# MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

Time : Three hours                                        Maximum : 100 marks

## SECTION A — (10 × 3 = 30 marks)

Answer ALL questions.

1. Construct the truth table for $(P \rightarrow Q) \wedge (Q \rightarrow P)$.

2. Write the predicate "$x$ is the father of the mother of $y$".

3. Show that for any two sets A and B, $A - (A \cap B) = A - B$.

4. Explain partial order relation with an example.

5. Define the compositions of functions and give an example.

6. Let $X = \{2, 3, 6, 12, 24, 36\}$ and the relation $\leq$ be such that $x \leq y$ if $x$ divides $y$. Draw the Hash diagram of $(\times, \leq)$.

7. Define normal subgroup and give an example.

8. Define a ring and give an example.

9. Define a path of a graph and the length of the path with an example.

10. Define adjacency matrix of a graph with an example.

## SECTION B — (4 × 10 = 40 marks)

Answer any FOUR questions.

11. Obtain the principal conjunctive normal form of the formula $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$.

12. Show that $(x)(P(x) \vee Q(x)) \Rightarrow xP(x) \vee (\exists x)Q(x)$.

13. Explain relation matrix and the graph of a relation.

14. Define characteristics function of a set and using that prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

15. Let T be the set of all even integers. Show that the semigroup $(Z, +)$ and $(T, +)$ are isomorphic.

16. Write the Warshall algorithm to produce the path matrix $A^+$ from a given adjacency matrix A.

SECTION C — (2 × 15 = 30 marks)

Answer any TWO questions.

17. (a) Verify the validity of the following arguments :

All men are mortal

Socrates is a man

Therefore Socrates is a mortal. (8)

(b) Let $X = \{1, 2, 3, 4, 5, 6, 7\}$ $R : \{\langle x, y \rangle / x - y$ is divisible by 3}. Show that $R$ is an equivalence relation. Draw the graph of $R$.
(7)

18. (a) Let $f : A \to B$ and $g : B \to C$ be both one-to-one and onto functions. Then prove that $(g \text{o} f)^{-1} = f^{-1} \text{o} g^{-1}$. (8)

(b) Prove that for any commutative monoid (M, $*$) the set of idempotent element of M forms a submonoid. (7)

19. (a) Prove that in a simple digraph $G = (V, E)$ every node of the digraph lies on exactly one strong component. (7)

(b) State and prove Lagranges theorem. (8)