MATHEMATICS

Questions - 75   **Paper - I I**   **Time - 90 Min**

151. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = \beta - 3$ then the equation having α/β and β/α as its roots is

     a) $3x^2 - 19x + 3 = 0$ b) $3x^2 + 19x - 3 = 0$ c) $3x^2 - 19x - 3 = 0$ d) $x^2 - 5x + 3 = 0$

152. If $y = (x + \sqrt{1+x^2})^n$, then $1 + x^2 \dfrac{d^2 y}{dx^2} + x \dfrac{dy}{dx}$ is

     a) $n^2 y$ b) $-n^2 y$ c) $-y$ d) $2x^2 y$

153. If $1, \log_9(3^{1-x} + 2), \log_3(4.3^{3x} - 1)$, are in A.P then x equals

     a) $\log_3 4$ b) $1 + \log_3 4$ c) $1 - \log_4 3$ d) $\log_4 3$

154. A problem in mathematics is given to three students A, B, C and their respective probability of solving

     the problem is $\dfrac{1}{2}, \dfrac{1}{3}$ and $\dfrac{1}{4}$. Probability that the problem is solved is

     a) $\dfrac{3}{4}$ b) $\dfrac{1}{2}$ c) $\dfrac{2}{3}$ d) $\dfrac{1}{3}$

155. The period of $\sin^2 \theta$ is

     a) π^2 b) π c) 2π d) $\pi/2$

156. l, m, n are the pth, q^{th} and r^{th} term of a G. P all positive, then $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ equals

     a) -1   b) 2   c) 1   d) 0

157. $\lim\limits_{x \to 0} \dfrac{\sqrt{1 - \cos 2x}}{\sqrt{2}x}$ is

     a) 1   b) -1   c) zero   d) does not exist

158. A triangle with vertices (4, 0), (-1, -1), (3, 5) is

     a) isosceles and right angled   b) isosceles but not right angled

     c) right angled but not isosceles   d) neither right angled nor isoceles

159. In a class of 100 students there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class is 72, then what is the average of the girls?

     a) 73   b) 65   c) 68   d) 74

160. $\cot^{-1}(\sqrt{\cos \alpha}) = \tan^{-1}(\sqrt{\cos \alpha}) = x$, then $\sin x =$

     a) $\tan^2\left(\dfrac{\alpha}{2}\right)$ b) $\cot^2\left(\dfrac{\alpha}{2}\right)$ c) $\tan \alpha$ d) $\cot\left(\dfrac{\alpha}{2}\right)$

161. The order and degree of the differential equation $\left(1 + 3\dfrac{dy}{dx}\right)^{2/3} = 4\dfrac{d^3 y}{dx^3}$ are

     a) $\left(1, \dfrac{2}{3}\right)$ b) (3, 1)   c) (3, 3)   d) (1, 2)

162. A plane which passes through the point (3, 2, 0) and the line $\dfrac{x-4}{1} = \dfrac{y-7}{5} = \dfrac{z-4}{4}$ is

     a) x - y + z = 1   b) x + y + z = 5   c) x + 2y - z = 1   d) 2x - y + z = 5

163. The solution of the equation $\dfrac{d^2 y}{dx^2} = e^{-2x}$

     a) $\dfrac{e^{-2x}}{4}$ b) $\dfrac{e^{-2x}}{4} + cx + d$ c) $\dfrac{1}{4}e^{-2x} + cx^2 + d$ d) $\dfrac{1}{4}e^{-4x} + cx + d$

164 $\lim\limits_{x \to} \left(\dfrac{x^2 + 5x + 3}{\infty x^2 + x + 3} \right)^{\frac{1}{x}}$

    a) e^4 b) e^2 c) e^3 d) 1

165. The domain of $\sin^{-1} [\log_3 (x/3)]$ is

    a) [1, 9]   b) [-1, 9]   c) [-9, 1]   d) [-9, -1]

166.   The value of $2^{1/4}.4^{1/8}.8^{1/6} + ... \infty$ is

    a) 1   b) 2   c) 3/2   d) 4

167. Fifth term of an GP is 2, then the product of its 9 terms is

    a) 256   b) 512   c) 1024   d) none of these

168. $\int_0^{10\pi} |\sin x| \, dx$ is

    a) 20   b) 8   c) 10   d) 18

169. $I_n = \int\limits_0^{\pi/4} \tan^n x \, dx$ then $\lim\limits_{n \to \infty} n[I_n + I_{n-2}]$ equals

