

E

Sr.No.

SUBJECT CODE BOOKLET CODE

2011 (I)  
PHYSICAL SCIENCES  
TEST BOOKLET

5

A

Time : 3:00 Hours

Maximum Marks : 200

**INSTRUCTIONS**

1. **You have opted for English as medium of Question Paper.** This Test Booklet contains sixty five (20 Part 'A' + 20 Part 'B' + 25 Part 'C') Multiple Choice Questions (MCQs). You are required to answer a maximum of 15, 20 and 10 (compulsory) + 10 (out of remaining 15) questions from part 'A' 'B' and 'C' respectively. If more than required number of questions are answered, only first 15, 20 and 10 + 10 questions in Parts 'A' 'B' and 'C' respectively, will be taken up for evaluation.
2. Answer sheet has been provided separately. Before you start filling up your particulars, please ensure that the booklet contains requisite number of pages and that these are not torn or mutilated. If it is so, you may request the Invigilator to change the booklet. Likewise, check the answer sheet also. Sheets for rough work have been appended to the test booklet.
3. Write your Roll No., Name, Your address and Serial Number of this Test Booklet on the Answer sheet in the space provided on the side 1 of Answer sheet. Also put your signatures in the space identified.
4. **You must darken the appropriate circles with a pencil related to Roll Number, Subject Code, Booklet Code and Centre Code on the OMR answer sheet. It is the sole responsibility of the candidate to meticulously follow the instructions given on the Answer Sheet, failing which, the computer shall not be able to decipher the correct details which may ultimately result in loss, including rejection of the OMR answer sheet.**
5. Each question in Part 'A' carries 2 marks, Part 'B' 3.5 marks and Part 'C' 5 marks respectively. Some questions in Part 'C' have two sub-parts, A and B carrying 3 and 2 marks each, respectively (total 5 marks). There will be negative marking @ 25% for each wrong answer.
6. Below each question in Part 'A', 'B' and 'C' four alternatives or responses are given. Only one of these alternatives is the "correct" option to the question. You have to find, for each question, the correct or the best answer.
7. **Candidates found copying or resorting to any unfair means are liable to be disqualified from this and future examinations.**
8. Candidate should not write anything anywhere except on answer sheet or sheets for rough work.
9. After the test is over, you MUST hand over the answer sheet (OMR) to the invigilator.
10. Use of calculator is not permitted.

Roll No. ....

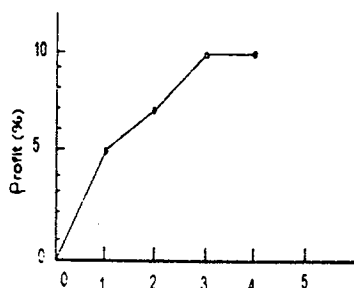
I have verified all the information filled in by the candidate.

Name .....

.....  
Signature of the Invigilator

**PART A**

1.



The cumulative profits of a company since its inception are shown in the diagram. If the net worth of the company at the end of 4<sup>th</sup> year is 99 crores, the principal it had started with was

1. 9.9 crores
2. 91 crores
3. 90 crores
4. 9.0 crores

2. Popular use of which of the following fertilizers increases the acidity of soil?

1. Potassium Nitrate
2. Ammonium sulphate
3. Urea
4. Superphosphate of lime

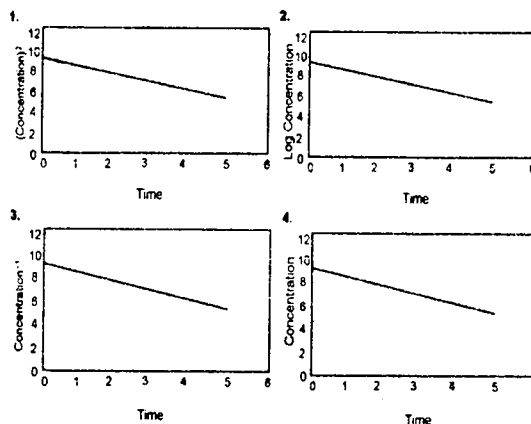
3. Exposing an organism to a certain chemical can change nucleotide bases in a gene, causing mutation. In one such mutated organism if a protein had only 70% of the primary amino acid sequence, which of the following is likely?

