DO NOT OPEN TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Resonance
Educating for better tomorrow
IIT-JEE | AIPMT | AIEEE | OLYMPIADS | KVPY | NTSE

T.B.C. : Q–TDSB–M–NDT **Test Booklet Series**

Serial No. **A**

TEST BOOKLET
MATHEMATICS
PAPER -III

Time Allowed : Two Hours **Maximum Marks : 200**

INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS BOOKLET DOES ***NOT*** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.

2. ENCODE CLEARLY THE TEST BOOKLET SERIES A, B, C OR D AS THE CASE MAY BE IN THE APPROPRIATE PLACE IN THE ANSWER SHEET.

3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside.
DO NOT write *anything else* on the Test Booklet.

4. This Test Booklet contains 100 items (questions). Each item comprises several responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose ONLY ONE response for each item.

5. You have to mark all your responses ONLY on the separate Answer Sheet provided. See directions in the Answer Sheet.

6. All items carry equal marks.

7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.

8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator *only the Answer Sheet.* You are permitted to take away with you the Test Booklet.

9. Sheet for rough work is appended in the Test Booklet at the end.

10. **Penalty for wrong answers :**
THERE WILL BE PENALTY FOR WRONG ANSWER MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
(i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, one-third (0.33) of the marks assigned to that question will be deducted as penalty.
(ii) If a candidate gives more than one answer, it will be treated as a wrong answer even if one of the given answer happens to be correct and there will be same penalty as above to that question.
(iii) If a question is left blank, i.e., no answer is give by the candidate, there will be no penalty for that questions.

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

1. What is the coefficient of x^n in the expansion of $\left(1+\frac{x}{1}+\frac{x^2}{2!}+\frac{x^3}{3!}+.....+\frac{x^n}{n!}\right)^2$?

(a) $\frac{2^n}{n!}$ (b) 2^n (c) n! (d) $\frac{1}{n!}$

Ans. **(a)**

Sol. coeff. of x^n in

$$\left[e^x-\frac{x^{n+1}}{n+1!}.....\right]^2$$

$$= e^{2x}+\left(\frac{-x^{n+1}}{n+1!}....\right)^2+2e^x\left(\frac{-x^{n+1}}{n+1}....\right)$$

coeff of x^n in $\left[1+\frac{2x}{1!}+\frac{(2x)^2}{2!}...+\frac{(2x)^2}{n!}....\right]$

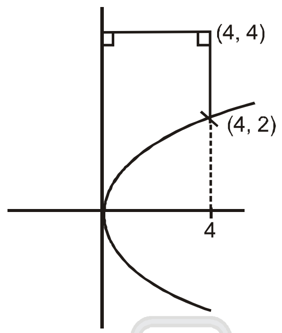
$$= \frac{2^n}{n!}$$

2. Two real numbers x and y are selected from the closed interval [0, 4]. What is the probability that the selected numbers satisfy the inequation $y^2 \le x$?

(a) $\frac{3}{16}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

Ans. **(c)**

Sol.



Total Area = 4 × 4 = 16

favorable area = $\int_0^4 \sqrt{x}\,dx$

$$= \left[\frac{2.4\times 2}{3}\right]$$

$$= \frac{16}{3}$$

Required Probability = $\frac{16}{3\times 16} = \frac{1}{3}$

3. What is $\int_{10}^{100} (x - [x])\,dx$ equal to, where [x] denotes the greatest integer function ?

(a) 90 (b) 45 (c) 0 (d) – 1

Ans. **(b)**

Sol. $\int_{10}^{100} \{x\}\,dx$

$= 90\int_{0}^{1} x\,dx$

$= \frac{90}{2} = 45$

4. Let f(x) = $x^2 - 1$ for $0 < x < 2$ and $2x + 3$ for $2 \le x < 3$. The quadratic equation whose roots are $\lim_{x \to 2-} f(x)$ and $\lim_{x \to 2+} f(x)$ is

(a) $x^2 - 4x + 21 = 0$ (b) $x^2 - 6x + 9 = 0$

(c) $x^2 - 10x + 21 = 0$ (d) $x^2 + 10x - 21 = 0$

Ans. **(c)**

Sol. $\alpha = \lim_{x \to 2^-} f(x)$

= 3

$\beta = \lim_{x \to 2^+} f(x)$

= 7

equation $x^2 - 10x + 21 = 0$

5. Let f(x) be a continuous function defined for $1 \le x \le 3$. If f(x) takes rational values for all x and f(2) = 10, then what is f(1.5) equal to ?

(a) 0 (b) 1

(c) 10 (d) Cannot be determined as the data is insufficient

Ans. **(c)**

Sol. takes rational values

$\Rightarrow$ f(x) = costant

$\Rightarrow$ f(2) = 10 = f(1·5)

6. If A = {–2, –1, 0, 1, 2} and

f : A $\to$ Z, f(x) = $x^2 - 2x - 3$,

then what is the pre-image(s) of – 3 ?

(a) 0 only (b) 2 only (c) 0, 2 (d) ϕ

Ans. **(c)**

Sol. f(x) = – 3

$x^2 - 2x - 3 = -3$

$\Rightarrow$ x = 0, 2

For the next two (02) questions that follow :

Consider the following determinant :

$$\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix}$$

7. What is the value of the determinant if $b^2 - ac < 0$ and $a > 0$?

(a) Positive (b) Negative (c) Zero (d) Sign cannot be determined

Ans. **(b)**

Sol. $c_3 \to c_3 - c_1 x - c_2 y$

$$\begin{vmatrix} a & b & 0 \\ b & c & 0 \\ ax+by & bx+cy & -x(ax+by)-y(bx+cy) \end{vmatrix}$$

$= [-ax^2 - bxy - bxy - cy^2]\,[(ac - b^2]$

$= [ax^2 + 2bxy + cy^2]\,[b^2 - ac]$

'D' of Quadratic $= 4(b^2 - ac) < 0$

and $a > 0$

Also $c > 0$

So $ax^2 + 2bxy + cy^2 > 0\ \forall\ x \in R$

and $b^2 - ac < 0$

$\Rightarrow$ value of determinant = – ve

8. What is the value of the determinant if $b^2 - ac = 0$ and $a > 0$?

(a) 0 (b) 1 (c) $b^2 + ac$ (d) abc

Ans. **(a)**

Sol. $b^2 - ac = 0 \quad \Rightarrow \quad$ value of determinant = 0

9. If f(x) is continuous for all real values of x, then what is $\sum_{r=1}^{n} \int_0^1 f(r-1+x)\,dx$ equal to ?

(a) $\int_0^n f(x)\,dx$ (b) $\int_0^1 f(x)\,dx$ (c) $n\int_0^1 f(x)\,dx$ (d) $(n-1)\int_0^1 f(x)\,dx$

Ans. **(a)**

Sol. sum $= \int_0^1 [f(x) + f(1 + x) + f(2 + x) \dots f(n - 1 + x]\,dx$

$= \int_0^1 f(x) + \int_0^1 f(1+x)\,dx + \int_0^1 f(2+x)\,dx \dots + \int_0^1 f(n-1+x)\,dx$

$= \int_0^1 f(x)dx + \int_1^2 f(x)dx + \int_2^3 f(x)dx + \dots\dots \int_{n-1}^{n} f(x)dx = \int_0^n f(x)dx$

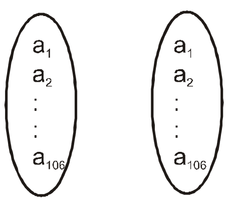
10. What is the number of bijective functions from a set A to itself when |A| = 106 ?

(a) 106 (b) $(106)^2$ (c) 106! (d) 2^{106}

Wrong question. How ever considering n(A) = 106.

Ans. **(c)**

Sol.



no. of bijective functions = 106 × 105 × 104 ......1 = 106!

11. Ten coins are thrown simultaneously. What is the probability of getting at least seven heads ?

(a) 3/64 (b) 5/64 (c) 7/64 (d) 11/64

Ans. (d)

Sol. [7(H) + 3(T)] + [8(H) + 3(T)] + [9(H) + 1(T)] + [10(H) + 0 (T)]

$$= {}^{10}C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^3 + {}^{10}C_3\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 + {}^{10}C_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right) + {}^{10}C_{10}\left(\frac{1}{2}\right)^{10}$$

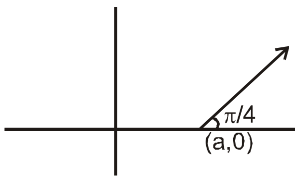
$$= \frac{176}{2^{10}} = \frac{11}{2^6} = \frac{11}{64}$$

12. What is the locus of a complex number $z = x + iy$ where $i = \sqrt{-1}$ in the Argand plane satisfying the relation $\arg(z - a) = \pi/4$, where 'a' is a real number ?

