SOLUTIONS

SECTION – A

1. (A) (1)
2. (C) (1)
3. (B) (1)

Since -3 is the root of quadratic polynomial, we have

$(K-1)(-3)^2 + 1 = 0 \Rightarrow 9(k-1) = -1 \Rightarrow k-1 = \frac{-1}{9} \Rightarrow k = 1 - \frac{1}{9} = \frac{8}{9}$

4. (A) $\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = 1$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ intersect at point. (1)

5. (D) $\tan x = \frac{CD}{AC}, \tan y = \frac{BC}{AC} = \frac{2CD}{AC} \Rightarrow \frac{\tan x}{\tan y} = \frac{1}{2}$ (1)

6 (B) (1)

7. (C)

$x = 3\sec^2\theta - 1, y = \tan^2\theta - 2$

$x - 3y = 3\sec^2\theta - 1 - 3\tan^2\theta + 6$

$= 3(\sec^2\theta - \tan^2\theta) + 5$ (½)

$= 3 + 5$

$= 8$ (½)

8. (B) $\cos\theta + \cos^2\theta = 1 \Rightarrow 1 - \cos^2\theta = \cos\theta$

$\sin^2\theta + \sin^4\theta = \left(1 - \cos^2\theta\right) + (1 - \cos^2\theta)^2$ (½)

$= \cos\theta + \cos^2\theta$

$= 1$ (½)

9. (C) $\Delta ABC \sim \Delta RQP$

$\angle A = \angle R = 80^o$

$\angle B = \angle Q = 60^o$ (½)

$\therefore \angle P = 180 - 140 = 40^o$ (½)

10. (D) $\cos A - \sin A = \frac{12}{13} - \frac{5}{13} = \frac{7}{13}$ (1)

SECTION – B

11. Since, $870 = 225 \times 3 + 195$ (½)

$225 = 195 \times 1 + 30$ (½)

$195 = 30 \times 6 + 15$ (½)

$30 = 15 \times 2 + 0$

$\therefore$ HCF (870,225) = 15 (½)

12.

$$\begin{array}{ll} 37x + 43y = 123 & 37x + 43y = 123 \\ 43x + 37y = 117 & 43x + 37y = 117 \\ (+)\quad (+) \qquad (+) & (-)\quad (-) \qquad (-) \\ \hline 80x + 80y = 240 & -6x + 6y = 6 \end{array}$$

(1)

$\Rightarrow$ x + y = 3 -x + y = 1

$\left.\begin{array}{l} x + y = 3 \\ -x + y = 1 \end{array}\right\}$ Solving $x = 1, y = 2$ (1)

OR

We have $\left(x + \frac{6}{y} = 6\right)3 \Rightarrow 3x + \frac{18}{y} = 18$ ... (1)

Subtracting equation (1) from $3x - \frac{8}{y} = 5$, we get

$-\frac{26}{y} = -13 \Rightarrow y = 2$ (1)

From equation (1), x = 3 (1)

13. α, β are roots of $x^2 - (k + 6)x + 2(2k - 1)$

$\alpha + \beta = k + 6, \ \alpha\beta = 2\left(2k - 1\right)$ (1)

Now $\alpha + \beta = \frac{1}{2}\alpha\beta \Rightarrow k + 6 = \frac{1}{2} \times 2\left(2k - 1\right)$

$\Rightarrow k + 6 = 2k - 1$ (1)

$\Rightarrow k = 7$

14. Cot $\theta = \frac{7}{8}$ (given)

$\frac{\left(1 + \sin\theta\right)\left(1 - \sin\theta\right)}{\left(1 + \cos\theta\right)\left(1 - \cos\theta\right)} = \frac{1 - \sin^2\theta}{1 - \cos^2\theta}$ (½)

$= \frac{\cos^2\theta}{\sin^2\theta}$ (½)

$= \cot^2\theta$

$= \frac{49}{64}$ (1)

15.