1. Mutation broke the protein
2. The organism could not make amino acids
3. Mutation created a terminator codon
4. The gene was not transcribed

4. A reference material is required to be prepared with 4 ppm calcium. The amount of CaCO<sub>3</sub> (molecular weight = 100) required to prepare 1000 g of such a reference material is

1. 10 µg
2. 4 µg
3. 4 mg
4. 10 mg

5. Identify the figure which depicts a first order reaction.



6. The speed of a car increases every minute as shown in the following Table. The speed at the end of the 19<sup>th</sup> minute would be

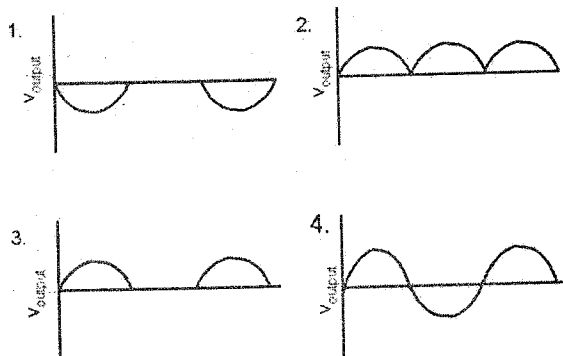
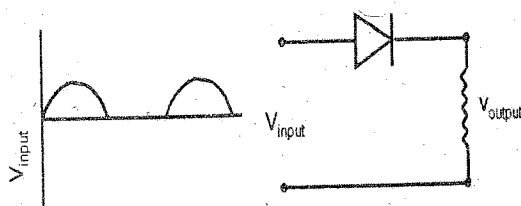
Time (minutes)	Speed (m/sec)
1	1.5
2	3.0
3	4.5
·	·
·	·
24	36.0
25	37.5

1. 26.5
2. 28.0
3. 27.0
4. 28.5

7. If the atmospheric concentration of carbon dioxide is doubled and there are favourable conditions of water, nutrients, light and temperature, what would happen to water requirement of plants?

1. It decreases initially for a short time and then returns to the original value
2. It increases
3. It decreases
4. It increases initially for a short time and then returns to the original value

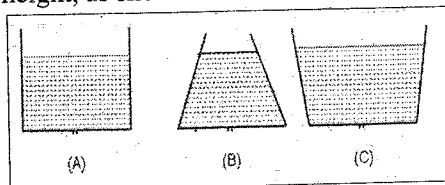
8. If  $V_{input}$  is applied to the circuit shown, the output would be



9. A physiological disorder X always leads to the disorder Y. However, disorder Y may occur by itself. A population shows 4% incidence of disorder Y. Which of the following inferences is valid?

1. 4% of the population suffers from both X & Y
2. Less than 4% of the population suffers from X
3. At least 4% of the population suffers from X
4. There is no incidence of X in the given population

10. Water is dripping out of a tiny hole at the bottom of three flasks whose base diameter is the same, and are initially filled to the same height, as shown



Which is the correct comparison of the rate of fall of the volume of water in the three flasks?

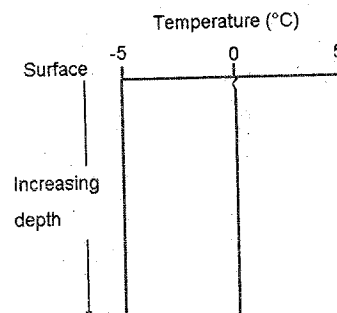
1. A fastest, B slowest
2. B fastest, A slowest

3. B fastest, C slowest
4. C fastest, B slowest

11. How many  $\sigma$  bonds are present in the following molecule?  
 $HC \equiv CCH = CHCH_3$

1. 4
2. 10
3. 6
4. 13

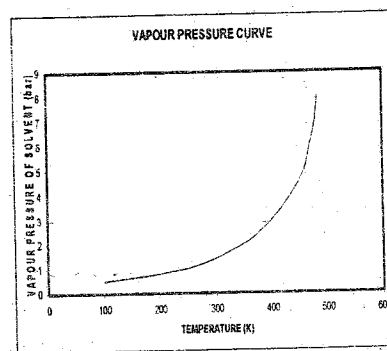
- 12.



The graph represents the depth profile of temperature in the open ocean; in which region this is likely to be prevalent?

1. Tropical region
2. Equatorial region
3. Polar region
4. Sub-tropical region

- 13.