(a) $x^2 - y^2 = a^2$ (b) $x^2 + y^2 = a^2$ (c) $x + y = a$ (d) $x - y = a$

Ans. (d)

Sol.



$y = x + c$

passes (a, 0)

$0 = a + c$

$\Rightarrow c = -a$

so locus is $y = x - a$

$x - y = a$

13. If $|x^2 - 5x + 6| > x^2 - 5x + 6$, then which one of the following is correct ?

(a) $x > 3$ (b) $x < 2$ (c) $2 < x < 3$ (d) $-3 < x < -2$

Ans. (c)

Sol. Case-I : $x^2 - 5x + 6 > 0$

+ve > +ve

no solution

Case-II : $x^2 - 5x + 6 < 0$

$-(x^2 - 5x + 6) > (x^2 - 5x + 6)$

$\Rightarrow x^2 - 5x + 6 < 0 \Rightarrow x \in (2, 3)$

14. If $3^{49}(x + iy) = \left(\frac{3}{2} + i\frac{\sqrt{3}}{2}\right)^{100}$ and $x = ky$, then what is the value of k ? $(i = \sqrt{-1})$

(a) $-\frac{1}{3}$ (b) $\sqrt{3}$ (c) $-\sqrt{3}$ (d) $-\frac{1}{\sqrt{3}}$

Ans. (d)

Sol. $$3^{49}[ky + iy] = \left[\frac{\sqrt{3}}{2}\right]^{100}\left[\sqrt{3} + i\right]^{100}$$

$$= 3^{50}\left[\frac{\sqrt{3}}{2} + \frac{i}{2}\right]^{100}$$

$\Rightarrow\ 3^{49}(k + i)y = 3^{50}(e^{i\,\pi/6})^{100}$

$\Rightarrow (k + i)y = 3\left(e^{i\frac{50\pi}{3}}\right)$

$= 3\left[\cos\frac{50\pi}{3} + i\sin\frac{50\pi}{3}\right]$

$(k + i)y = 3\left[-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right]$ $\Rightarrow ky = -\frac{3}{2}$ and $y = \frac{3\sqrt{3}}{2}$

$\Rightarrow k\left(\frac{3\sqrt{3}}{2}\right) = -\frac{3}{2}$ $\Rightarrow k = -\frac{1}{\sqrt{3}}$

15. If A and B are two events such that $P(A) = \frac{3}{5}$, $P(B) = \frac{7}{10}$, then which one of the following is correct ?

(a) $P(A \cap B) \geq \frac{3}{10}$ (b) $P(A \cap B) \leq \frac{7}{10}$

(c) $\frac{3}{5} < P(A \cup B) < \frac{7}{10}$ (d) $\frac{3}{5} < P(A \cup B) \leq \frac{7}{10}$

Ans. **(a)**

Sol. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{5} + \frac{7}{10} - P(A \cup B)$

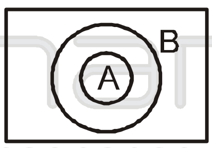
$P(A \cup B) = \frac{13}{10} - P(A \cap B)$ ...(i)

$P(A \cup B) \leq 1$

$\frac{13}{10} - p(A \cap B) \leq 1$

$P(A \cup B) \geq \frac{3}{10}$

$P(A \cap B) \leq \frac{3}{5}$



so $\frac{3}{10} \leq P(A \cap B) \leq \frac{3}{5}$

$-\frac{3}{5} \leq -P(A \cap B) \leq -\frac{3}{10}$

$\frac{13}{10} - \frac{3}{5} \leq \frac{13}{10} - P(A \cap B) \leq \frac{13}{10} - \frac{3}{10}$

$\frac{7}{10} \leq P(A \cup B) \leq 1$

For the next two (02) questions that follow :

Consider the function $f(x) = [x] \sin(\pi x)$ where $[x]$ is the greatest integer not exceeding x.

16. What is the limit of $f(x)$ as $x \to k$, where k is an integer ?

(a) -1 (b) 0 (c) $k - 1$ (d) none of the above

Ans. **(b)**

Sol. $\lim\limits_{x\to k} [x] \sin \pi x = 0$

17. What is the left hand derivative of $f(x)$ at $x = k$, where k is an integer ?

(a) $(-1)^k (k-1)\pi$ (b) $(-1)^{k-1}(k-1)\pi$ (c) $(-1)^k k\pi$ (d) $(-1)^{k-1} k\pi$

Ans. **(a)**

Sol. $\lim\limits_{h\to 0^+} \dfrac{f(k-h)-f(k)}{-h}$

$\lim\limits_{h\to 0^+} \dfrac{(k-1)\sin(\pi k - \pi h) - k(0)}{-h}$

$(-1)^{k-1}(k-1) \lim\limits_{h\to 0} \dfrac{\sin \pi h}{-h} = (-1)^k (k-1)\pi$

18. A determinant of the second order is made with the elements 0 and 1. What is the probability that the determinant made is non-negative ?

(a) 13/16 (b) 3/16 (c) 3/4 (d) 7/8

Ans. **(a)**

Sol. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \qquad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = 0$

$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} = 0$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = -1$

$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = 1 \qquad \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = 1 \qquad \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = 1 \qquad \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = -1$

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 0$

$\dfrac{13}{16}$

19. What is the sum of real roots of the equation $x^2 + 4|x| - 5 = 0$?

(a) 4 (b) 1 (c) 0 (d) -1

Ans. **(c)**

Sol.

$x > 0$	$x < 0$
$x^2 + 4x - 5 = 0$	$x^2 - 4x - 5 = 0$
$(x + 5)(x - 1) = 0$	$(x - 0)(x + 1) = 0$
$x = 1$	$x = -1$

Sum = 0

20. If $y^2 = P(x)$, a polynomial of degree $n \geq 3$, then what is $2\frac{d}{dx}\left(y^3\frac{d^2y}{dx^2}\right)$ equal to ?

(a) $-P(x)\,P'''(x)$ (b) $P(x)\,P'''(x)$
(c) $P(x)\,P''(x)$ (d) $-P(x)\,P''(x)$

Ans. **(b)**
Sol. $y^2 = P(x)$
$2yy' = p'(x)$
$2yy'' + 2(y')^2 = p''(x)$
$2y^3\,y'' + 2y^2\,(y')^2 = y^2\,p''(x)$

$$2y^3\,y'' + 2\left(\frac{p'(x)}{2}\right)^2 = y^2\,p''(x)$$

$$2y^3\,y'' = y^2\,p''(x) - \frac{(p''(x))^2}{2} = p(x)\,.p''(x) - \frac{1}{2}(p'(x))^2$$

$$\frac{d}{dx}(2y^3\,y'') = p(x).p'''(x)$$

21. How many natural numbers less than a million can be formed using the digits 0,7 and 8?
(a) 728 (b) 726 (c) 730 (d) 724
Ans. **(a)**
Sol. 0, 7, 8
Single digit $\boxed{2} = 2$
$\boxed{2}\boxed{3} = 6$
$\boxed{2}\boxed{3}\boxed{3} = 18$
$\boxed{2}\boxed{3}\boxed{3}\boxed{3} = 6 \times 9 = 54$
$\boxed{2}\boxed{3}\boxed{3}\boxed{3}\boxed{3} = 9 \times 6 \times 3 = 162$
$\boxed{2}\boxed{3}\boxed{3}\boxed{3}\boxed{3}\boxed{3} = 6 \times 9 \times 9 = 486$
Total = 2 + 6 + 18 + 54 + 162 + 486 = 728

22. Three persons A,B, C are to speak at a function along with five others. If they all speak in random order, what is the probability that A speaks before B and B speaks before C ?
(a) 3/8 (b) 1/6 (c) 3/5 (d) 5/6
Ans. **(b)**

Sol. Probability = $\frac{8!/3!}{8!} = \frac{1}{3!} = \frac{1}{6}$

23. If A is a skew-symmetric matrix of order 3, then matrix A^3 is a/an
(a) Orthogonal matrix (b) Diagonal matrix
(c) Symmetric matrix (d) Skew-symmetric matrix
Ans. **(d)**
Sol. $(A^3)^T = A^T \cdot A^T \cdot A^T = -A^3$

24. The solution of $\frac{dy}{dx} = \frac{ax+g}{by+f}$ represents a circle if

(a) a = b (b) a = – b (c) a = – 2b (d) a = 2b

Data in sufficient. Assuming a, b $\neq$ 0.