C.I	f	c.f.
135 – 140	4	4
140 – 145	7	11
145 – 150	11	22
150 – 155	6	28
155 – 160	7	35
160 – 165	5	40

$n = 40 \Rightarrow \frac{n}{2} = 20$

Median class is 145 – 150 (½)

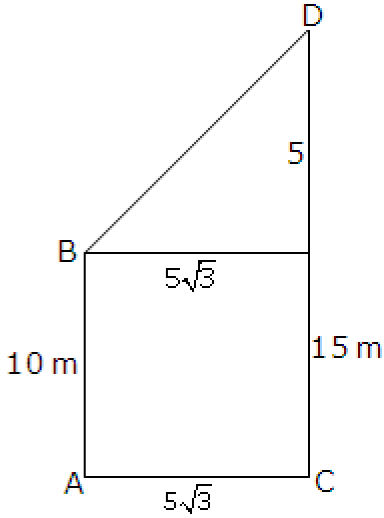
Also, since highest frequency is 11, Modal class is 145 – 150. (1)

16.

More than type	f
more than 50	50
more than 55	48
more than 60	42
more than 65	34
more than 70	20
more than 75	5

(2)

17. Diagram and correct marking (1)



$BD^2 = 5^2 + \left(5\sqrt{5}\right)^2$

$= 25 + 75$

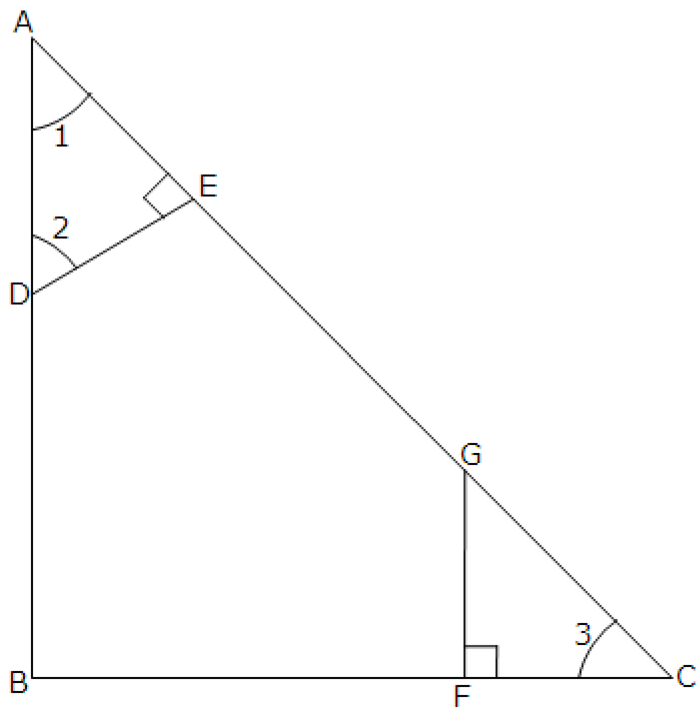
$= 100$

$BD = \sqrt{100} = 10$ m

Distance between their top is 10 m. (1)

18. From $\triangle ABC$, $\angle 1 + \angle 3 = 90^o$

From $\triangle ADE$, $\angle 1 + \angle 2 = 90^o$



$\angle 1 + \angle 3 = \angle 1 + \angle 2 \Rightarrow \angle 3 = \angle 2$ (1)

$\therefore$ In $\Delta ADE \sim \Delta GCF$ by AA rule as $\angle E = \angle F = 90^{o}$ and $\angle 2 = \angle 3$ (1)

SECTION – C

19. To prove $5 + \sqrt{2}$ is irrational, let us assume $5 + \sqrt{2}$ is rational.

$\therefore$ We can find integers a and b where a, b are co-prime, $b \neq 0$ (1)

Such that, $5 + \sqrt{2} = \frac{a}{b} \Rightarrow \sqrt{2} = \frac{a}{b} - 5$

Now a, b are integers, $\frac{a}{b} - 5$ is rational. (1)

$\Rightarrow \sqrt{2}$ is rational.

Which is a contradiction. So $5 + \sqrt{2}$ is irrational. (1)

OR

Let us assume to the contrary, that $\sqrt{n-1} + \sqrt{n+1}$ is a rational number. (1/2)

$\Rightarrow (\sqrt{n-1}+\sqrt{n+1})^2$ is rational.