The normal boiling point of a solvent (whose vapour pressure curve is shown in the figure) on a planet whose normal atmospheric pressure is 3 bar, is about

1. 400 K
2. 273 K
3. 100 K
4. 500 K

14. Diabetic patients are advised a low glycaemic index diet. The reason for this is

1. They require less carbohydrate than healthy individuals
2. They cannot assimilate ordinary carbohydrates
3. They need to have slow, but sustained release of glucose in their blood stream
4. They can tolerate lower, but not higher than normal blood sugar levels

15. Glucose molecules diffuse across a cell of diameter  $d$  in time  $\tau$ . If the cell diameter is tripled, the diffusion time would

1. increase to  $9\tau$
2. decrease to  $\tau/3$
3. increase to  $3\tau$
4. decrease to  $\tau/9$

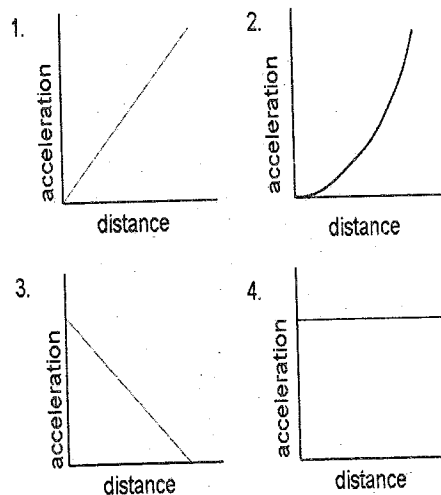
16. The reason for the hardness of diamond is

1. extended covalent bonding
2. layered structure
3. formation of cage structures
4. formation of tubular structures

17. Which of the following particles has the largest range in a given medium if their initial energies are the same?

1. alpha
2. gamma
3. positron
4. electron

18. A ball is dropped from a height  $h$  above the surface of the earth. Ignoring air drag, the curve that best represents its variation of acceleration is



19. Standing on a polished stone floor one feels colder than on a rough floor of the same stone. This is because

1. Thermal conductivity of the stone depends on the surface smoothness
2. Specific heat of the stone changes by polishing it
3. The temperature of the polished floor is lower than that of the rough floor
4. There is greater heat loss from the soles of the feet when in contact with the polished floor than with the rough floor

20. The acidity of normal rain water is due to

1.  $\text{SO}_2$
2.  $\text{CO}_2$
3.  $\text{NO}_2$
4.  $\text{NO}$

**PART B**

21. A particle of unit mass moves in a potential

$$V(x) = ax^2 + \frac{b}{x^2}, \text{ where } a \text{ and } b \text{ are}$$

positive constants. The angular frequency of small oscillations about the minimum of the potential is

1.  $\sqrt{8b}$
2.  $\sqrt{8a}$
3.  $\sqrt{8a/b}$
4.  $\sqrt{8b/a}$

22. A signal of frequency 10 kHz is being digitized by an A/D converter. A possible sampling time which can be used is

1. 100  $\mu$ s
2. 40  $\mu$ s
3. 60  $\mu$ s
4. 200  $\mu$ s

23. The electrostatic potential  $V(x, y)$  in free space in a region where the charge density  $\rho$  is zero is given by

$$V(x, y) = 4e^{2x} + f(x) - 3y^2. \text{ Given that}$$

the  $x$ -component of the electric field,  $E_x$ , and  $V$  are zero at the origin,  $f(x)$  is

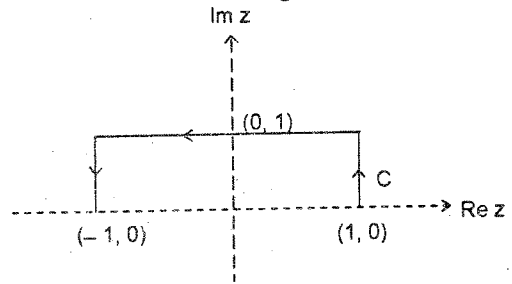
1.  $3x^2 - 4e^{2x} + 8x$
2.  $3x^2 - 4e^{2x} + 16x$
3.  $4e^{2x} - 8$
4.  $3x^2 - 4e^{2x}$

24. Consider the transition of liquid water to steam as water boils at a temperature of 100°C under a pressure of 1 atmosphere. Which one of the following quantities does not change discontinuously at the transition?

1. The Gibbs free energy
2. The internal energy
3. The entropy
4. The specific volume

25. The value of the integral  $\int_C dz z^2 e^z$ , where

$C$  is an open contour in the complex  $z$ -plane as shown in the figure below, is:



1.  $\frac{5}{e} + e$
2.  $e - \frac{5}{e}$
3.  $\frac{5}{e} - e$
4.  $-\frac{5}{e} - e$

26. Which of the following matrices is an element of the group  $SU(2)$ ?

1.  $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$
2.  $\begin{pmatrix} \frac{1+i}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1-i}{\sqrt{3}} \end{pmatrix}$
3.  $\begin{pmatrix} 2+i & i \\ 3 & 1+i \end{pmatrix}$
4.  $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$