Ans. (b)

Sol. (by + f)dy = (ax + g)dx

$$\frac{by^2}{2} + fy = a\frac{x^2}{2} + gx + C$$

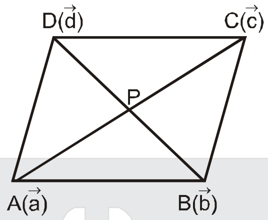
$a = -b \neq 0$, $g^2 + f^2 - 2C \geq 0$, for real circle.

25. Let ABCD be a parallelogram whose diagonals intersect at P. If O is the origin, then what is $\vec{OA}+\vec{OB}+\vec{OC}+\vec{OD}$ equal to ?

(a) $2\vec{OP}$ (b) $3\vec{OP}$ (c) $4\vec{OP}$ (d) $6\vec{OP}$

Ans. (c)

Sol. $\vec{OP} = \frac{\vec{a}+\vec{c}}{2} = \frac{\vec{b}+\vec{d}}{2}$



$2\vec{OP} = \frac{\vec{a}+\vec{c}+\vec{b}+\vec{d}}{2}$

$\vec{a}+\vec{c}+\vec{b}+\vec{d} = 4\vec{OP}$

26. Let f : R $\to$ R be a differentiable function and f(1) = 4. What is the value of $\lim_{x\to1}\int_4^{f(x)}\frac{2t\,dt}{x-1}$ equal to ?

(a) 8 f' (1) (b) 4 f' (1) (c) 2 f' (1) (d) f' (1)

Ans. (a)

Sol. $\int_4^{f(x)} 2t\,dt = \lim_{x\to1}\frac{(f(x))^2-16}{x-1}$

$\lim_{x\to1}\frac{2f(x)f'(x)}{1}$

8 f '(1)

27. All curves in the xy-plane having the property that the tangents pass through the origin are

(a) $y = cx^2$ (b) y = cx (c) $x = cy^2$ (d) xy = c

Ans. (b)

Sol. (x, y)

Y – y = m(X – x)

Passing through (0, 0) i.e. – y = –m x

$y = x\frac{dy}{dx} \Rightarrow \frac{1}{x}dx = \frac{1}{y}dy$

lnx = lny – lnc

y = cx

28. Consider the following statements :

1. If A = {(x, y) ∈ [R × R : $x^3 + y^3 = 1$] and B = {(x, y) ∈ [R : x – y = 1]}, then A ∩ B contains exactly one elements.

2. If A = {(x, y) ∈ [R × R : $x^3 + y^3 = 1$] and B = {(x, y) ∈ [R : x + y = 1]}, then A ∩ B contains exactly two elements.

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only (c) Both 1 and 2 (d) Neither 1 and 2

Ans. **(c)**

Sol. $x^3 + (x-1)^3 = 1$

$x^3 + x^3 - 3x^2 + 3x - 1 = 1$

$2x^3 - 3x^2 + 3x - 2 = 0$

$(x-1)(2x^2 - x + 2) = 0$

$x = 1$

$y = 0$ (1, 0)

Statement 1 is True

Statement 2 :

$X^3 + (1 - x)^3 = 1 \Rightarrow X^3 + 1 - 3x + 3x^2 - x^3 = 1$

$\Rightarrow x^2 - x = 0$

$x = 0, 1$ (0, 1) (1, 0)

29. For any n ≥ 2, let M_n (R) denote the set of all n × n matrices over the set of real numbers. Consider the following statements :

1. If A ∈ M_n (R) is a non-zero matrix with det A = 0, then det(adj A) = 0.

2. For any A ∈ M_n (R), det (adj A) = $(\det A)^{n-1}$

which of the above statements is/are correct ?

(a) 1 only (b) 2 only (c) Both 1 and 2 (d) Neither 1 nor 2

Ans **(c)**

Sol. Obivious

30. If the set of integers with the operation defined by $a * b = a + b - 1$ forms a group, what is the inverse of a ?

(a) –a (b) 2a (c) 2 – a (d) 1 – a

Ans **(c)**

Sol. $a * b = a + b - 1$

Let Identity = e

$a * e = a$

$a + e - 1 = a \Rightarrow e = 1$

now let inverse of a be b

$a * b = e = 1$

$a + b - 1 = 1$

$b = 2 - a$

31. Consider the following statements :

1. If for some positive integers p, q, r (p < q < r), $2^{2p+1}, 2^{2q+1}, 2^{2r+1}$ are in G.P., then p^{-1}, q^{-1}, r^{-1} will be in HP.

2. If x, y, z are positive integers such that x^{-1}, y^{-1}, z^{-1} are in HP , then for any real number u ≠ 0, $(xu)^{-1}, (yu)^{-1}, (zu)^{-1}$ will also be in HP.

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only (c) Both 1 and 2 (d) Neither 1 nor 2

Ans. **(c)**

Sol. Statement-1 : $2^{2P+1}, 2^{2q+1}, 2^{2r+1}$ are in G.P.

⇒ 2p+1, 2q+1, 2r+1 are in A.P.

⇒ p, q, r are in A.P.

$\Rightarrow \frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ are in H.P. as (0 < p < q < r)

Statement-2 : $\frac{1}{x},\frac{1}{y},\frac{1}{z}$ are in H.P.

$\Rightarrow \frac{1}{ux},\frac{1}{uy},\frac{1}{uz}$ are in H.P. $(u \neq 0)$

Hence Statement-1 and Statement-2 both are true.

32. Let us define the length of a vector $a\hat{i}+b\hat{j}+c\hat{k}$ = a + b + c. The definition coincides with the usual definition of length of a vector $a\hat{i}+b\hat{j}+c\hat{k}$ iff

(a) a = b = c = 0
(b) any two of a, b, c are zero
(c) any one of a, b, c is zero
(d) a + b + c = 0

Ans. **(a)**

Sol. $\left|a\hat{i}+b\hat{j}+c\hat{k}\right| = a + b + c$

$\sqrt{a^2+b^2+c^2} = a + b + c \quad \Leftrightarrow \quad ab + bc + ca = 0 \quad \Leftrightarrow \quad a = b = c = 0$

For the next three (03) questions that follow :
Consider the following integral :

$$I_n = \int_0^{\pi/4} \tan^n x\, dx \qquad \text{where } n \in N,\ n > 1$$

33. What is $I_n + I_{n-2}$ equal to ?

(a) $\frac{1}{n-1}$ (b) $\frac{1}{n}$ (c) $\frac{1}{n+1}$ (d) $\frac{1}{n+2}$

Ans. **(a)**

Sol. $I_n = \int_0^{\pi/4} \tan^n x\, dx \quad n \in N,\ n > 1$

$I_n = \int_0^{\pi/4} \tan^{n-2} x\left(\sec^2 x - 1\right)dx$

$I_n = \int_0^{\pi/4} \tan^{n-2} x \sec^2 x\, dx - \int_0^{\pi/4} \tan^{n-2} x\, dx$

$I_n + I_{n-2} = \int_0^{\pi/4} \tan^{n-2} x \sec^2 x\, dx$

$= \left.\frac{\tan^{n-1} x}{n-1}\right|_0^{\pi/4}$

$= \frac{1}{n-1}$

34. What is $I_{n-1} + I_{n+1}$ equal to ?

(a) $\frac{1}{n-1}$ (b) $\frac{1}{n}$ (c) $\frac{1}{n+1}$ (d) $\frac{1}{n+2}$

Ans. **(b)**

Sol. $I_{n+1} = \int_0^{\pi/4} \tan^{n+1} x\,dx$

$= \int_0^{\pi/4} \tan^{n-1} x(\sec^2 x - 1)\,dx$

$= \int_0^{\pi/4} \tan^{n-1} x \sec^2 dx - I_{n-1}$

$\Rightarrow I_{n+1} + I_{n-1} = \int_0^{\pi/4} \tan^{n-1} x \sec^2 x dx$

$= \left.\frac{\tan^n x}{n}\right|_0^{\pi/4}$

$= \frac{1}{n}$

35. Consider the following statements :

1. $\frac{1}{n+1} < 2I_n < \frac{1}{n-1}$ 2. $\frac{1}{n} < 2I_{n-1} < \frac{1}{n-2}$

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only (c) Both 1 and 2 (d) Neither 1 nor 2

Ans. **(c)**

Sol. $\tan^n x < \tan^{n-2} x$ $x \in (0, \pi/4)$

$\Rightarrow \int_0^{\pi/4} \tan^n x\,dx < \int_0^{\pi/4} \tan^{n-2} x\,dx$

$\Rightarrow I_n < I_{n-2}$

$\Rightarrow 2I_n < I_n + I_{n-2} = \frac{1}{n-1}$ ----------(1)

Similarly $I_{n+2} < I_n$

$I_{n+2} + I_n < 2I_n$

$\frac{1}{n+1} < 2I_n$ -----------(2)

by (1) and (2)

$\frac{1}{n+1} < 2I_n < \frac{1}{n-1}$ so statement (1) is correct

$n \to n - 1$ in statement 1

$\Rightarrow \frac{1}{n} < 2\,I_{n-1} < \frac{1}{n-2}$

So statement 2 is also correct

36. Out of 13 applicants for a job, there are 5 women and 8 men. It is desired to select 2 persons for this job. What is the probability that at least one of the selected persons will be a woman ?