$\Rightarrow (n-1)+(n+1)-2(\sqrt{n-1}\times\sqrt{n+1})$ is rational (1/2)

$\Rightarrow 2n+2\sqrt{n^2-1}$ is rational

But we know that $\sqrt{n^2-1}$ is an irrational number (1/2)

So $2n+2\sqrt{n^2-1}$ is also an irrational number (1/2)

So our basic assumption that the given number is rational is wrong.

Hence, $\sqrt{n-1} + \sqrt{n+1}$ is an irrational number. (1)

20. If the number 15^n where $n \in N$, were to end with a zero, then its prime factorisation must have 2 and 5 as its factors. (1)
But $15 = 5 \times 3$
$15^n = (5 \times 3)^n = 5^n \times 3^n$ (1)
So Prime factors of 15^n includes only 5 but not 2
Also from the Fundamental theorem of Arithmetic, the prime factorisation of a number is unique.
Hence a number of the form 15^n where $n \in N$, will never end with a zero. (1)

21. $f(x) = x^2 - 2x + 1$

Zeroes of f(x) are α, β

Sum of zeroes $\alpha + \beta = 2$ and $\alpha.\beta = 1$ $(\frac{1}{2})$

Now $\frac{2\alpha}{\beta} + \frac{2\beta}{\alpha} = 2\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = 2\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right)$ (1)

$$= 2\frac{((\alpha+\beta)^2 - 2\alpha\beta)}{\alpha\beta} = \frac{2 \times 2}{1} = 4$$

Also, $\frac{2\alpha}{\beta} \times \frac{2\beta}{\alpha} = 4$ $(\frac{1}{2})$

Required polynomial $= k(x^2 - 4x + 4)$, where k is any integer. (1)

22. Given: $\frac{\cos\alpha}{\cos\beta} = m,$ $\frac{\cos\alpha}{\sin\beta} = n$

$\Rightarrow m^2 = \frac{\cos^2\alpha}{\cos^2\beta},$ $n^2 = \frac{\cos^2\alpha}{\sin^2\beta}$ (1)

L.H.S. $= (m^2 + n^2)\cos^2\beta$

$= \left[\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right]\cos^2\beta$

$= \cos^2\alpha\left[\frac{\sin^2\beta + \cos^2\beta}{\sin^2\beta\cos^2\beta}\right]\cos^2\beta$

$= \cos^2\alpha\left(\frac{1}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$ (1)

$= \frac{\cos^2\alpha}{\sin^2\beta} = n^2 = \text{R.H.S.}$

Therefore, $(m^2 + n^2)\cos^2\beta = n^2$

(1)

OR

Using $\sec(90^\circ - \theta) = \operatorname{cosec}\theta,\ \tan(90^\circ - \theta) = \cot\theta$

and $\cos(90^\circ - \theta) = \sin\theta$ (1)

$$\frac{\sec(90^\circ - \theta).\operatorname{cosec}\theta - \tan(90^\circ - \theta)\cot\theta + \cos^2 25^\circ + \cos^2 65^\circ}{3\tan 27^\circ \tan 63^\circ}$$

$$= \frac{\operatorname{cosec}\theta.\operatorname{cosec}\theta - \cot\theta.\cot\theta + \cos^2(90^\circ - 65^\circ) + \cos^2 65^\circ}{3\tan(90^\circ - 63^\circ)\tan 63^\circ} \quad (1)$$

$$= \frac{\operatorname{cosec}^2\theta - \cot^2\theta + \sin^2 65^\circ + \cos^2 65^\circ}{3\cot 63^\circ \tan 63^\circ}$$

[Since, $\sin^2\theta + \cos^2\theta = 1$ and $\operatorname{cosec}^2\theta - \cot^2\theta = 1$]

$$= \frac{1+1}{3} = \frac{2}{3} \quad (1)$$

23. In ΔABC, $\angle B = 90°$, we have

$$\frac{AB}{AC} = \sin 30° = \frac{1}{2} \Rightarrow \frac{5}{AC} = \frac{1}{2} \Rightarrow AC = 10 \text{ cm} \quad (1)$$

And, $\frac{BC}{AC} = \cos 30° = \frac{\sqrt{3}}{2}$ (1)

$$\Rightarrow \frac{BC}{10} = \frac{\sqrt{3}}{2} \Rightarrow BC = 5\sqrt{3} \text{ cm} \quad (1)$$

24.

