (3) $[Co(NH_3)_3Cl_3]$ (4) $[Co(en)(NH_3)_2Cl_2]^{+}$

31. (3)
$[Co(NH_3)_3Cl_3]$ will not exhibit optical isomerism due to presence of plane of symmetry.

32. Which one of the following molecules is expected to exhibit diamagnetic behaviour ?
(1) C_2 (2) N_2 (3) O_2 (4) S_2

32. (3)
O_2 is expected to diamagnetic in nature but actually it is paramagnetic.

33. A solution of (–)–1 – chloro – 1– phenylethane in toluene racemises slowly in the presence of a small amount of $SbCl_5$,due to the formation of :
(1) carbanion (2) carbene (3) carbocation (4) free radical

33. (3)
Racemises slowly due to formation of intermediate carbocation.

34. Given :

$E^0_{Cr^{3+}/Cr} = -0.74\text{ V}$; $E^0_{MnO_4^-/Mn^{2+}} = 1.51\text{ V}$

$E^0_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33\text{ V}$; $E^0_{Cl/Cl^-} = 1.36\text{ V}$

Based on the data given above, strongest oxidising agent will be :
(1) Cl^- (2) Cr^{3+} (3) Mn^{2+} (4) MnO_4^-

34. (4)
MnO_4^- is strongest oxidizing agent.

35. A piston filled with 0.04 mol of an ideal gas expands reversibly from 50.0 mL to 375 mL at a constant temperature of 37.0° C. As it does so, it absorbs 208 J of heat. The values of q and w for the process will be :
(R = 8.314 J / mol K) (ln 7.5 = 2.01)
(1) q = +208 J , w = –208 J (2) q = –208 J, w = – 208 J
(3) q = –208 J , w = +208 J (4) q = +208 J , w = +208 J

35. (1)
q = + 208 J (as it absorb heat)

$$w_{Rew} = -\,2.303\text{ nRT }\log_{10}\left(\frac{v_2}{v_1}\right)$$

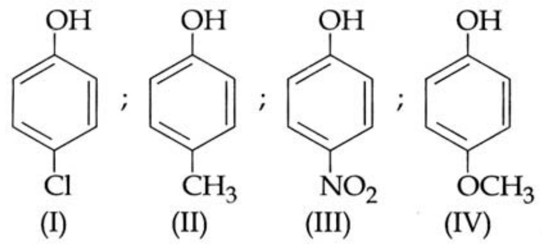
$$= -\,2.303 \times (0.04) \times 8.314 \times (310)\ \log_{10}\left(\frac{375}{50}\right) = -\,208\text{ J}$$

36. The molarity of a solution obtained by mixing 750 mL of 0.5(M)HCl with 250 mL of 2(M)HCl will be :
(1) 0.875 M (2) 1.00 M (3) 1.75 M (4) 0.975 M

36. (1)

$$(\text{Molarity})_{\text{Mix}} = \frac{0.5\times750+250\times2}{1000} = 0.875\,\text{M}$$

37. Arrange the following compounds in order of decreasing acidity :



(1) II > IV > I > III (2) I > II > III > IV
(3) III > I > II > IV (4) IV > III > I > II

37. (3)
Acidic nature order (III) > (I) > (II) > (IV)

38. For a gaseous state, if most probable speed is denoted by C^*, average speed by $\overline{C}$ and mean square speed by C, then for a large number of molecules the ratios of these speeds are :
(1) $C^* : \overline{C} : C = 1.225 : 1.128 : 1$ (2) $C^* : \overline{C} : C = 1.128 : 1.225 : 1$
(3) $C^* : \overline{C} : C = 1 : 1.128 : 1.225$ (4) $C^* : \overline{C} : C = 1 : 1.225 : 1.128$

38. (3)

$$\overset{*}{C} : \overline{C} : C = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{3RT}{M}}$$

$$\sqrt{2} : \sqrt{\frac{8}{3.14}} = \sqrt{3}$$

1 : 1.128 : 1.225

39. The rate of a reaction doubles when its temperature changes from 300 K to 310 K. Activation of such a reaction will be :
($R = 8.314\ JK^{-1}$ and log 2 = 0.301)
(1) $53.6\ kJ\ mol^{-1}$ (2) $48.6\ kJ\ mol^{-1}$ (3) $58.5\ kJ\ mol^{-1}$ (4) $60.5\ kJ\ mol^{-1}$

39. (1)

$$0.3010 = \frac{Ea}{2.303R}\left[\frac{T_2 - T_1}{T_1 T_2}\right]$$

$$\Rightarrow\ 0.3010 = \frac{Ea}{2.303 \times 8.314 \times 10^{-3}}\left(\frac{310-300}{310 \times 300}\right)$$

$E_a = 53.6$ kJ mol

40. A compound with molecular mass 180 is acylated with CH_3COCl to get a compound with molecular mass 390. The number of amino groups present per molecule of the former compound is :
(1) 2 (2) 5 (3) 4 (4) 6