(a) 5/13 (b) 10/13 (c) 14/39 (d) 25/39

Ans. **(d)**

Sol. Probability that at least on of the selected persons will be a woman

= 1 – probability that both are men

$$= 1 - \frac{^8C_2}{^{13}C_2}$$

$$= 1 - \frac{8 \times 7}{13 \times 12}$$

$$= 1 - \frac{14}{39} = \frac{25}{39}$$

37. If p is chosen at random is the closed interval [0, 5], the probability that the equation $x^2 + px + \frac{p+2}{4} = 0$ will have real roots is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{5}$ (c) $\frac{2}{3}$ (d) $\frac{3}{5}$

Ans. **(d)**

Sol. $x^2 + px + \frac{p+2}{4} = 0$ will have real roots

$\Rightarrow p^2 - 4\left(\frac{p+2}{4}\right) \geq 0$ $\Rightarrow p^2 - p - 2 \geq 0$

$\Rightarrow (p-2)(p+1) \geq 0$ $\Rightarrow p \in (-\infty, -1] \cup [2, \infty)$

but $p \in [0, 5]$ $\Rightarrow$ Required probability $= \frac{3}{5}$

38. The set of matrices

$S = \left\{\begin{bmatrix} x & -x \\ -x & x \end{bmatrix} \text{ such that } 0 \neq x \in R\right\}$ forms a group under multiplication with identify element

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

Ans. **(d)**

Sol. Let e is the identity matrix then

$\begin{bmatrix} x & -x \\ -x & x \end{bmatrix} \times e = \begin{bmatrix} x & -x \\ -x & x \end{bmatrix}$ $\therefore e \in S$

$\Rightarrow \begin{bmatrix} x & -x \\ -x & x \end{bmatrix}\begin{bmatrix} a & -a \\ -a & a \end{bmatrix} = \begin{bmatrix} x & -x \\ -x & x \end{bmatrix}$

$\Rightarrow a + a = 1 \Rightarrow a = \frac{1}{2}$ $\Rightarrow e = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}$

39. If A, B, C are acute positive angles such that A + B + C = π and cotA . cot B . cot C = k, then

(a) $k \le \frac{1}{3\sqrt{3}}$ (b) $k \ge \frac{1}{3\sqrt{3}}$ (c) $k < \frac{1}{9}$ (d) $k > \frac{1}{3}$

Ans. **(a)**

Sol. $\because \tan A + \tan B + \tan C \ge 3(\tan A \tan B \tan C)^{1/3}$

$\Rightarrow \tan A + \tan B + \tan C \ge 3\left(\frac{1}{k}\right)^{1/3}$

$\frac{1}{k} \ge 3\left(\frac{1}{k}\right)^{1/3}$

$\frac{1}{k^3} \ge \frac{27}{k}$ $\Rightarrow$ $\frac{1}{27} \ge k^2$

$\frac{1}{3\sqrt{3}} \ge k$

40. If A = $\sin^2\theta + \cos^4\theta$, then for all real θ, which one of the following is correct ?

(a) $1 \le A \le 2$ (b) $\frac{3}{4} \le A \le 1$ (c) $\frac{13}{16} \le A \le 1$ (d) $\frac{3}{4} \le A \le \frac{13}{16}$

Ans. **(b)**

Sol. $A = \sin^2\theta + \cos^4\theta$

$\Rightarrow A = (1 - \sin^2\theta)^2 + \sin^2\theta$

$\Rightarrow A = \sin^4\theta - \sin^2\theta + 1$

$\Rightarrow A = \left(\sin^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4}$

$\because 0 \le \sin^2\theta \le 1$

$\Rightarrow -\frac{1}{2} \le \sin^2\theta - \frac{1}{2} \le \frac{1}{2}$

$\Rightarrow 0 \le (\sin^2\theta - \frac{1}{2})^2 \le \frac{1}{4}$

$\Rightarrow \frac{3}{4} \le (\sin^2\theta - \frac{1}{2})^2 + \frac{3}{4} \le 1$

$\Rightarrow \frac{3}{4} \le A \le 1$

41. If S is finite set containing n elements, then what is the total number of binary operations on S ?

(a) n^n (b) 2^{n^2} (c) n^{n^2} (d) n^2

Ans. **(c)**

Sol. S is finite set containing n elements

Number of elements in S × S is n^2

Now binary operation on S is a function f : S × S $\to$ S

Number of binary operations on S = n^{n^2}

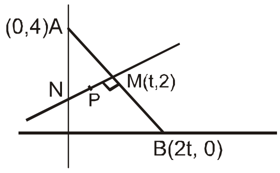
42. Let A be the fixed point (0, 4) and B be a moving point (2t, 0). Let the mid-point of AB be M. Let the perpepependicular bisector of AB meet the y-axis at N. What is the locus of mid-point P of MN ?

(a) $x^2 + (y-2)^2 = 1/2$ (b) $-x^2 + (y-2)^2 = 1/2$

(c) $x^2 + y = 2$ (d) $x + y^2 = 2$

Ans. **(c)**

Sol.



Equation of perpendicular bisector of AB is $y - 2 = \frac{t}{2}(x - t)$

$\therefore$ N $\left(0, 2 - \frac{t^2}{2}\right)$

$\therefore$ P$\left(\frac{t}{2}, 2 - \frac{t^2}{4}\right)$

$\therefore$ Locus of P is $x^2 + y = 2$

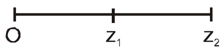
43. If z_1 , z_2 are two non zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then what is arg $(z_1) - \arg(z_2)$ equal to ?

(a) $-\pi$ (b) $-\frac{\pi}{2}$ (c) 0 (d) $\frac{\pi}{2}$

Ans. **(c)**

Sol. $|z_1 + z_2| = |z_1| + |z_2|$

$\therefore$ z_1 and z_2 are collinear and are at same side w.r.t. origin



$\therefore$ $\arg(z_1) - \arg(z_2) = 0$

44. What is the equation to the line passing through a $\hat{i}$ and perpendicular to $\hat{j}$ and $\hat{k}$?

(a) x-axis (b) y-axis (c) z-axis (d) None of these

Ans. **(a)**

Sol. Passing through (a,0,0) and perpendicular to $\hat{j}$ and $\hat{k}$ is x-axis

45. If $\vec{a},\vec{b},\vec{c}$ are non-zero vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$, then consider the following statements :

1. $\vec{a},\vec{b},\vec{c}$ are orthogonal in pairs

2. Each of $\vec{a},\vec{b},\vec{c}$ are unit vectors, Which of the above statements is/are correct ?

(a) 1 only (b) 2 only (c) Both 1 and 2 (d) Neither 1 nor 2

Ans. **(a)**

Sol. Since $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$

$\therefore$ $\vec{c} \perp \vec{a}$ and $\vec{b}$ $\therefore$ $\vec{a} \perp \vec{b}$ and $\vec{c}$

But $\vec{a},\vec{b},\vec{c}$ may be unit vector but not compulsarily

46. Consider the following statements :

1. The sum of two unit vectors can be a unit vector.
2. The magnitude of the difference between two unit vectors can be greater than the magnitude of a unit vector.