CI	50-60	60-70	70-80	80-90	90-100	100-110	Total
f_i	5	3	4	p	2	13	27+p
x_i	55	65	75	85	95	105	
$f_i x_i$	275	195	300	85p	190	1365	2325+85p

(1)

Mean = $\frac{\sum f_i x_i}{\sum f_i}$ (1)

Substituting the values we get

$$\Rightarrow 86 = \frac{2325 + 85p}{27 + p}$$

$\Rightarrow 86p + 2322 = 2325 + 85p$

$\Rightarrow p = 3$ (1)

25.

Age in yrs. (more than or equal to)	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of persons (f_i)	10	15	25	22	13	10	5

(1)

Since the maximum frequency is 25 and it lies in the class interval 20-30.
Therefore, modal class = 20 - 30
$\ell = 20$, $h = 10$, $f_0 = 15$, $f_1 = 25$, $f_2 = 22$

mode $= \ell + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$ (1)

$= 20 + \left(\frac{25-15}{2(25)-15-22}\right) \times 10$

= 20+7.69 = 27.69 years (approx.) (1)

26.
Let the length and breadth of the rectangle be x and y respectively.
So the original area of the rectangle=xy
According to question,

(x+2)(y-2)=xy-28 (1/2)

i.e. xy-2x+2y-4=xy-28

2x-2y=24 ...(i)

Next, (x-1)(y+2)=xy+33 (1/2)

i.e. xy+2x-y-2=xy+33

2x-y=35 ..(ii)

Now we need to solve (i) and (ii)
From (ii) we get,

y=2x-35 (1/2)

substituting this value in (i) we get,
2x-4x+70=24
-2x=-46
x=23 (1/2)
substituting this value in (ii)
we get,
y=11 (1/2)
So the length and breadth of the rectangle are 23 metres and 11 metres respectively. (1/2)

OR

Let 40 % acids in the solution be x litres
Let 60 % of other solution be y litres
Total Volume in the mixture = x + y
Given volume is 10 litres

x + y = 10 ---(i) (1)

Also, $\frac{40}{100}x + \frac{60}{100}y = \frac{50}{100} \times 10$

So, 40 x + 60 y =500 or 2x+3y =25 ...(ii) (1)
Solving (i) and (ii) we get x = y = 5 litres (1)

27. By BPT

$\frac{PX}{XQ} = \frac{PY}{YR}$ (1)

$\Rightarrow \frac{PX}{XQ} + 1 = \frac{PY}{YR} + 1$

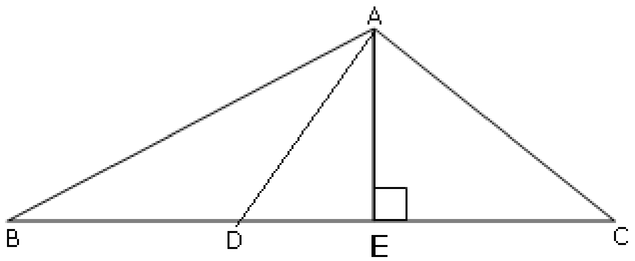
$\Rightarrow \frac{PX + XQ}{XQ} = \frac{PY + YR}{YR}$

$\Rightarrow \frac{PQ}{XQ} = \frac{PR}{YR}$ (1)

$\Rightarrow \frac{7}{3} = \frac{6.3}{YR}$

$\Rightarrow YR = \frac{6.3 \times 3}{7} = 2.7$ cm (1)

28.