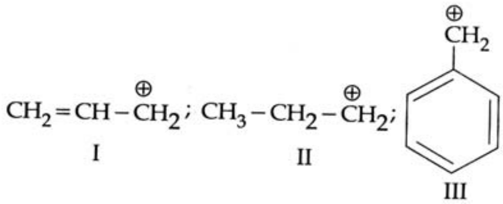
40. (2)

No. of amino group $= \frac{390-180}{42} = 5$

41. Which of the following arrangements does not represent the correct order of the property stated against it ?
(1) $V^{2+} < Cr^{2+} < Mn^{2+} < Fe^{2+}$: paramagnetic behaviour
(2) $Ni^{2+} < Co^{2+} < Fe^{2+} < Mn^{2+}$: ionic size
(3) $Co^{3+} < Fe^{3+} < Cr^{3+} < Sc^{3+}$: stability in aqueous solution
(4) Sc < Ti < Cr < Mn : number of oxidation states

41. (1)
Number of unpaired e^- in Fe^{2+} is less than Mn^{+2}.

42. The order of stability of the following carbocations :



is :
(1) III > II > I (2) II > III > I (3) I > II > III (4) III > I > II

42. (4)
On the basis of number of resonating structure (III) > (I) > (II)

43. Consider the following reaction :

$$xMnO_4^- + yC_2O_4^{2-} + zH^+ \rightarrow xMn^{2+} + 2yCO_2 + \frac{z}{2}H_2O$$

The values of x, y and z in the reaction are, respectively :
(1) 5, 2 and 16 (2) 2, 5 and 8 (3) 2, 5 and 16 (4) 5, 2 and 8

43. (3)

$$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \rightarrow 2Mn^{+2} + 10CO_2 + 8H_2O$$

44. Which of the following is the wrong statement ?
(1) ONCl and ONO^- are not isoelectronic (2) O_3 molecule is bent
(3) Ozone is violet – black in solid state (4) Ozone is diamagnetic gas.

44. (1)
ONCl and $ONCl^-$ are isoelectronic in nature.

45. A gaseous hydrocarbon gives upon combustion 0.72 g. of water and 3.08 g of CO_2. The empirical formula of the hydrocarbon is :
(1) C_2H_4 (2) C_3H_4 (3) C_6H_5 (4) C_7H_8

45. (4)

$$C_xH_y + \left(x + \frac{y}{4}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$$

$$x : \frac{y}{2} = \frac{3.08}{44} : \frac{0.72}{18}$$

$$\Rightarrow \frac{2x}{y} = \frac{3.08}{44} \times \frac{18}{0.72} = \frac{7}{4} \quad \text{i.e. } \frac{x}{y} = \frac{7}{8}$$

46. In which of the following pairs of molecules / ions , both the species are not likely to exist ?
(1) H_2^+ , He_2^{2-} (2) H_2^- , He_2^{2-} (3) H_2^{2+} , He_2 (4) H_2^- , He_2^{2+}

46. (3)
H_2^{2+} and H_2 does not exist.

47. Which of the following exists as covalent crystals in the solid state ?
(1) Iodine (2) Silicon (3) Sulphur (4) Phosphorus

47. (2)
Silicon exists as covalent crystals in the solid state.

48. Synthesis of each molecule of glucose in photosynthesis involves :

(1) 18 molecules of ATP (2) 10 molecules of ATP
(3) 8 molecules of ATP (4) 6 molecules of ATP

48. (4)

$C_6H_{12}O_6 + 36ADP + 36H_3PO_4 + 6O_2 \rightarrow 6CO_2 + 36ATP + 42H_2O$

49. The coagulating power of electrolytes having ions Na^+ , Al^{3+} and Ba^{2+} for arsenic sulphide sol increases in the order :

(1) $Al^{3+} < Ba^{2+} < Na^+$ (2) $Na^+ < Ba^{2+} < Al^{3+}$
(3) $Ba^{2+} < Na^+ < Al^{3+}$ (4) $Al^{3+} < Na^+ < Ba^{2+}$

49. (2)

Factual

50. Which of the following represents the correct order of increasing first ionization enthalpy for Ca, Ba, S, Se and Ar ?

(1) Ca < S < Ba < Se < Ar (2) S < Se < Ca < Ba < Ar
(3) Ba < Ca < Se < S < Ar (4) Ca < Ba < S < se < Ar

50. (3)

Factual

51. Energy of an electron is given by $E = -2.178 \times 10^{18} J\left(\frac{Z^2}{n^2}\right)$. Wavelength of light required to excite an electron in an hydrogen atom from level n = 1 to n = 2 will be :

($h = 6.62 \times 10^{-34}$ Js and $c = 3.0 \times 10^{-8}$ ms^{-1})

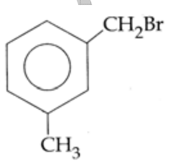
(1) 1.214×10^{-7} m (2) 2.816×10^{-7} m
(3) 6.500×10^{-7} m (4) 8.500×10^{-7} m

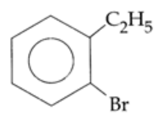
51. (1)

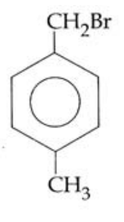
$$\frac{1}{\lambda} = R_H Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

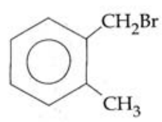
$Z = 1$
$n_1 = 1$
$n_2 = 2$
$\lambda = 1.214 \times 10^{-7}$ m

52. Compound (A), C_8H_9Br, gives a white precipitate when warmed with alcoholic $AgNO_3$. Oxidation of (A) gives an acid (B) , $C_8H_6O_4$. (B) easily forms anhydride on heating. Identify the compound (A) .