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only
(c) both 1 and 2 (d) Neither 1 nor 2

Ans. **(c)**

Sol. (1) $|\vec{a}+\vec{b}| = \sqrt{|\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta}$

For $\theta = \frac{2\pi}{3}$

$= \sqrt{1+1+2\left(-\frac{1}{2}\right)} = 1$

(2) $|\vec{a}-\vec{b}| = \sqrt{|\vec{a}|^2+|\vec{b}|^2-2|\vec{a}||\vec{b}|\cos\theta}$

For $\theta = \frac{2\pi}{3}$

$= \sqrt{1+1-2\left(-\frac{1}{2}\right)} = \sqrt{3}$

47. If $\vec{a}$ and $\vec{b}$ are two non-zero non-colinear vectors and m, n are scalars such that $m\vec{a}+n\vec{b}=\vec{0}$, then

(a) $m \neq 0, n \neq 0$ (b) $m = 0, n \neq 0$ (c) $m \neq 0, n = 0$ (d) $m = 0, n = 0$

Ans. **(d)**

Sol. $\because$ $m\vec{a}+n\vec{b}=\vec{0}$

$\vec{a} = -\frac{n}{m}\vec{b}$

But $\vec{a}$ and $\vec{b}$ are non-collinear, non-zero vectors

so $\vec{a} \neq \lambda\vec{b}$

$\therefore$ m = 0 and n = 0

48. What is $\int e^x(2+\sin 2x)$ $\sec^2 x\,dx$ equal to ?

(a) $e^x(1 + \cos 2x)\sin x + c$ (b) $e^x(\sec x + \tan x) + c$
(c) $2e^x \tan x + c$ (d) $2e^x \sec x + c$

Ans. **(c)**

Sol. $\int e^x(2+\sin 2x)\sec^2 x\,dx$

$= \int e^x\left(2\sec^2 x + 2\tan x\right)dx$

$= e^x \cdot 2\tan x + C$

For the next two (02) question that follow :

The last term in the binomial expansion of

$$\left(2^{1/3}-\frac{1}{\sqrt{2}}\right)^n \text{ is } \left(\frac{1}{3\sqrt[3]{9}}\right)^{\log_3 8}$$

49. What is n equal to ?

(a) 8 (b) 9 (c) 10 (d) 11

Ans. **(c)**

Sol. $\left(2^{1/3}-\frac{1}{\sqrt{2}}\right)^n$

Last term = $\left(-\frac{1}{\sqrt{2}}\right)^n = \left(\frac{1}{3(9)^{1/3}}\right)^{\log_3 8}$

$-\frac{1}{2^{n/2}} = \left(\frac{1}{3.3^{2/3}}\right)^{\log_3(2^3)}$

$-\frac{1}{2^{n/2}} = 3^{-\frac{5}{3}(3\log_3 2)}$

$-\frac{1}{2^{n/2}} = 3^{\log_3(2)^{-5}}$

$-\frac{1}{2^{n/2}} = 2^{-5}$

$\therefore\ \frac{n}{2} = 5$

n = 10

50. What is the 5^{th} term in the expansion ?

(a) 840 (b) 720 (c) 360 (d) 210

Ans. **(d)**

Sol. $T_{4+1} = {}^{10}C_4\,(2^{1/3})^6\left(-\frac{1}{\sqrt{2}}\right)^4$

$= \frac{10.9.8.7}{4.3.2.1}.\,2^2.\,\frac{1}{2^2} = 210$

For the next two (02) questions that follow :

Let f be a twice differentiable function such that f ''(x) = –f(x) and f '(x) = g(x).

Let h(x) = $\{f(x)\}^2 + \{g(x)\}^2$ where h(5) = 11

51. What is h'(x) equal to ?

(a) 0 (b) 1 (c) x (d) x^2

Ans. **(a)**

Sol. $\because f''(x) = -f(x)$

$\therefore h(x) = (f(x))^2 + (f'(x))^2$

$= 2f(x)\,f'(x) + 2f'(x)\,f''(x)$

$= 2f(x)\,f'(x) - 2f'(x)\,f(x)$

$= 0$

52. What is h(10) equal to ?

(a) 0 (b) 11 (c) 22 (d) 44

Ans. **(b)**

Sol. Let $f(x) = \sqrt{11}\ \sin x$

$f'(x) = \sqrt{11}\cos x = g(x)$

$f''(x) = -\sqrt{11}\sin x$

$\because\ h(x) = \{f(x)\}^2 + \{g(x)\}^2$

$h(x) = 11(\sin^2 x + \cos^2 x)$

$h(x) = 11$

$h(10) = 11$

53. If $B = B^2$ and $I - B = A$, then which one of the following is correct ?

(a) $A^2 = B$ (b) $A^2 = A$ (c) $A^2 = I$ (d) $A^2 = -A$

Ans. **(b)**

Sol. $\because\ B = B^2,\ I - B = A$

$\therefore\ A^2 = (I - B)(I - B)$

$= I - B - B + B^2$

$= A$

54. What is the equation of the curve through the point (1, 1) and whose is $\frac{2ay}{x(y-a)}$?

(a) $y^a x^{2a} = e^{y-1}$ (b) $y^a x^{2a} = e^y$

(c) $y^{2a} x^a = e^{y-1}$ (d) $y^{2a} x^a = e^y$

Ans. **(a)**

Sol. $y^a.\ x^{2a} = e^{y-1}$ ....(i)

Differentiate both side w.r.t. x

$y^a.\ 2ax^{2a-1} + ay^{a-1}.x^{2a}.\ y' = e^{y-1}.\ y'$

$$y' = \frac{2a.y^a.\frac{x^{2a}}{x}}{e^{y-1} - a.\frac{y^a x^{2a}}{y}}$$

from (i)

$$y' = \frac{y.2a\,e^{y-1}}{x.(ye^{y-1} - a.e^{y-1})} = \frac{2ay}{x(y-a)}$$

55. What is the degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{2/3} + 4 - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0$?

(a) 1 (b) 2 (c) 3 (d) 4

Ans. **(b)**

Sol. $$\left(\frac{d^3y}{dx^3}\right) = \left(3\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 4\right)^3$$

Power of highest order derivative is 2.

so degree is 2.

56. Consider the following statements in respect of the function f(x) = 1 + |sin x|
1. f(x) is continuous everywhere.
2. f(x) is differentiable at x = 0
Which of the above statements is/are correct ?
(a) 1 only (b) 2 only (c) both 1 and 2 (d) Neither 1 nor 2

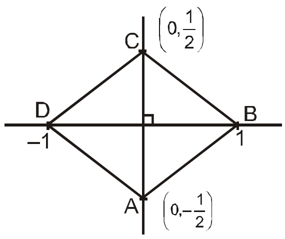
Ans. **(a)**

Sol. f(x) = 1 + |sin x|
only (1) is true but (2) is not true
since it is not differentiable at x = 0

57. What do the lines |x| + 2|y| = 1 represent ?
(a) sides of a triangle (b) sides of a rhombus
(c) sides of a square (d) None of the above

Ans. **(b)**

Sol.



AB = BC = CD = DA
|x| + 2|y| = 1
sides of Rhombus

58. The function $\frac{a\sin x + b\cos x}{c\sin x + d\cos x}, x \in R$ attains neither maximum nor minimum if

(a) $\frac{a}{c} = \frac{b}{d}$ (b) $\frac{a}{d} = \frac{b}{c}$ (c) $\frac{a}{d} = \frac{c}{b}$ (d) None of these

Ans. **(d)**

Sol. $f(x) = \frac{a\sin x + b\cos x}{c\sin x + d\cos x}$

$$f'(x) = \frac{(c\sin x + d\cos x)\,(a\cos x - b\sin x) - (a\sin x + b\cos x)(c\cos x - d\sin x)}{(c\sin x + d\cos x)^2}$$

$f'(x) = \frac{ad - bc}{(c\sin x + d\cos x)^2}$ $f'(x) \neq 0$ $\Rightarrow$ $ad \neq bc$

59. What is $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ equal to ?

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi^2}{12}$ (d) None of these

Ans. **(b)**

Sol. $I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ ...(i)

Here a + b = π/2 using property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{2} - x\right)} + \sqrt{\sin\left(\frac{\pi}{2} - x\right)}} dx \quad ...(ii)$$

equation (i) and (ii)

$$2I = \int_{\pi/6}^{\pi/3} 1dx = (x)_{\pi/6}^{\pi/3} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

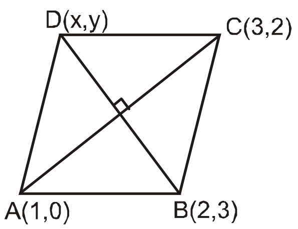
$$I = \frac{\pi}{12}$$

60. If the three vertices of a parallelogram ABCD are A(1, 0), B (2, 3) , C(3, 2) what are the coordinates of the fourth point ?

(a) (2, 1) (b) (2, – 1) (c) (–1, 2) (d) (–1, – 2)

Ans. **(b)**

Sol.



mid point of AC = mid point of BD

$$\frac{3+1}{2} = \frac{2+x}{2}$$

x = 2

and $\frac{0+2}{2} = \frac{3+y}{2}$

y = –1 $\Rightarrow$ (2, – 1)

61. If one of the lines represented by $6x^2 + kxy + y^2 = 0$ is $2x + y = 0$, then

(a) 3 (b) 4 (c) 5 (d) –5

Ans. **(c)**

Sol. $6x^2 + kxy + y^2 = 0$, $y = -2x$

$\Rightarrow$ $6x^2 + kx(-2x) + (-2x)^2 = 0$

$\Rightarrow$ $x^2(6 - 2k + 4) = 0$

$\Rightarrow$ $x^2(-2k + 10) = 0$

k = 5

62. If H is an orthogonal square matrix, then what is the determinant of H ?
(a) 0 (b) 1 (c) 2 (d) 4

Ans. **(b)**

Sol. $AA^T = I$
$|AA^T| = |I| = 1$
$|A|\,|A^T| = 1$
$|A|^2 = 1$
$|A| = \pm 1$
No correct option. Suitable answer is (b).