27. For constant uniform electric and magnetic fields  $\vec{E} = \vec{E}_0$  and  $\vec{B} = \vec{B}_0$ , it is possible to choose a gauge such that the scalar potential  $\phi$  and vector potential  $\vec{A}$  are given by

1.  $\phi = 0$  and  $\vec{A} = \frac{1}{2}(\vec{B}_0 \times \vec{r})$
2.  $\phi = -\vec{E}_0 \cdot \vec{r}$  and  $\vec{A} = \frac{1}{2}(\vec{B}_0 \times \vec{r})$
3.  $\phi = -\vec{E}_0 \cdot \vec{r}$  and  $\vec{A} = 0$
4.  $\phi = 0$  and  $\vec{A} = -\vec{E}_0 t$

28. Let  $\vec{a}$  and  $\vec{b}$  be two distinct three-dimensional vectors. Then the component of  $\vec{b}$  that is perpendicular to  $\vec{a}$  is given by

1.  $\frac{\vec{a} \times (\vec{b} \times \vec{a})}{a^2}$
2.  $\frac{\vec{b} \times (\vec{a} \times \vec{b})}{b^2}$
3.  $\frac{(\vec{a} \cdot \vec{b})\vec{b}}{b^2}$
4.  $\frac{(\vec{b} \cdot \vec{a})\vec{a}}{a^2}$

29. The wavefunction of a particle is given by

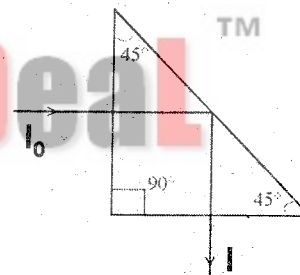
$\psi = \left( \frac{1}{\sqrt{2}} \phi_0 + i \phi_1 \right)$ , where  $\phi_0$  and  $\phi_1$  are the normalized eigenfunctions with energies  $E_0$  and  $E_1$  corresponding to the ground state and first excited state, respectively. The expectation value of the Hamiltonian in the state  $\psi$  is

1.  $\frac{E_0}{2} + E_1$
2.  $\frac{E_0}{2} - E_1$
3.  $\frac{E_0 - 2E_1}{3}$
4.  $\frac{E_0 + 2E_1}{3}$

30. A particle is confined to the region  $x \geq 0$  by a potential which increases linearly as  $u(x) = u_0 x$ . The mean position of the particle at temperature  $T$  is

1.  $\frac{k_B T}{u_0}$
2.  $(k_B T)^2 / u_0$
3.  $\sqrt{\frac{k_B T}{u_0}}$
4.  $u_0 k_B T$

31. Circularly polarized light with intensity  $I_0$  is incident normally on a glass prism as shown in the figure. The index of refraction of glass is 1.5. The intensity  $I$  of light emerging from the prism is



1.  $I_0$
2.  $0.96 I_0$
3.  $0.92 I_0$
4.  $0.88 I_0$

32. The acceleration due to gravity ( $g$ ) on the surface of Earth is approximately 2.6 times that on the surface of Mars. Given that the radius of Mars is about one half the radius of Earth, the ratio of the escape velocity on Earth to that on Mars is approximately

1. 1.1
2. 1.3
3. 2.3
4. 5.2

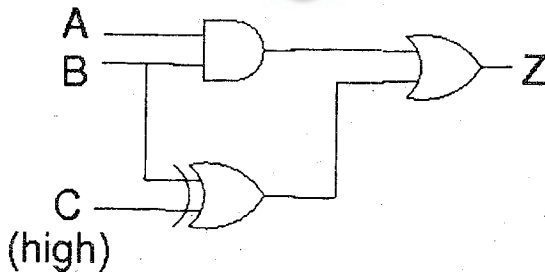
33. A plane electromagnetic wave is propagating in a lossless dielectric. The electric field is given by

$$\vec{E}(x, y, z, t) = E_0(\hat{x} + A\hat{z}) \exp\left[ik_0\left\{-ct + (x + \sqrt{3}z)\right\}\right],$$

where  $c$  is the speed of light in vacuum,  $E_0$ ,  $A$  and  $k_0$  are constants and  $\hat{x}$  and  $\hat{z}$  are unit vectors along the  $x$ - and  $z$ -axes. The relative dielectric constant of the medium,  $\epsilon_r$ , and the constant  $A$  are