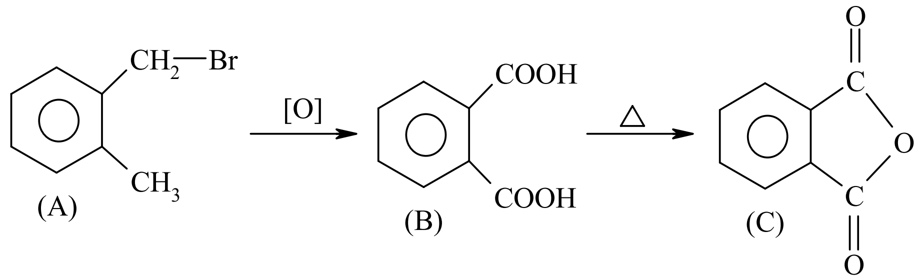
(1) 

(2) 

(3) 

(4) 

52. (4)



53. Four successive members of the first row transition elements are listed below with atomic numbers. Which one of them is expected to have the highest $E^0_{M^{3+}/M^{2+}}$ value ?

(1) Cr(Z = 24) (2) Mn(Z = 25) (3) Fe(Z = 26) (4) Co(Z = 27)

53. (4)
Factual

54. How many litres of water must be added to 1 litre of an aqueous solution of HCl with a pH of 1 to create an aqueous solution with pH of 2 ?

(1) 0.1 L (2) 0.9 L (3) 2.0 L (4) 9.0 L

54. (4)

$0.1 \times 1 = (1 + v) \times 0.01$

$1 + v = \dfrac{0.1}{0.01}$

$\Rightarrow\ 1 + v = 10 \Rightarrow v = 10 - 1 = 9L$

55. The first ionisation potential of Na is 5.1 eV. The value of electron gain enthalpy of Na^+ will be :

(1) −2.55 eV (2) −5.1 eV (3) −10.2 eV (4) +2.55 eV

55. (2)

E.A. = Ionisation potential

$\therefore$ EA of $Na^+ = -5.1$ eV

56. An organic compound A upon reacting with NH_3 gives B. On heating, B gives C. C in presence of KOH reacts with Br_2 to give $CH_3CH_2NH_2$. A is :

(1) CH_3COOH (2) $CH_3CH_2CH_2COOH$

(3) $CH_3-\underset{\underset{CH_3}{|}}{CH}-COOH$ (4) CH_3CH_2COOH

56. (4)

$$CH_3-CH_2-\overset{\overset{O}{\|}}{C}-OH \xrightarrow[\Delta]{NH_3} CH_3-CH_2-\overset{\overset{O}{\|}}{C}-NH_2 \xrightarrow{Br_2/KOH} CH_3-CH_2-NH_2$$

57. Stability of the species Li_2 , Li_2^- and Li_2^+ increases in the order of :

(1) $Li_2 < Li_2^+ < Li_2^-$ (2) $Li_2^- < Li_2^+ < Li_2$

(3) $Li_2 < Li_2^- < Li_2^+$ (4) $Li_2^- < Li_2 < Li_2^+$

57. (2)

B.O. of $Li_2^+ = 0.5$

B.O. of $Li_2^- = 0.5$

Hence stability order = $Li_2^- < Li_2^+ < Li_2$

58. An unknown alcohol is treated with the "Lucas reagent" to determine whether the alcohol is primary, secondary or tertiary. Which alcohol reacts fastest and by what mechanism :

(1) secondary alcohol by S_N1 (2) tertiary alcohol by S_N1
(3) secondary alcohol by S_N2 (4) tertiary alcohol by S_N2

58. (2)

Fastest reaction with Lucas reagent given by 3° R−OH and it follow SN^1 mech.

59. The gas leaked from a storage tank of the Union Carbide plant in Bhopal gas tragedy was :

(1) Methylisocyanate (2) Methylamine
(3) Ammonia (4) Phosgene

59. (1)

Factual

60. Experimentally it was found that a metal oxide has formula $M_{0.98}O$. Metal M, is present as M^{2+} and M^{3+} in its oxide. Fraction of the metal which exists as M^{3+} would be :

(1) 7.01 % (2) 4.08 % (3) 6.05 % (4) 5.08 %

60. (2)

3x + 2 (0.98 − x) = 2
x + 1.96 = 2
x = 0.04

$\% \text{ of } M^{+3} = \frac{0.04}{0.98}\times 100 = 4.08\%$

PART- C : MATHEMATICS

61. Distance between two parallel planes 2x + y +2z = 8 and 4x + 2y + 4z + 5 = 0 is :

(1) $\frac{3}{2}$ (2) $\frac{5}{2}$ (3) $\frac{7}{2}$ (4) $\frac{9}{2}$

61. (3)