63. A family of curves involves four parameters. Let the order of diferential equation representing the family be m. What is 'm' equal to ?
(a) 1 (b) 2 (c) 3 (d) 4

Ans. **(d)**

Sol. Curves involves four parameter
so order is 4

64. The number of pairs of parallel tangents that an ellipse has, is
(a) zero (b) 1 (c) 2 (d) Infinite

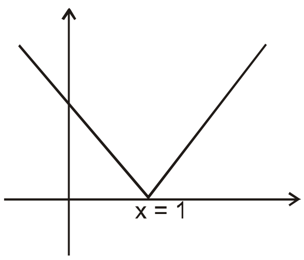
Ans. **(c)**

Sol. Obvious it has two parallel tangents.

65. Consider the function f(x) = |x – 1| defined on an interval [–1, 2]. The point x_0 where f′(x) = 0 on that interval
(a) is 0 (b) is 1 (c) is –1 (d) does not exist

Ans. **(d)**

Sol.



$f(x) = |x - 1|$
$f'(x) = 0$
No possible so does not exist.

66. Let $x^m + y^m = 1$. If $\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{1/3}$, then what is the value of m ?
(a) 1/3 (b) 2/3 (c) 1 (d) 3/2

Ans. **(b)**

Sol. $x^m + y^m = 1$

$mx^{m-1} + my^{m-1}\frac{dy}{dx} = 0$

$$\left(\frac{dy}{dx}\right) = -\frac{mx^{m-1}}{my^{m-1}} = -\left(\frac{x}{y}\right)^{m-1} = -\left(\frac{y}{x}\right)^{1-m}$$

$\Rightarrow 1 - m = \frac{1}{3}$

$m = 1 - \frac{1}{3}$

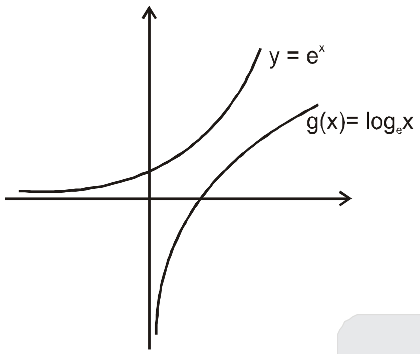
$m = \frac{2}{3}$

67. Consider the functions $f(x) = e^x$, $g(x) = \log_e x$. Which one of the following statements is not correct ?

(a) f(x) is always positive
(b) f(x) > g(x) for all values of x
(c) g(x) is always positive
(d) f(x) and g(x) curves never intersect

Ans. **(c)**

Sol.



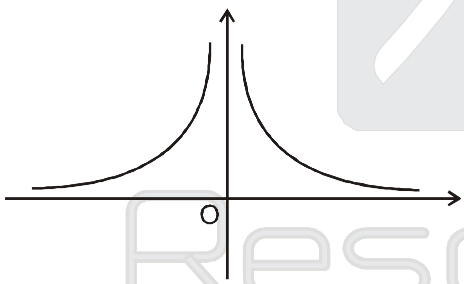
By graph g(x) is not always positive.

68. Let $f(x) = \frac{1}{|x|}$. In which one of the following intervals is the function discontinuous ?

(a) (0, 1) (b) (–1, 0) (c) [0, 1] (d) (0, 1]

Ans. **(c)**

Sol.



$y = \frac{1}{|x|}$ graph is

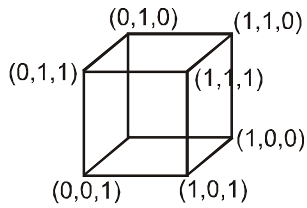
clearly it is discontinuous at x = 0

69. A line makes equal angles with the diagonals of a cube. What is the sine of the angle ?

(a) $\sqrt{\frac{2}{3}}$ (b) $\sqrt{\frac{1}{3}}$ (c) $\sqrt{\frac{1}{2}}$ (d) none of the above

Ans. **(a)**

Sol. Let DC's of line are l,m,n
DR's of diagonals of a cube are
(1, 1, 1), (–1, 1, 1), (1, 1, –1) and (1, –1, 1)
Now



$$\cos\theta = \left|\frac{\ell+m+n}{\sqrt{3}}\right| = \left|\frac{-\ell+m+n}{\sqrt{3}}\right| = \left|\frac{\ell+m-n}{\sqrt{3}}\right| = \left|\frac{\ell-m+n}{\sqrt{3}}\right|$$

This is only possible when
(ℓ, m, n) are (±1, 0, 0) or (0, ±1, 0) or (0, 0, ±1)

$$\therefore \cos\theta = \left|\pm\frac{1}{\sqrt{3}}\right|$$

$$\sin\theta = \sqrt{1-\frac{1}{3}} = \sqrt{\frac{2}{3}}$$

70. In an arithmetic series of 16 terms with first term 16, the sum is equal to the square of the last term. What is the common difference ?
(a) 8/5 (b) –4/5 (c) –8/5 (d) None of the above

Ans. **(c)**

Sol. $a = 16$
$n = 16$
$S_{16} = (T_{16})^2$

$$\frac{16}{2}[32 + (16-1)d] = (16 + 15d)^2$$

$8(32 + 15d) = (16 + 15d)^2$
$256 + 120d = 256 + 225d^2 + 480d$
$225d^2 + 360d = 0$

$$d = 0 \quad \text{or} \quad d = -\frac{360}{225} \quad \Rightarrow \quad d = -\frac{8}{5}$$

71. The solution of $|x-2| < 5$ is all the real numbers satisfying
(a) $-2 < x < 5$ (b) $-3 < x < 7$ (c) $-5 < x < 7$ (d) $-3 < x < 5$

Ans. **(b)**

Sol. $|x-2| < 5$
$-5 < x - 2 < 5$
$-3 < x < 7$

72. Let $\bar{z}$ be the complex conjugate of z. The curve represented by $\text{Re}(\bar{z})^2 = k^2$ is a
(a) parabola (b) rectangular hyperbola
(c) hyperbola but not rectangular (d) pair of straight lines
In sufficient data. $k \neq 0, k \in R$

Ans. **(b)**

Sol. $\text{Re}\,(\bar{z})^2 = k^2$
$\text{Re}(x - iy)^2 = k^2$
$x^2 - y^2 = k^2$
Rectangular hyperbola

73. Which one of the following functions is inverse of itself ?

(a) $\frac{1-\sin x}{1+\sin x}$ (b) $\frac{1-x^2}{2+x^2}$

(c) $\frac{1}{2}\ell n\left(\frac{e^x+e^{-x}}{e^x-e^{-x}}\right)$ (d) none of the above

Ans. **(c)**

Sol. (a) f(x) = $\frac{1-\sin x}{1+\sin x}$ many one function so not invertible

(b) f(x) = $\frac{1-x^2}{2+x^2}$ many one function so not invertible

(c) f(x) = $\frac{1}{2}\ell n\frac{e^x+e^{-x}}{e^x-e^{-x}}$

$$2y = \ell n\frac{e^x+e^{-x}}{e^x-e^{-x}}$$

$$\frac{e^{2y}}{1} = \frac{e^x+e^{-x}}{e^x-e^{-x}}$$

$$\frac{e^{2y}+1}{e^{2y}-1} = \frac{2e^x}{2e^{-x}}$$

$$\frac{e^y+e^{-y}}{e^y-e^{-y}} = e^{2x}$$

$$2x = \ell n\left(\frac{e^y+e^{-y}}{e^y-e^{-y}}\right)$$

$$f^{-1}(x) = \frac{1}{2}\ell n\left(\frac{e^x+e^{-x}}{e^x-e^{-x}}\right) = f(x)$$

74. Let N_1 be the total number of injective mappings from a set with m elements to a set with n elements when $m \le n$. When m > n, let the number of injective mappings be N_2. Then

(a) $N_1 > N_2$ (b) $N_1 < N_2$ (c) $N_1 = N_2$ (d) $N_1 + N_2 = m + n$

Ans. **(a)**

Sol. $f : A \to B$

n(A) = m

n(B) = n $\quad m \le n$

when m > n then no injective mapping is possible

so $\quad N_2 = 0$

$\therefore \quad N_1 > N_2$

75. Let A_1 be the area enclosed in between an ellipse and the circle drawn with the major axis as a diameter and A_2 be the area enclosed between the same ellipse and the circle drawn with the minor axis as diameter. Then which one of the following is correct ?