To prove $AB^2 = AC^2 = 2AD^2 + 2\left(\frac{1}{2}BC\right)^2$

Draw AE ⊥ BC (1/2)

In ΔABD since ∠D > 90°

∴ $AB^2 = AD^2 + BD^2 + 2BD \times DE$ ...(1) (using Obtuse angle property) (1/2)

ΔACD = since ∠D < 90°

$AC^2 = AD^2 + DC^2 - 2DC \times DE$ ...(2) (using acute angle property) (1/2)

Adding (1) and (2)

$AB^2 + AC^2 = 2(AD^2 + BD^2)$

$=2(AD^2 + \left(\frac{1}{2}BC\right)^2)$ (1)

Or $AB^2 + AC^2 = 2(AD^2 + BD^2)$ (1/2)

Hence proved.

SECTION - D

29. To solve the equations, make the table corresponding to each equation.

$2x - y + 6 = 0$

$\Rightarrow y = 2x + 6$

x	−1	−2	−3
y	4	2	0

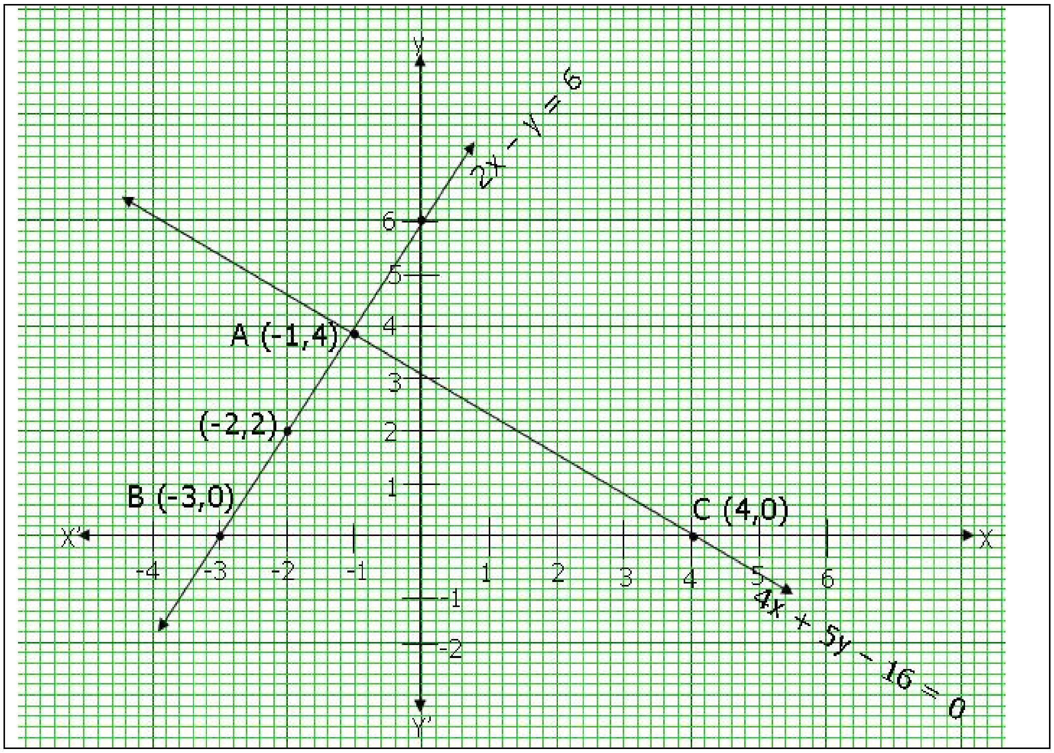
$4x + 5y - 16 = 0$

$\Rightarrow y = \frac{16 - 4x}{5}$

x	4	−1
y	0	4

(1)

Now plot the points and draw the graph.