Planes are $2x + y + 2z - 8 = 0$ & $2x + y + 2z + \frac{5}{2} = 0$

$$\text{Distance is} = \frac{\frac{5}{2}-(-8)}{\sqrt{2^2+1^2+2^2}} = \frac{\frac{21}{2}}{3} = \frac{7}{2}$$

62. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dp}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is :

(1) 2500 (2) 3000 (3) 3500 (4) 4500

62. (3)

$$\frac{dp}{dx} = 100 - 12\sqrt{x}$$

$$P = 100x - 12\cdot\frac{2}{3}x^{3/2} + C$$

$$P = 100x - 8x^{3/2} + C \qquad \ldots (1)$$

Given if x = 0, P = 2000

$\Rightarrow$ C = 2000

So, (1) becomes

$P = 100x - 8x^{3/2} + 2000$

If $x = 25$ (is 25 move workers)

$P = 100 \times 25 - 8 \times (5^2)^{3/2} + 2000$

$= 2500 - 1000 + 2000 = 3500$

63. Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is :

(1) 256 (2) 220 (3) 219 (4) 211

63. (3)

$A = \{x, y\}$

$B = \{a, b, c, d\}$

$A \times B$ having $2 \times 4 = 8$ elements

Total subsets of $A \times B$ is $2^8 = 256$

$\therefore$ Total no. of subsets of $A \times B$ having 3 or more elements

$$= 256 - \left(\underset{\substack{\text{null}\\\text{set}}}{1} + \underset{\substack{\text{single ton}\\\text{set}}}{8} + \underset{\substack{\text{subset having}\\\text{2 elements}}}{{}^8C_2}\right)$$

$= 256 - 1 - 8 - 28 = 219$

64. If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, then k can have :

(1) any value (2) exactly one value

(3) exactly two values (4) exactly three values

64. (3)

Lines are co−planer $\Rightarrow$ S. D. = 0

$$\Rightarrow \begin{vmatrix} 2-1 & 3-4 & 4-5 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0 \quad \Rightarrow \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$(1 + 2k) + (1 + k^2) - (2 - k) = 0$

$3k + k^2 = 0$

$k(k + 3) = 0$

$k = 0$ or $k = -3$

Exactly two values.

65. If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is :

(1) $\sqrt{18}$ (2) $\sqrt{72}$ (3) $\sqrt{33}$ (4) $\sqrt{45}$

65. (3)

Median through ‘A’ is $\frac{(3\hat{i} + 3\hat{k}) + (5\hat{i} - 2\hat{j} + 4\hat{k})}{2}$

$= 4\hat{i} - \hat{j} + 4\hat{k}$

So, Length $= \sqrt{4^2 + 1^2 + 4^2} = \sqrt{33}$

66. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in [0, 1]

(1) lies between 1 and 2 (2) lies between 2 and 3

(3) lies between −1 and 0 (4) does not exist

66. (4)

$f(x) = 2x^3 + 3x + k$

$f'(x) = 6x^2 + 3$

$f'(x) = 0$

$\Rightarrow x^2 = -\frac{1}{2}$

Not Possible.

As condition for two distinct real root is $f(\alpha)\ f(\beta) = 0$

(where α, β are roots of $f'(x) = 0$)

67. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ……, is :

(1) $\frac{7}{81}\left(179-10^{-20}\right)$ (2) $\frac{7}{9}\left(99-10^{-20}\right)$

(3) $\frac{7}{81}\left(179+10^{-20}\right)$ (4) $\frac{7}{9}\left(99+10^{-20}\right)$

67. (3)

$0.7 + 0.77 + 0.777 + \ldots\ldots + 0.777 \ldots 7$

$= \frac{7}{9}[0.9+0.99+0.999+\ldots+0.999\ldots9]$

$= \frac{7}{9}[(1-0.1)+(1-0.01)+(1-0.001)+\ldots+(1-0.000\ldots1)]$

$= \frac{7}{9}\left[20-\left(\frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3}+\ldots+\frac{1}{10^{20}}\right)\right]$

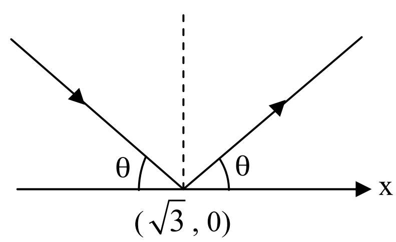
$= \frac{7}{9}\left[20-\frac{1}{10}\cdot\frac{1-\frac{1}{10^{20}}}{1-\frac{1}{10}}\right] = \frac{7}{9}\left[20-\frac{1}{9}\cdot\left(\frac{10^{20}-1}{10^{20}}\right)\right]$

$= \frac{7}{81}\left[180-\left(1-\frac{1}{10^{20}}\right)\right] = \frac{7}{81}[179+10^{-20}]$

68. A ray of light along $x+\sqrt{3}y=\sqrt{3}$ gets reflected upon reaching x–axis, the equation of the reflected ray is :