(a) $A_1 = A_2$ (b) $A_1 < A_2$ (c) $A_1 > A_2$ (d) $A_1 - A_2 = \pi(b^2 - a^2)$

Ans. **(c)**

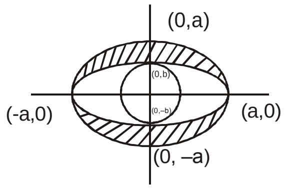
Sol. Area $A_1 = \pi\ a^2 - \pi ab$

$A_1 = \pi\ a\ (a-b)$.........................1

and $A_2 = \pi ab - \pi b^2$

$A_2 = \pi\ b\ (a-b)$..........................2

$A_1 > A_2$



76. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b}$ be a vector such that $\vec{a} \bullet \vec{b} = 0$ and $\vec{a} \times \vec{b} = \vec{0}$. Then which one of the following is correct?

(a) $\vec{b} = \lambda\vec{a}$ for some scalar λ (b) $\vec{b} = \vec{0}$

(c) $\vec{b}$ is perpendicular to $\vec{a}$ (d) $\vec{b}$ is non-zero vector

Ans. **(b)**

Sol. Option (a) is incomplete. It should be $\lambda \neq 0$.

$$\vec{a} \times \vec{b} = \vec{0}$$
$$\vec{b} = \lambda\vec{a}$$
$$\vec{b} = \lambda(\hat{i} + \hat{j} + \hat{k})$$
$$\vec{a}.\vec{b} = \lambda(1+1+1) = 3\lambda = 0$$
$$\therefore \lambda = 0$$
$$\therefore \vec{b} = \vec{0}$$

77. What is $\int_0^1 \frac{x^2dx}{\sqrt{x^6+1}}$ equal to?

(a) $\frac{1}{3}\ell n(\sqrt{2}+1)$ (b) $\frac{1}{2}\ell n(\sqrt{2}+1)$ (c) $\frac{1}{3}\ell n(\sqrt{2}-1)$ (d) $\frac{1}{2}\ell n(\sqrt{2}-1)$

Ans. **(a)**

Sol. $\int_0^1 \frac{x^2dx}{\sqrt{x^6+1}}$

$x^3 = t$

$3x^2dx = dt$

$$I = \frac{1}{3}\int_0^1 \frac{dt}{\sqrt{t^2+1}}$$

$$I = \frac{1}{3}\left[\ln\left|t + \sqrt{t^2+1}\right|\right]$$

$$I = \frac{1}{3}\left[\ln\left|1+\sqrt{2}\right| - \ln 1\right]$$

$$I = \frac{1}{3}\ln(\sqrt{2}+1)$$

78. Let f(x) be a function defined in the closed interval I where I = [0, 1].

If $f(x) = \begin{cases} 0, \text{ when x is rational} \\ 1, \text{ when x is irrational} \end{cases}$

Then which one of the following is correct?

(a) f(x) is continuous on I

(b) f(x) is continuous on I except for a finite number of points

(c) f(x) is continuous no where on I

(d) None of the above

Ans. **(c)**

Sol. $f(x) = \begin{cases} 0 \;, \; x \in Q \\ 1 \;, \; x \in Q^c \end{cases}$ $\quad f : [0,1] \to [0,1]$

f (x) is not continuous at any point in given intervals

79. If A is a 3×3 matrix and det(3A) = k det(A), then what is the value of k?

(a) 3 (b) 9 (c) 27 (d) 81

Ans. **(c)**

Sol. det (3A) = k (det A)

$|3A| = 3^3|A|$

$= 27|A|$

$\therefore$ k = 27

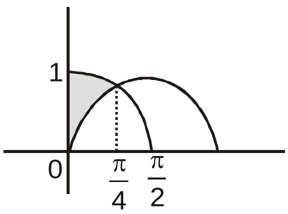
80. What is the area of the region in the first quadrant bounded by the y-axis and the curves y=sin x and y = cos x?

(a) $\sqrt{2}$ square units (b) $\sqrt{2}$ + 1 square units

(c) $\sqrt{2}$ – 1 square units (d) $2\sqrt{2}$ + 1 square units

Ans **(c)**

Sol.



$$I = \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx$$

$$= (\sin x + \cos x)_0^{\pi/4}$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1$$

$$= \sqrt{2} - 1$$

81. What is $\int_{-1}^{1} \frac{|x|}{x} dx$ equal to?

(a) 2 (b) 0 (c) 1 (d) 1/2

Ans. (b)

Sol.

$$\int_{-1}^{1} \frac{|x|}{x} dx = \int_{-1}^{0} -1 dx + \int_{0}^{1} dx$$

$$= (-x)_{-1}^{0} + (x)_{0}^{1}$$

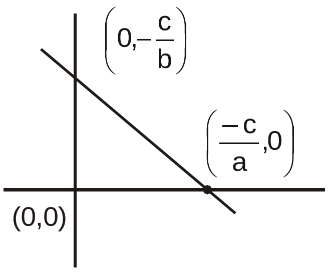
$$= -1 + 1 = 0$$

82. The straight line ax + by + c = 0 and the co-ordinate axes form an isosceles triangle when

(a) |a| = |b| (b) |a| = |c| (c) |b| = |c| (d) None of the above

Ans. (a)

Sol.



$$\left|-\frac{c}{a}\right| = \left|-\frac{c}{b}\right|$$

$$|a| = |b|$$

83. If the planes $\vec{r} \bullet (2\hat{i} - \lambda\hat{j} + \hat{k}) = 3$ and $\vec{r} \bullet (4\hat{i} + \hat{j} - \mu\hat{k}) = 5$ are parallel, then what is λ equal to?

(a) 1/2 (b) 1 (c) –1/2 (d) –1

Ans. (c)

Sol. $\bar{r}.(2\hat{i} - \lambda\hat{j} + \hat{k}) = 3$ ----------- (i)

and $\bar{r}.(4\hat{i} + \hat{j} - \mu\hat{k}) = 5$ ----------- (ii)

both are parallel, then

$\Rightarrow \quad \frac{2}{4} = -\frac{\lambda}{1} = \frac{1}{-\mu}$

$\Rightarrow \quad \lambda = -\frac{1}{2}$ and $\mu = -2$

84. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ and $m = \vec{a} \bullet \vec{b} + \vec{b} \bullet \vec{c} + \vec{c} \bullet \vec{a}$, then which one of the following is correct?

(a) m < 0 (b) m > 0

(c) m = 0 (d) Cannot be determined

Ans. (a)

Sol. $\vec{a}+\vec{b}+\vec{c} = 0$

dot product with $\vec{a}$

$\Rightarrow \quad \vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{a}.\vec{c} = 0$

$\Rightarrow \quad \vec{a}.\vec{b}+\vec{a}.\vec{c} = -a^2$

similarly $\vec{a}.\vec{b}+\vec{b}.\vec{c} = -b^2$

and $\quad \vec{a}.\vec{c}+\vec{b}.\vec{c} = -c^2$

$\Rightarrow \quad \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a} = -\frac{1}{2}(\vec{a}^2+\vec{b}^2+\vec{c}^2)$

$\Rightarrow \quad m < 0$

85. The least positive integer n for which $\left(\frac{1+i}{1-i}\right)^{2n}$, where $i=\sqrt{-1}$, has a negative value is

(a) 1 (b) 2 (c) 3 (d) 4

Ans. **(a)**

Sol. $\left(\frac{1+i}{1-i}\right)^{2n} = \left(\frac{2i}{-2i}\right)^{n} = (-1)^n < 0$

n is least positive integer $\Rightarrow$ n = 1

86. The factors of $\sin\theta + \sin\phi - \cos\theta\sin(\theta+\phi)$ are

(a) $\sin\theta$ and $1+\sin(\theta+\phi)$ (b) $\sin\theta$ and $1-\cos(\theta+\phi)$

(c) $\sin\phi$ and $1-\cos(\theta+\phi)$ (d) None of the above

Ans. **(b)**

Sol. $\sin\theta + \sin\phi - \cos\theta\sin(\theta+\phi)$

$= 2\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right) - 2\cos\theta\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta+\phi}{2}\right)$

$= 2\sin\left(\frac{\theta+\phi}{2}\right)\left[\cos\left(\frac{\theta-\phi}{2}\right) - \cos\theta\cos\left(\frac{\theta+\phi}{2}\right)\right]$