    a) $\dfrac{1}{2}$ b) 1   c) ∞ d) zero

170. $\int\limits_0^{\sqrt{2}} [x^2] \, dx$ is

    a) 2- $\sqrt{2}$ b) 2+ $\sqrt{2}$ c) $\sqrt{2}$ -1   d) $\sqrt{2}$ - 2

171. $\int_{-\pi}^{\pi} \dfrac{2x(1+\sin x)}{1+\cos^2 x} \, dx$ is

    a) $\dfrac{\pi^2}{4}$ b) π^2 c) zero   d) $\dfrac{\pi}{2}$

172. Let f(x) = 4 and f'(x) = 4. Then $\lim\limits_{x \to 2} \dfrac{xf(2) - 2f(x)}{x-2}$ is given by

    a) 2   b) -2   c) -4   d) 3

173. z and w are two nonzero complex numbers such that $| z | = | w |$ and Arg z + Arg w = π then z equals

    a) $\overline{w}$ b) - $\overline{w}$ c) w   d) -w

174. If $| z - 4 | < | z - 2 |$, its solution is given by

    a) Re(z)>0   b) Re(z)<0   c) Re(z)>3   d) Re(z)>2

175. The locus of the centre of a circle which touches the circle $|| z - z_1 | = a$ and $| z - z_2 | = b$ externally (z, z_1 & z_2 are complex numbers) will be

    a) an ellipse   b) a hyperbola   c) a circle   d) none of these

176. Sum of infinite number of terms of GP is 20 and sum of their square is 100. The common ratio of GP is

    a) 5   b) 3/5   c) 8/5   d) 1/5

177. $1^3 - 2^3 + 3^3 - 4^3 + + ... + 9^3 =$

    a) 425   b) -425   c) 475   d) -475

178. Difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and a $\neq$ b. then

    a) a + b + 4 = 0   b) a + b - 4 = 0   c) a - b - 4 = 0   d) a - b + 4 = 0

179. Product of real roots of the equation $t^2 x^2 + |x| + 9 = 0$

    a) is always positive   b) is always negative   c) does not exist   d) none of these

180. If p and q are the roots of the equation $x^2 + px + q = 0$, then

    a) p=1, q= -2   b) p=0, q=1   c) p=-2, q=0   d) p=-2, q=1

181. If a, b, c, are distinct +ve real numbers and $a^2 + b^2 + c^2 = 1$ then ab + bc + ca is

    a) less than 1   b) equal to 1   c) greater that 1   d) any real no.

182. Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 (using repetition allowed) are

    a) 216   b) 375   c) 400   d) 720

183. Number greater than 1000 but less than 4000 is formed using the digits 0, 1, 2, 3, 4, (repetition allowed) is

    a) 125   b) 105   c) 375   d) 625

184. Five digit number divisible by 3 is formed using 0, 1, 2, 3, 4, 6 and 7 without repetition. Total number of such numbers are

    a) 312   b) 3125   c) 120   d) 216

185. The sum of integers from 1 to 100 that are divisible by 2 or 5 is

    a) 3000   b) 3050   c) 3600   d) 3250

186. The coefficients of x^p and x^q in the expansion of $(1+x)^{p+q}$ are.

    a) equal   b) equal with opposite signs

    c) reciprocals of each other   d) none of these

187. If the sum of the coefficients in the expansion of $(a + b)^n$ is 4096, then the greatest coefficient in the expansion is

    a) 1594   b) 792   c) 924   d) 2924

188. The positive integer just greater than $(1+0.0001)^{10000}$ is

    a) 4   b) 5   c) 2   d) 3

189. r and n are positive integers r > 1, r > 2 and coefficient of $(r + 2)^{th}$ term and $3r^{th}$ term in the expansion of $(1+x)^{2n}$ are equal, then n equals

    a) 3r   b) 3r + 1   c) 2r   d) 2r + 1

190. If a > 0 discriminant of $ax^2 + 2bx + c$ is -ve, then $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ is

    a) +ve   b) $(ac-b^2)(ax^2+2bx+c)$ c) -ve   d) 0

191. If $a_n = \sqrt{7 + \sqrt{7 + \sqrt{7 + ...}}}$ haing n radical signs then by methods of mathematical induction which is true

    a) $a_n > 7 \ \forall n \geq 1$ b) $a_n > 7 \ \forall n \geq 1$ c) $a_n < 4 \ \forall n \geq 1$ d) $a_n < 3 \forall n \geq 1$