(1) $y = x+\sqrt{3}$ (2) $\sqrt{3}y = x-\sqrt{3}$ (3) $y=\sqrt{3}x-\sqrt{3}$ (4) $\sqrt{3}y = x-1$

68. (2)



Slope of $x+\sqrt{3}y = \sqrt{3}$ is $-\frac{1}{\sqrt{3}} = m_1$ (let)

So, $\tan\theta = -\frac{1}{\sqrt{3}}$

$\theta = 150°$

So, slope of reflected ray is $\tan 30° = \frac{1}{\sqrt{3}}$

So, equation of reflected ray is $y - 0 = \frac{1}{\sqrt{3}}(x-\sqrt{3})$

$$\sqrt{3}y = x-\sqrt{3}$$

69. The number of values of k, for which the system of equations :
$(k + 1)x + 8y = 4k$
$kx + (k + 3)y = 3k - 1$
has no solution, is :
(1) infinite (2) 1 (3) 2 (4) 3

69. (2)

$$\Delta = \begin{vmatrix} k+1 & 8 \\ k & k+3 \end{vmatrix} = k^2 + 4k + 3 - 8k$$
$$= k^2 - 4k + 3$$
$$= (k - 3)(k - 1)$$

$$\Delta_1 = \begin{vmatrix} 4k & 8 \\ 3k-1 & k+3 \end{vmatrix} = 4k^2 + 12k - 24k + 8$$
$$= 4k^2 - 12k + 8$$
$$= 4(k^2 - 3k + 2)$$
$$= 4(k - 2)(k - 1)$$

$$\Delta_2 = \begin{vmatrix} k+1 & 4k \\ k & 3k-1 \end{vmatrix} = 3k^2 + 2k - 1 - 4k^2$$
$$= -k^2 + 2k - 1$$
$$= -(k - 1)^2$$

As given no solution $\Rightarrow \Delta_1 \ \& \ \Delta_2 \neq 0$
But $\Delta = 0$
$\Rightarrow k = 3$

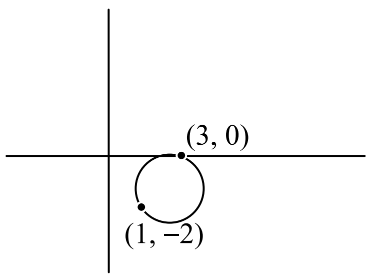
70. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, a, b, $c \in$ R, have a common root, then a : b : c is
(1) 1 : 2 : 3 (2) 3 : 2 : 1 (3) 1 : 3 : 2 (4) 3 : 1 : 2

70. (1)
$x^2 + 2x + 3 = 0$
$D = 2^2 - 4 \cdot 1 \cdot 3 < 0$
$\Rightarrow$ Both roots complex
$\Rightarrow$ Both roots are common of $x^2 + 2x + 3 = 0$
& $ax^2 + bx + c = 0$

$$\frac{a}{1} = \frac{b}{2} = \frac{c}{3}$$

71. The circle passing through (1, −2) and touching the axis of x at (3, 0) also passes through the point :
(1) (−5, 2) (2) (2, −5) (3) (5, −2) (4) (−2, 5)

71. (3)
Let centre C(3, k)
As touches X−axis
$\Rightarrow r = k$
So, circle is $(x - 3)^2 + (y - k)^2 = k^2$
Given it passes (1, −7)
$4 + (k + 2)^2 = k^2$
$4 + k^2 + 4k + 4 = k^2$



$4k = -8$
$k = -2$
Circle is $(x - 3)^2 + (y + 2)^2 = 4$
Obviously $(5, -2)$ satisfy

72. If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then :
(1) $x = y = z$ (2) $2x = 3y = 6z$ (3) $6x = 3y = 2z$ (4) $6x = 4y = 3z$

72. (1)
$2y = x + z$ … (1)
As $\tan^{-1} x$, $\tan^{-1} y$, $\tan^{-1} z$ in AP

$$\Rightarrow 2\tan^{-1} y = \tan^{-1}\frac{x+z}{1-xz}$$
$$\frac{2y}{1-y^2} = \frac{x+z}{1-xz}$$
$$\frac{x+z}{1-y^2} = \frac{x+z}{1-xz} \quad \text{… by (1)}$$
$$(x+z)\left\{\frac{1}{1-y^2} - \frac{1}{1-xz}\right\} = 0$$
$x + z = 0$ or $1 - xz = x - y^2$
$y^2 = xz$
$\Rightarrow$ x, y, z in GP.
As x, y, z AP & GP
$\Rightarrow x = y = z$

73. Consider :
Statement – I : $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.
Statement II : $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a tautology.
(1) Statement – I is true; Statement – II is true; Statement – II is a correct explanation for Statement I.
(2) Statement – I is true; Statement – II is true; Statement – II is not a correct explanation for Statement – I.
(3) Statement – I is true; Statement – II is false.
(4) Statement – I is false; Statement – II is true.

