(Pages : 4) **4167**

Reg. No. : ....

Name : ....

First Semester M. Sc. Computer Science Degree Examination, July 2009
(I.D.E)
MATHEMATICAL FOUNDATIONS OF INFORMATION TECHNOLOGY

Time : 3 Hours Max. Marks : 80

Instructions : *The question paper contains **two** Parts, Part **A** and Part **B**. Part **A** carries **32** marks and Part **B** carries **48** marks.*

PART – A

Answer any **eight** questions. **All** questions carry **equal** marks. **(4×8=32 Marks)**

1. Define a tautology. Is the

   proposition $((\neg P \to Q) \to (Q \to P))$ a tautology ?

2. Check whether the set of elements of R^3, $[x_1, x_2, x_3]'$ with each $x_i \geq 0$ is a vector space or not.

3. Define a power set. List the elements of the power set of A = $\{\{\phi\}\}$.

4. For what truth value will the following statement be true. "It is not the case that houses are cold or haunted and it is false that cottages are warm or houses ugly."

5. Define a lattice. Are all partially ordered sets lattices ? Justify your answer.

6. Prove that a group of prime order must be cyclic.

7. Construct a grammar for the language $L = \{a^{zi}, b^{3i} \mid i \geq 1\}$.

8. Simplify the Boolean expression :

   $[a * (b' \oplus c]' * [b' \oplus (a * c')']'$.

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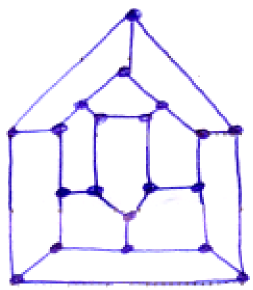
9. Let $< L, \leq >$ be a lattice in which $*$ denote the operation of meet. Prove that for $a, b \in L,\ a \leq b \Leftrightarrow a * b = a$.

10. Define a connected planar graph. Is K_4 planar ? Explain.

11. Distinguish between a Rooted tree and a Binary tree.

12. Find an Hamiltonian circuit for the given graph.



PART – B

Answer **any six** questions in full. **All** questions carry **equal** marks. **(8×6=48 Marks)**

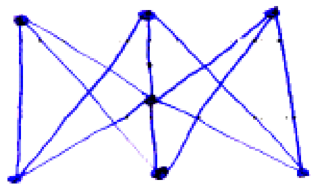
13. Use truth tables and laws of logic to show that the given propositions are logically equivalent :

    i) $P \rightarrow (Q \rightarrow R)$ ii) $P \rightarrow (\neg Q \vee R)$ iii) $(P \wedge Q) \rightarrow R$

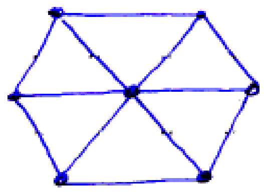
14. Check whether the given vectors $x_1 = [3\ 2\ 7]'$; $x_2 = [2\ 4\ 1]'$; $x_3 = [1\ -2\ 6]'$ are linearly dependent. If so find a relation between them.

15. Show that the graphs G_1 and G_2 are isomorphic by defining a 1–1 correspondence between the vertex sets and the edge sets.

G_1 :



G_2 :



16. Let A_1, A_2, A_3 and A_4 denote the set of integers from 1 to 250 (both inclusive) that are divisible by 2, 3, 5 and 7 respectively. Represent each of the following using a Venn diagram and hence find the number of integers that are :

a) divisible by 2 and 7 but not by 5

b) divisible by 3 and 2 but not by 7 or 5.

17. Define a semigroup. Let $(A, *)$ be a semigroup and $a \in A$. Consider a binary operation $\oplus$ on A such that $\forall x, y \in A, \ \ x \oplus y = x * a * y$. Check whether $\oplus$ is

a) associative b) commutative.

18. Illustrate the procedure to determine the minimal spanning tree from a weighted graph. Support your illustration using an example.

19. a) Construct a digital network using AND-gate, OR-gate and NOT-gate to realize the Boolean expression given by $\overline{(\overline{x_1 \vee x_2}) \vee (\overline{x}_1 \wedge x_3)}$

b) Write the above expression in both disjunctive and conjunctive normal form.

20. Give a brief discription on phase structure grammar.

21. Let $*$ be a binary operation on set $A = \{0,1,2,3,4\}$ defined by $a * b$ = {remainder obtained when a+b is divided by 5 }.

   a) Find the identity element(s) if it exists.

   b) Find the inverse of each element if it exists.

   c) Prove or disprove that $*$ is commutative.

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