192. The sides of a triangle are 3x + 4y, 4x + 37 and 5x + 57 where x, y > 0 then the triangle is

    a) right angled   b) obtuse angled   c) equilateral   d) none of these

193. Locus of mid point of the portion between the axes of $x \cos\alpha + y \sin\alpha = p$ where p is constant is

    a) $x^2 + y^2 = \dfrac{4}{p^2}$ b) $x^2 + y^2 = 4p^2$ c) $\dfrac{1}{x^2} + \dfrac{1}{y^2} = \dfrac{2}{p^2}$ d) $\dfrac{1}{x^2} + \dfrac{1}{y^2} = \dfrac{4}{p^2}$

194. If the pair of lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ intersect on the y - axis then

    a) 2fgh = bg^2+ch^2 b) $bg^2 \neq ch^2$ c) abc = 2fgh   d) none of these

195. The point of lines represented by $3ax^2 + 5xy + (a^2 - 2)y^2 = 0$ and perpendicular to each other for

    a) two values of a   b) $\forall$ a   c) for one value of a   d) for no values of a

196. If the chord y = mx +1 of the circle $x^2 + y^2 = 1$ subtends an angle of measure 45^0 at the major segment of the circle then value of m is

    a) $2 \pm \sqrt{2}$ b) $-2 \pm \sqrt{2}$ c) $-1 \pm \sqrt{2}$ d) none of these

197. The centres of a set of circle, each of radius 3, lie on the circle $x^2 + y^2 = 25.$ The locus of any point in the set is

a) $4 \le x^2 + y^2 \le 64$ b) $x^2 + y^2 \le 25$ c) $x^2 + y^2 \ge 25$ d) $3 \le x^2 + y^2 \le 9$

198. The centre of the circle passing through (0, 0) and (1, 0) and touching the circle $x^2 + y^2 = 9$ is

a) $\left(\frac{1}{2}, \frac{1}{2}\right)$ b) $\left(\frac{1}{2}, -\sqrt{2}\right)$ c) $\left(\frac{3}{2}, \frac{1}{2}\right)$ d) $\left(\frac{1}{2}, \frac{3}{2}\right)$

199. The equation of a circle with origin as a centre and passing through equilateral triangle whose median is of length 3a is

a) $x^2 + y^2 = 9a^2$ b) $x^2 + y^2 = 16a^2$ c) $x^2 + y^2 = 4a^2$ d) $x^2 + y^2 = a^2$

200. Two common tangents to the circle $x^2 + y^2 = 2a^2$ and parabola $y^2 = 8ax$ are

a) $x = \pm(y + 2a)$ b) $y = \pm(x + 2a)$ c) $x = \pm(y + a)$ d) $y = \pm(x + a)$

201. In a triangle with sides a, b, c, $r_1 > r_2 > r_3$ (which are the ex-radii) then

a) a>b>c   b) a<b<c   c) a>b and b<c   d) a<b and b<c

202. The number of solution of tanx + secx = 2cosx in $[0, 2\pi]$ is

a) 2   b) 3   c) 0   d) 1

203. Which one is not periodic

a) $|\sin 3x| + \sin^2 x$ b) $\cos\sqrt{x} + \cos^2 x$ c) $\cos4x + \tan^2 x$ d) $\cos2x + \sin xs$

204. $\lim_{n\to\infty} \frac{1^p + 2^p + 3^p + ...n^p}{n^{p+1}}$ is

a) $\frac{1}{p+1}$ b) $\frac{1}{1-p}$ c) $\frac{1}{p} - \frac{1}{p-1}$ d) $\frac{1}{p+2}$

205. $\lim_{x\to0} \frac{\log x^n - [x]}{[x]}, n \in N,$ ([x] denotes greatest integer less than or equal to x)

a) has value -1   b) has value 0   c) has value 1   d) does not exist

206. If f(1) = 1, f'(1) = 2, then $\lim_{x\to1} \frac{\sqrt{f(x)-1}}{\sqrt{x}-1}$ is

a) 2   b) 4   c) 1   d) 1/2

207. f is defined in [-5, 5] as f(x) = x if x is rational and = -x if x is irrational. Then

a) f(x) is continuous at every x, except x = 0
b) f(x) is discontinuous at every x, except x = 0
c) f(x) is continuous everywhere
d) f(x) is discontinuous everywhere.