73. (2)

p	q	~p	~q	P ^ ~q	~p ^ q	
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	F	F

It is Fallacy

p	q	~p	~q	$P \to q$	$\sim q \to \sim p$	$(p \to q) \leftrightarrow (\sim q \to \sim p)$
T	T	F	F	T	T	T
T	F	F	T	T	T	T
F	T	T	F	F	F	T
F	F	T	T	T	T	T

It is tautology

74. If $\int f(x)\,dx = \psi(x)$, then $\int x^5 f(x^3)\,dx$ is equal to :

(1) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^2\psi(x^3)\,dx\right] + C$

(2) $\frac{1}{3}x^3\psi(x^3) - 3\int x^3\psi(x^3)\,dx + C$

(3) $\frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)\,dx + C$

(4) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^3\psi(x^3)\,dx\right] + C$

74. (3)

$\int f(x)\,dx = \psi(x)$, then $\int x^5 f(x^3)\,dx$

$I = \int x^3 \cdot x^2 f(x)^3\,dx$ Let $x^3 = t$

$3x^2\,dx = dt$

$I = \int t f(t) \frac{dt}{3}$

$I = \frac{1}{3}[t\int f(t)\,dt - \int f(t)\,dt]$

$I = \frac{x^3}{3}\psi(x^3) - \frac{1}{3}\int \frac{d}{dt}(t)\int f(t)\,dt + c$

$I = \frac{x^3}{3}\psi(x^3) - \frac{1}{3}\int 3x^2\psi(x^3)\,dx + c$

$I = \frac{x^3}{3}\psi(x^3) - x^2\psi(x^3)\,dx + c$

75. $\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to :

(1) $-\frac{1}{4}$ (2) $\frac{1}{2}$ (3) 1 (4) 2

75. (4)

$\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$

$= \lim_{x\to 0}\frac{(2\sin^2 x)(3+\cos x)}{x\left(\frac{\tan 4x}{4x}\right)\times 4x}$

$= \lim_{x\to 0}\frac{2\sin^2 x(3+\cos x)}{4x^2} = \frac{2}{4}(3+1) = 2$

76. Statement – I :

The value of the integral $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$.

Statement – II :

$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

(1) Statement – I is true; Statement – II is true; Statement – II is a correct explanation for Statement – I.

(2) Statement – I is true; Statement – II is true; Statement – II is a not a correct explanation for Statement – I.
(3) Statement – I is true; Statement – II is false.
(4) Statement – I is false; Statement – II is true.

76. (4)

$$I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}} \qquad \ldots \text{(i)}$$

$$I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}} \qquad \ldots \text{(ii)}$$

Adding (i) and (ii)

$$\Rightarrow 2I = \int_{\pi/6}^{\pi/3} 1\,dx \qquad \Rightarrow 2I = \frac{\pi}{3}-\frac{\pi}{6}$$

$$2I = \frac{\pi}{6} \qquad \Rightarrow I = \frac{\pi}{12}$$

77. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, and having centre at (0, 3) is :

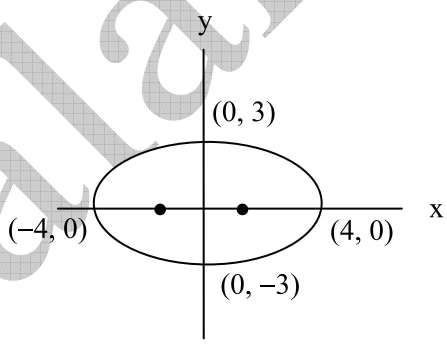
(1) $x^2 + y^2 - 6y - 7 = 0$ (2) $x^2 + y^2 - 6y + 7 = 0$
(3) $x^2 + y^2 - 6y - 5 = 0$ (4) $x^2 + y^2 - 6y + 5 = 0$

77. (1)

Co-ordinate of Foci are (ae, 0) ; (–ae, 0)

$$e = \sqrt{1-\frac{9}{16}}$$

$$\Rightarrow \qquad e = \frac{\sqrt{7}}{4}$$



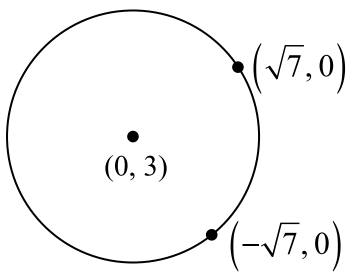
Co-ordinate of Foci are $(\sqrt{7}, 0)\,;(-\sqrt{7}, 0)$

$$R = \sqrt{7+9} = 4$$

$$(x-0)^2 + (y-3)^2 = 4^2$$

$$x^2 + y^2 - 6y + 9 = 16$$

$$\therefore \quad x^2 + y^2 - 6y - 7 = 0$$



78. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is :

(1) $\frac{17}{3^5}$ (2) $\frac{13}{3^5}$ (3) $\frac{11}{3^5}$ (4) $\frac{10}{3^5}$

78. (3)

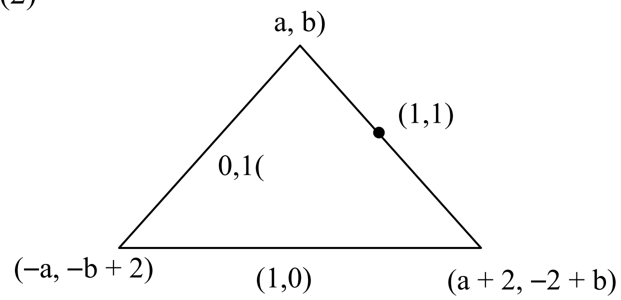
Total No. of ways of attempting the question = 3^5

$$5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^1 + 5C_5\left(\frac{1}{3}\right)^5 \Rightarrow \frac{11}{3^5}$$