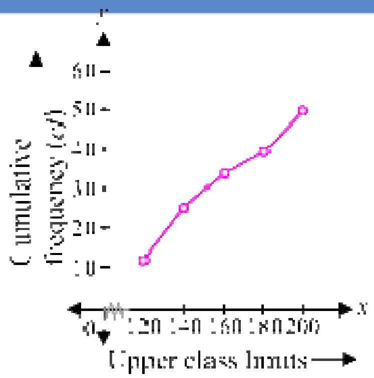
(2)

Since the lines intersect at the point (−1, 4), so x = −1 and y = 4 be the solution. Also by observation vertices of triangle formed by lines and x-axis are A (−1, 4), B (−3, 0) and C (4, 0). (1)

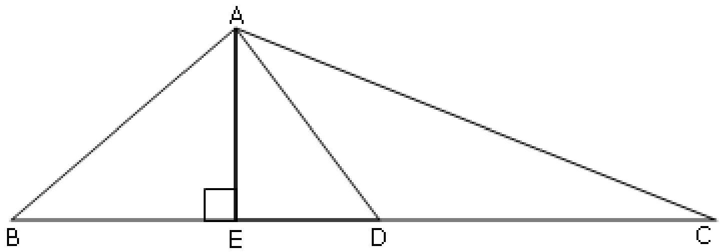
30. We can find frequency distribution table of less than type as following –

Daily income (in Rs) (upper class limits)	Cumulative frequency
Less than 120	12
Less than 140	12 + 14 = 26
Less than 160	26 + 8 = 34
Less than 180	34 + 6 = 40
Less than 200	40 + 10 = 50

Now taking upper class limits of class intervals on *x*-axis and their respective frequencies on *y*-axis we can draw its ogive as following –



31. AD is the median of ΔABC since D is mid point of BC



(1)

$\Rightarrow$ BD = DC= $\frac{BC}{2}$ ....(i)

In right triangle AEB,

$AB^2 = AE^2 + BE^2$ ...Pythagoras theorem

$= (AD^2 - DE^2) + (BD-DE)^2$ (1)

Using Pythagoras theorem for right triangle AED and BE=BD-DE

$= AD^2 - DE^2 + \left(\frac{BC}{2} - DE\right)^2$ ....from (i)

$AB^2 = AD^2 - DE^2 + \frac{BC^2}{4} + DE^2 - 2\left(\frac{BC \times DE}{2}\right)$ (1)

$\Rightarrow AB^2 = AD^2 - BC \times DE + \frac{BC^2}{4}$ (1)

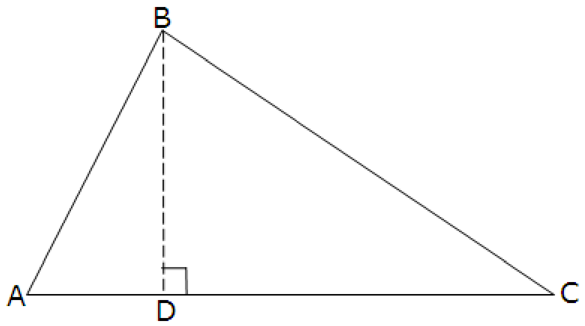
Hence proved.

OR

Given: A right triangle ABC right angled at B. (1/2)

To prove: that $AC^2 = AB^2 + BC^2$ (1/2)

Construction:Let us draw BD $\perp$ AC (See fig.) (1/2)



(1/2)

Proof : (2)

Now, $\Delta ADB \sim \Delta ABC$ (Using Theorem:If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse ,then triangles on both sides of the perpendicular are similar to the whole triangle and to each other)

So, $\frac{AD}{AB} = \frac{AB}{AC}$ (Sides are proportional)

Or, $AD.AC = AB^2$ (1)

Also, $\Delta BDC \sim \Delta ABC$ (Theorem)

So, $\frac{CD}{BC} = \frac{BC}{AC}$

Or, $CD.\ AC = BC^2$

Adding (1) and (2),

$AD.\ AC + CD.\ AC = AB^2 + BC^2$

OR, $AC\ (AD + CD) = AB^2 + BC^2$

OR, $AC.AC = AB^2 + BC^2$

OR $AC^2 = AB^2 + BC^2$

Hence Proved.