208. f(x) and g(x) are two differentiable functions on [0, 2] such that f''(x) - g''(x) = 0, f'(1) = 2g'(1) = 4 f(2) = 3g(2) = 9 then f(x)-g(x) at x = 3/2 is

a) 0   b) 2   c) 10   d) 5

209. If f(x +y) = f(x). f(y) $\forall$ x.y and f(5) = 2, f'(0) = 3, then f'(5) is

a) 0   b) 1   c) 6   d) 2

210. The maximum distance from origin of a point on the curve x = a sin t-b sin $\left(\frac{at}{b}\right)$, y = a cos t-b cos $\left(\frac{at}{b}\right)$, both a, b > 0 is

a) a-b   b) a+b   c) $\sqrt{a^2 + b^2}$ d) $\sqrt{a^2 - b^2}$

211. If $2a+3b+6c = 0$, $(a,b,c \in R)$ then the quadratic equation $ax^2 + bx\ c = 0$ has

    a) at least one root in $[0, 1]$ b) at least one root in $[2, 3]$

    c) at least one root in $[4, 5]$ d) none of these

212. If $y = f(x)$ makes +ve intercept of 2 and 0 unit on x and y axes and encloses an area of 3/4 square unit

    with the axes then $\int\limits_{0}^{2} xf'(x)dx$ is

    a) 3/2   b) 1   c) 5/4   d) -3/4

213. The area bounded by the curves $y = \ln x$, $y = \ln |x|$, $y = |\ln x|$ and $y = |\ln |\ |x|$ is

    a) 4sq. units   b) 6sq. units   c) 10sq. units   d) none of these

214. If $|\vec{a}| = 4\ |\vec{b}| = 2$ and the angle between $\vec{a}$ and $\vec{b}$ is $\pi/6$ then $(\vec{a} \times \vec{b})^2 = 2$ is equal to

    a) 48   b) 16   c) $\vec{a}$ d) none of these

215. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $[\vec{a}.\ \vec{b}.\ \vec{c}] = 4$ then $[\vec{a} \times \vec{b}\ \ \vec{b} \times \vec{c}\ \ \vec{c} \times \vec{a}] =$

    a) 16   b) 64   c) 4   d) 8

216. If $\vec{a}, \vec{b}, \vec{c}$ are vectors show that $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 7$, $|\vec{b}| = 5$, $|\vec{c}| = 3$ then angle between vector

    $\vec{b}$ and $\vec{c}$ is

    a) 60^0 b) 30^0 c) 45^0 d) 90^0

217. If $|a| = 5$, $|b| = 4$, $|c| = 3$, thus what will be the value of $|a\ b + b.c + c.a|$, given that $\vec{a} + \vec{b} + \vec{c} = 0$

    a) 25   b) 50   c) -25   d) -50

218. $3\lambda\vec{c} + 2\mu(\vec{a} \times \vec{b}) = 0$ then

    a) $3\lambda + 2\mu = 0$ b) $3\lambda = 2\mu$ c) $\lambda = \mu$ d) $\lambda + \mu = 0$

219. $\vec{a} = 3\hat{i} - 5\hat{j}$ and $\vec{b} = 6\hat{i} - 3\hat{j}$ are two vectors and $\vec{c}$ is a vector such that $\vec{c} = \vec{a} \times \vec{b}$ then $|\vec{a}| : |\vec{b}| : |\vec{c}|$

    a) $\sqrt{34} : \sqrt{45} : \sqrt{39}$ b) $\sqrt{34} : \sqrt{45} : 39$ c) 34 : 39 : 45   d) 39 : 35 : 34

220. If $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$ then $\vec{a} + \vec{b} + \vec{c} =$

    a) abc   b) -1   c) 0   d) 2

221. A and B are events such that $P(A \cup B) = 3/4$, $(A \cap B) = 1/4$, $P(\overline{A}) = 2/3$ then $P(\overline{A} \cap B)$ is

    a) 5/12   b) 3/8   c) 5/8   d) 1/4

222. A die is tossed 5 times. Getting an odd number is considered a success. Then the variance of distribution of success is

    a)8/3   b) 3/8   c) 4/5   d) 5/4

223. The d.r of normal to the plane through $(1, 0, 0,)$, $(0, 1, 0)$ which makes an angles $\pi/4$ with plane $x + y = 3$ are

    a) $1, \sqrt{2}, 1$ b) $1, 1, \sqrt{2}$ c) 1, 1, 2   d) $\sqrt{2}, 1,1$

224. The sum of two forces is 18 N and resultant whose direction is at right angles to the smaller force is 12 N. The magnitude of the two forces are

    a) 13, 5   b) 12, 6   c) 14, 4   d) 11, 7

225. A bead of weight w can slide on smooth circular wire in a vertical plane. The bead is attached by a light thread to the highest point of the wire and in equilibrium, the thread is taut and make an angle θ with the vertical then tension of the thread and reaction of the wire on the bead are.

    a) $T = w \cos\theta$ $R = w \tan\theta$ b) $T = 2w \cos\theta$ $R = w$

    c) $T = w$ $R = w \sin\theta$ d) $T = w \sin\theta$ $R = w\cot\theta$