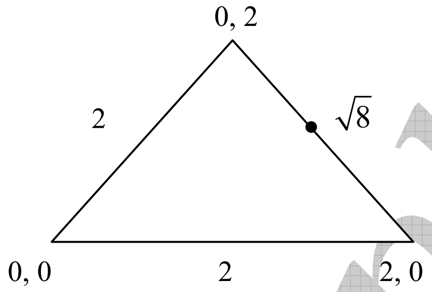
79. The x−coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1) (1, 1) and (1, 0) is :

(1) $2+\sqrt{2}$ (2) $2-\sqrt{2}$ (3) $1+\sqrt{2}$ (4) $1-\sqrt{2}$

79. (2)



On solving a = 0 and b = 2



$$I_x = \frac{0\times 2+0\times\sqrt{8}+2\times 2}{2+2+2\sqrt{2}}$$

$$I_x = \frac{4}{4+2\sqrt{2}}$$

$$I_x = \frac{2}{2+\sqrt{2}}\times\frac{2-\sqrt{2}}{2-\sqrt{2}}$$

$$I_x = \frac{2\left(2-\sqrt{2}\right)}{2} \Rightarrow 2-\sqrt{2}$$

80. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is :

(1) 4 (2) 120 (3) 210 (4) 310

80. (3)

$$\left[\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right]^{10}$$

$\Rightarrow \left[(x^{1/3}+1)-(1+x^{-1/2})\right]^{10}$

$\Rightarrow [x^{1/3}-x^{-1/2}]^{10}$

$\Rightarrow T_{r+1} = {}^{10}C_r\,(x^{1/3})^{10-r}\,(-x^{-1/2})^r$

$T_{r+1} = {}^{10}C_r\ x^{\frac{10-r}{3}-\frac{r}{2}}(-1)^r$

$T_{r+1} = {}^{10}C_r\ x^{\frac{20-5r}{6}}(-1)^r$

$\Rightarrow 20-5r = 0$

$\Rightarrow r = 4$

$\Rightarrow T_5 = 10C_4 = 210$

81. The area (in square units) bounded by the curves $y=\sqrt{x}$, $2y - x + 3 = 0$, x−axis, and lying in the first quadrant is :

(1) 9 (2) 36 (3) 18 (4) $\frac{27}{4}$

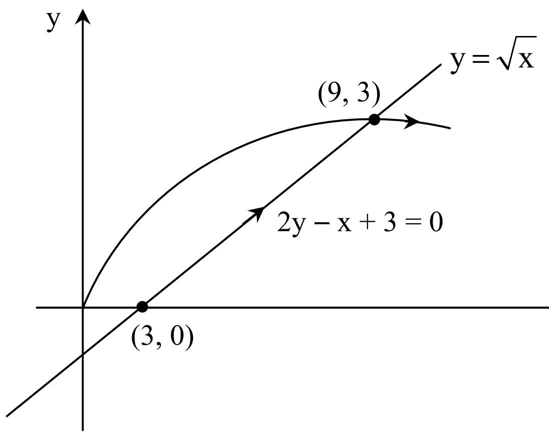
81. (1)

$y = \sqrt{x}$ and $2y - x + 3 = 0$

$$\int_0^9 \sqrt{x}\, dx - \int_3^9 \left(\frac{x-3}{2}\right) dx$$

$$\left(\frac{x^{3/2}}{3/2}\right)_0^9 - \left[\frac{\left(\frac{x^2}{2} - 3x\right)}{2}\right]_3^9$$

$\Rightarrow$ 9 square units



82. Let T_n be the number of all possible triangles formed by joining vertices of a n−sided regular polygon. If $T_{n+1} - T_n = 1$ then the value of n is :

(1) 7 (2) 5 (3) 10 (4) 8

82. (2)

$^{n+1}C_3 - {}^nC_3 = 10$

On solving n = 5

83. If z is a complex number of unit modulus and argument θ, then $\arg\left(\frac{1+z}{1+\bar{z}}\right)$ equals :

(1) $-\theta$ (2) $\frac{\pi}{2} - \theta$ (3) θ (4) $\pi - \theta$

83. (3)

Let $z = \omega$

Now $\frac{1+z}{1+\bar{z}} = \frac{1+\omega}{1+\omega^2} = \frac{-\omega^2}{-\omega} = \omega$

$\therefore \quad \arg\frac{1+z}{1+z} = \arg \omega = \theta$

(put $z = \cos\theta + i \sin\theta$)

84. ABCD is a trapezium such that AB and CD are parallel and BC ⊥ CD. If ∠ ADB = θ, BC = p and CD = q, then AB is equal to :