32. Let $p(x) = x^3 - 6x^2 - 15x + 80$

Let say that we subtracted ax + b so that it is exactly divisible by $x^2 + x - 12$

$s(x) = x^3 - 6x^2 - 15x + 80 - (ax + b)$

$= x^3 - 6x^2 - (15 + a)x + (80 - b)$ (1)

Dividend = Divisor x Quotient + Remainder

But remainder = 0

$\therefore$ Dividend = Divisor x Quotient (1)

s(x) = $(x^2 + x - 12)$ x quotient

s(x) = $x^3 - 6x^2 - (15 + a)x + (80 - b)$

$x(x^2 + x - 12) - 7(x^2 + x - 12)$

$= x^3 + x^2 - 7x^2 - 12x - 7x + 84$

$= x^3 - 6x^2 - 19x + 84$ (1)

Hence, $x^3 - 6x^2 - 19x + 84 = x^3 - 6x^2 - (15 + a)x + (80 - b)$

$-15 - a = -19 \quad \Rightarrow a = +4$

& $80 - b = 84 \quad \Rightarrow b = -4$ (1)

Hence if in p(x) we subtracted 4x - 4 then it is exactly divisible by $x^2 + x - 12$.

33. To prove: $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec}A + \cot A$

Using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$

L.H.S = $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} + \frac{1}{\sin A}} \quad (1)$$

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{\{(\cot A) - (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}{\{(\cot A) + (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}$$

$$= \frac{(\cot A - 1 + \operatorname{cosec} A)^2}{(\cot A)^2 - (1 - \operatorname{cosec} A)^2} \quad (1)$$

$$= \frac{\cot^2 A + 1 + \operatorname{cosec}^2 A - 2\cot A - 2\cos ecA + 2\cot A \operatorname{cosec} A}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A)}$$

$$= \frac{2\operatorname{cosec}^2 A + 2\cot A \operatorname{cosec} A - 2\cot A - 2\operatorname{cosec} A}{\cot^2 A - 1 - \operatorname{cosec}^2 A + 2\operatorname{cosec} A}$$

$$= \frac{2\operatorname{cosec} A(\operatorname{cosec} A + \cot A) - 2(\cot A + \operatorname{cosec} A)}{\cot^2 A - \operatorname{cosec}^2 A - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{-1 - 1 + 2\operatorname{cosec} A} \quad (1)$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{(2\operatorname{cosec} A - 2)}$$

$= \operatorname{cosec}A + \cot A$ (1)

= R.H.S

OR

$$\text{LHS} = \sqrt{\frac{1+\sin A}{1-\sin A}} = \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}} \quad (1)$$

$$= \sqrt{\frac{(1+\sin A)^2}{(1-\sin A)(1+\sin A)}} \quad (1)$$

$$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$$

$$= \frac{1+\sin A}{\sqrt{\cos^2 A}} \quad (1)$$

$$= \frac{1+\sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$= \sec A + \tan A = \text{RHS}$ (1)

34. $$\text{LHS} = \frac{P^2-1}{P^2+1} = \frac{(\sec\theta+\tan\theta)^2+1}{(\sec\theta+\tan\theta)^2+1}$$ (½)

$$= \frac{\sec^2\theta+\tan^2\theta+2\sec\theta\,.\,\tan\theta-1}{\sec^2\theta+\tan^2\theta+2\sec\theta\,.\,\tan\theta-1}$$ (½)

$$= \frac{(\sec^2\theta-1)+\tan^2\theta+2\sec\theta\tan\theta}{\sec^2\theta+(1+\tan^2\theta)+2\sec\theta\tan\theta}$$ (½)

$$= \frac{\tan^2\theta+\tan^2\theta+2\sec\theta\tan\theta}{\sec^2\theta+\sec^2\theta+2\sec\theta\tan\theta}$$ (½)

$$= \frac{2\tan^2\theta+2\sec\theta\tan\theta}{2\sec^2\theta+2\sec\theta\tan\theta}$$

$$= \frac{2\tan\theta(\tan\theta+\sec\theta)}{2\sec\theta(\tan\theta+\sec\theta)} = \frac{\tan\theta}{\sec\theta}$$ (½)

$$= \sin\theta = \text{RHS}$$ (½)