$= \sin\left(\frac{\theta+\phi}{2}\right)\left[2\cos\left(\frac{\theta-\phi}{2}\right) - \cos\left(\frac{3\theta+\phi}{2}\right) - \cos\left(\frac{\theta-\phi}{2}\right)\right]$

$= \sin\left(\frac{\theta+\phi}{2}\right)\left[\cos\left(\frac{\theta-\phi}{2}\right) - \cos\left(\frac{3\theta+\phi}{2}\right)\right]$

$= \sin\left(\frac{\theta+\phi}{2}\right)\left[2\sin\theta\,\sin\left(\frac{\theta+\phi}{2}\right)\right]$

$= \sin\theta\left(2\sin^2\left(\frac{\theta+\phi}{2}\right)\right)$

$= \sin(1-\cos(\theta+\phi))$

87. The inverse of a symmetric matrix, if it exists, is

(a) symmetric (b) skew-symmetric
(c) always unit matrix (d) None of the above

Ans. **(a)**

Sol. $A^T = A$ A is symmetric

$\Rightarrow (A^T)^{-1} = A^{-1} \Rightarrow (A^{-1})^T = A^{-1}$

Hence A^{-1} is also symmetric matrix

88. Which one of the following points does not lie on the circle with centre at (3, 4) and radius 5?

(a) (0, 0) (b) (–1, 1) (c) (2, 3) (d) (3, –1)

Ans. **(c)**

Sol. Equation of circle is

$(x - 3)^2 + (y - 4)^2 = 25$

(0, 0), (–1, 1) and (3, –1) satisfies the above equation

$S(2, 3) = (2 - 3)^2 + (3 - 4)^2 - 25 \neq 0$

$\Rightarrow$ (2, 3) not lies on the circle

89. The equation Ax + By + C = 0 involves only

(a) One arbitrary constant (b) Two arbitrary constants
(c) Three arbitrary constants (d) None of the above

Ans. **(b)**

Sol. $Ax + By + C = 0$

divided by C (if $C \neq 0$)

$\Rightarrow \frac{A}{C}x + \frac{B}{C}y + 1 = 0$

$\Rightarrow \lambda x + \mu y + 1 = 0$

λ & μ are two arbitrary constant.

90. Let X be the set of all persons living in a state. Elements x, y in X are said to be related if '$x < y$', whenever y is 5 years older than x. Which one of the following is correct?

(a) The relation is an equivalence relation
(b) The relation is transitive only
(c) The relation is transitive and symmetric, but not reflexive
(d) The relation is neither reflexive, nor symmetric, nor transitive

Ans. **(d)**

Sol. $(x, y) \in X \Leftrightarrow x < y$ and $y = x + 5$

$\Rightarrow (x, x) \notin X$, Hence not reflective

If $(x, y) \in X$

$\Rightarrow x < y$ and $y = x + 5$

$\not\Rightarrow (y, x) \in X$

$\Rightarrow$ X is not symmetric

Let $(x, y) \in X$ and $(y, z) \in X$

$\Rightarrow x < y;\ y = x + 5$ and $y < z;\ z = y + 5$

$\Rightarrow x < z$ and $z = x + 10$

$\Rightarrow (x, z) \notin X$

Not transitive

For the next three (03) question that follow

Consider $f(x) = \frac{1+x}{1-x}, (x \neq 0)$ and $g(x) = fofofo()x$

91. What is g(x)equal to?

(a) $\frac{1}{1-x}$ (b) $\frac{1}{x}$ (c) x (d) $\frac{1}{1+x}$

Ans. **(c)**

Sol. $f(x) = \frac{1+x}{1-x}$

$\Rightarrow fof(x) = \frac{1+f(x)}{1-f(x)} = \frac{1+\frac{1+x}{1-x}}{1-\frac{1+x}{1-x}}$

$= \frac{2}{-2x} = -\frac{1}{x}$

$\Rightarrow fofofof(x) = x$

$\Rightarrow g(x) = x$

92. What is $g(x)\ g\left(\frac{1}{x}\right)$ equal to?

(a) $\frac{1}{1-x}$ (b) $\frac{1}{x^2}$ (c) x^2 (d) 1

Ans. **(d)**

Sol. $g(x).g\left(\frac{1}{x}\right) = x.\frac{1}{x} = 1$

93. Consider the following statement

1. g(x) is an identity function
2. f o(x) is an identity function

Which of the above statements is/are correct?

(a) 1 only (b) 2 only (c) Both 1 and 2 (d) Neither 1 nor 2

Ans. **(a)**

Sol. $g(x) = fofofof(x) = x \qquad (x \neq 0)$

$\Rightarrow g(x) = x \qquad \forall\ x \in D_g$

Hence g(x) is an identity function

and $fof(x) = -\frac{1}{x}$. which is not a identity function

For the next three (03) question that follow consider the polynomial
p(x) = (x-α) (x-β) (x-γ) with
α = cos 75°, β = cos 45° and cos γ = cos 165°
In appropiate data. Assuming γ = cos 165º

94. What is the constant term of p(x) equal to?

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $\frac{1}{4\sqrt{2}}$ (d) $-\frac{1}{4\sqrt{2}}$

Ans. **(c)**
Sol. P(x) = (x – α) (x – β) (x – γ)
constant term in P(x) is – $\alpha\beta\gamma$
$= -\cos 75°\cos 45°\cos 165°$

$= \frac{1}{\sqrt{2}}\cos 15°\sin 15°$

$= \frac{1}{2\sqrt{2}}\sin 30° = \frac{1}{4\sqrt{2}}$

95. What is the coefficient of x^2 in p(x)?
(a) –1 (b) 1 (c) 0 (d) None of the above
Ans. **(c)**
Sol. Coefficient of x^2 in p(x) is – ($\alpha + \beta + \gamma$)
= – (cos 75º + cos45º + cos 165º)

$= -\left(\frac{\sqrt{3}-1}{2\sqrt{2}} + \frac{1}{\sqrt{2}} - \frac{\sqrt{3}+1}{2\sqrt{2}}\right)$

$= -\left(\frac{\sqrt{3}-1+2-\sqrt{3}-1}{2\sqrt{2}}\right)$

= 0

96. What is the coefficient of x in p(x)?

(a) $\frac{2\sqrt{2}+1}{4}$ (b) $-\frac{2\sqrt{2}+1}{4}$ (c) $\frac{\sqrt{2}+1}{4}$ (d) None of the above

Ans. **(d)**
Sol. Coefficient of x in p(x) is ($\alpha\beta + \beta\gamma + \gamma\alpha$)
= cos 75º cos45º + cos45º cos165º + cos 165º cos 75º

$= \cos 45º(\cos 75º + \cos 165º) - \frac{\sqrt{3}-1}{2\sqrt{2}}.\frac{\sqrt{3}+1}{2\sqrt{2}}$

$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}-1}{2\sqrt{2}} - \frac{\sqrt{3}+1}{2\sqrt{2}}\right) - \frac{(3-1)}{8}$

$= \frac{1}{\sqrt{2}}.\left(\frac{-2}{2\sqrt{2}}\right) - \frac{2}{8}$

$= -\frac{1}{2} - \frac{1}{4} = -\frac{3}{4}$

For the next four (04) question that follow

The series of natural number is divided into groups (1), (2,3,4), (5,6,7,8,9) and so on.

97. How many number are there in the n^{th} group?
(a) n (b) 2n – 1 (c) 2n (d) 2n + 1

Ans. **(b)**

Sol. No. of elements in each groups
1, 3, 5, 7 . . . .
$1 + (n-1) \times 2 = 2n - 1$

98. What is the first term in the n^{th} group?
(a) $2n - 1$ (b) $n^2 - 2n - 1$ (c) $n^2 - 2n + 2$ (d) None of the above

Ans. **(c)**

Sol. $(n-1)^2 + 1 = n^2 - 2n + 2$

99. What is the sum of the numbers in the n^{th} group?
(a) $(2n + 1)(n^2 - n + 1)$ (b) $n^3 - 3n^2 + 3n - 1$
(c) $n^3 + (n+1)^3$ (d) None of the above

Ans. **(d)**

Sol. $(n-1)^2 + 1, \ldots\ldots + n^2$

$$\frac{2n-1}{2}[2(n-1)^2 + 2 + (2n-2) \times 1]$$

$(2n-1)[(n-1)^2 + 1 + n - 1]$
$(2n-1)[n^2 - n + 1]$
$2n^3 - 3n^2 + 3n - 1$

100. What is the last term in the n^{th} group?
(a) $4n + 1$ (b) $2n^2$ (c) n^2 (d) None of the above

Ans. **(c)**

Sol. Obvious.