(1) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ (2) $\frac{p^2+q^2\cos\theta}{p\cos\theta + q\sin\theta}$ (3) $\frac{p^2+q^2}{p^2\cos\theta + q^2\sin\theta}$ (4) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$

84. (1)

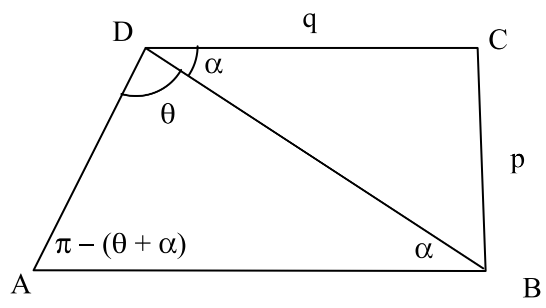
$BD = \sqrt{p^2 + q^2}$

$\angle ABD = \angle BDC = \alpha$

$\Rightarrow \angle DAB = \pi - (\theta + \alpha)$

$\tan\alpha = \frac{p}{q}$

Δ ABD



$$\frac{AB}{\sin\theta} = \frac{BD}{\sin(\pi - (\theta+\alpha))} = \frac{BD}{\sin(\theta+\alpha)}$$

$$\therefore AB = \frac{BD\sin\theta}{\sin(\theta+\alpha)} = \frac{BD^2\sin\theta}{BD\sin(\theta+\alpha)} = \frac{BD^2\sin\theta}{BD\sin\theta\cos\alpha + BD\cos\theta\sin\alpha} = \frac{(p^2+q^2)\sin\theta}{q\sin\theta + p\cos\theta}$$

85. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then α is equal to :

(1) 4 (2) 11 (3) 5 (4) 0

85. (2)

$$\because \; 1\begin{vmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{vmatrix}1 = |A|^2 = 4^2$$

$2\alpha - 6 = 16 \qquad \therefore \; \alpha = 11.$

86. The intercepts on x–axis made by tangents to the curve, $y = \int_0^x |t|dt,\ x \in R,$ which are parallel to the line y = 2x, are equal to

(1) ± 1 (2) ± 2 (3) ± 3 (4) ± 4

86. (1)

$y = \int_0^x |t|dt$

Case – I : If $t > 0$

$y = \left[\frac{t^2}{2}\right]_0^x = \frac{x^2}{2} = \frac{dy}{dx} = x = 2$

$\Rightarrow$ x = 2 and y = 2

$(y - 2) = 2(x - 2) \quad \Rightarrow \; y - 2x + 2 = 0.$ Hence x intercept = 1.

Case – II : $t < 0$

Similarly, x intercept = –1.

87. Given : A circle, $2x^2 + 2y^2 = 5$ and a parabola, $y^2 = 4\sqrt{5}\,x$.

Statement – I : An equation of a common tangent to these curves is $y = x + \sqrt{5}$.

Statement – II : If the line, $y = mx + \frac{\sqrt{5}}{m}\ (m \neq 0)$ is their common tangent, then 'm' satisfies $m^4 - 3m^2 + 2 = 0$.

(1) Statement–I is true; Statement–II is true; Statement – II is a **correct** explanation for Statement – I.
(2) Statement–I is true; Statement–II is true; Statement – II is **not** a correct explanation for Statement – I.
(3) Statement–I is true; Statement–II is false.
(4) Statement–I is false; Statement–II is true.

87. (2)

Obviously $y = x + \sqrt{5}$ is common tangent. Statement I is true.

Statement 2 : Let $y = mx + \frac{\sqrt{5}}{m}$ is common tangent circle & parabola.

$\Rightarrow 2x^2 + 2y^2 = 5$

As tangent

$$\left|\frac{m\times 0-0+\frac{\sqrt{5}}{m}}{\sqrt{1+m^2}}\right|=\frac{\sqrt{5}}{2}$$

$m^4 + m^2 - 2 = 0$

$\Rightarrow m^2 = 1$

$m = \pm 1$

So, Statement − II is true.

Hence (2).

88. If $y=\sec\left(\tan^{-1}x\right)$, then $\frac{dy}{dx}$ at x = 1 is equal to :

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$ (3) 1 (4) $\sqrt{2}$

88. (1)

$y = \sec(\tan^{-1}x)$

$\Rightarrow \frac{dy}{dx} = \sec(\tan^{-1}x)\cdot\tan(\tan^{-1}x)\cdot\frac{1}{1+x^2}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = \frac{\sqrt{2}}{1+1} = \frac{1}{\sqrt{2}}$

89. The expression $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$ can be written as :

(1) sinA cosA + 1 (2) secA cosecA + 1 (3) tanA + cotA (4) secA + cosecA

89. (2)

$$\text{Exp.} = \frac{\tan^2 A}{\tan A-1}+\frac{1}{\tan A-\tan^2 A}=\frac{1}{\tan A-1}\left[\tan^2 A-\frac{1}{\tan A}\right]$$

$$=\frac{\tan^2 A+\tan A+1}{\tan A} = \tan A+\cot A+1$$

$$= \sec A\cdot\text{cosec } A + 1$$

90. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given ?

(1) mean (2) median (3) mode (4) variance

90. (4)

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