1.  $\epsilon_r = 4$  and  $A = -\frac{1}{\sqrt{3}}$
2.  $\epsilon_r = 4$  and  $A = +\frac{1}{\sqrt{3}}$
3.  $\epsilon_r = 4$  and  $A = \sqrt{3}$
4.  $\epsilon_r = 4$  and  $A = -\sqrt{3}$

34. Consider the digital circuit shown below in which the input  $C$  is always high (1).



The truth table for the circuit can be written as

A	B	Z
0	0	
0	1	
1	0	
1	1	

The entries in the  $Z$  column (vertically) are

1. 1010
2. 0100
3. 1111
4. 1011

35. The energy levels of the non-relativistic electron in a hydrogen atom (i.e. in a Coulomb potential  $V(r) \propto -1/r$ ) are given by  $E_{nlm} \propto -1/n^2$ , where  $n$  is the principal quantum number, and the corresponding wave functions are given by  $\psi_{nlm}$ , where  $\ell$  is the orbital angular momentum quantum number and  $m$  is the magnetic quantum number. The spin of the electron is not considered. Which of the following is a correct statement?

1. There are exactly  $(2\ell + 1)$  different wave functions  $\psi_{nlm}$ , for each  $E_{nlm}$ .
2. There are  $\ell(\ell + 1)$  different wave functions  $\psi_{nlm}$ , for each  $E_{nlm}$ .
3.  $E_{nlm}$  does not depend on  $\ell$  and  $m$  for the Coulomb potential.
4. There is a unique wave function  $\psi_{nlm}$  for each  $E_{nlm}$ .

36. The Hamiltonian of an electron in a constant magnetic field  $\vec{B}$  is given by  $H = \mu\vec{\sigma} \cdot \vec{B}$  where  $\mu$  is a positive constant and  $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$  denotes the Pauli matrices. Let  $\omega = \mu B/\hbar$  and  $I$  be the  $2 \times 2$  unit matrix. Then the operator  $e^{iHt/\hbar}$  simplifies to

1.  $I \cos \frac{\omega t}{2} + \frac{i\vec{\sigma} \cdot \vec{B}}{B} \sin \frac{\omega t}{2}$
2.  $I \cos \omega t + \frac{i\vec{\sigma} \cdot \vec{B}}{B} \sin \omega t$
3.  $I \sin \omega t + \frac{i\vec{\sigma} \cdot \vec{B}}{B} \cos \omega t$
4.  $I \sin 2\omega t + \frac{i\vec{\sigma} \cdot \vec{B}}{B} \cos 2\omega t$

37. The Hamiltonian of a system with  $n$  degrees of freedom is given by

$H(q_1, \dots, q_n; p_1, \dots, p_n; t)$ , with an explicit dependence on the time  $t$ . Which of the following is correct?

1. Different phase trajectories cannot intersect each other.
2.  $H$  always represents the total energy of the system and is a constant of the motion.
3. The equations  $\dot{q}_i = \partial H / \partial p_i$ ,  $\dot{p}_i = -\partial H / \partial q_i$  are not valid since  $H$  has explicit time dependence.
4. Any initial volume element in phase space remains unchanged in magnitude under time evolution.

38. If the perturbation  $H' = ax$ , where  $a$  is a constant, is added to the infinite square well potential

$$V(x) = \begin{cases} 0 & \text{for } 0 \leq x \leq \pi \\ \infty & \text{otherwise.} \end{cases}$$

The correction to the ground state energy, to first order in  $a$ , is

1.  $\frac{a\pi}{2}$
2.  $a\pi$
3.  $\frac{a\pi}{4}$
4.  $\frac{a\pi}{\sqrt{2}}$

39. Let  $p_n(x)$  (where  $n = 0, 1, 2, \dots$ ) be a polynomial of degree  $n$  with real coefficients, defined in the interval  $2 \leq n \leq 4$ . If  $\int_2^4 p_n(x)p_m(x) dx = \delta_{nm}$ , then

1.  $p_0(x) = \frac{1}{\sqrt{2}}$  and  $p_1(x) = \sqrt{\frac{3}{2}}(3-x)$
2.  $p_0(x) = \frac{1}{\sqrt{2}}$  and  $p_1(x) = \sqrt{3}(3+x)$

$$3. p_0(x) = \frac{1}{2} \text{ and } p_1(x) = \sqrt{\frac{3}{2}}(3-x)$$

$$4. p_0(x) = \frac{1}{\sqrt{2}} \text{ and } p_1(x) = \sqrt{\frac{3}{2}}(3-x)$$

40. A cavity contains blackbody radiation in equilibrium at temperature  $T$ . The specific heat per unit volume of the photon gas in the cavity is of the form  $C_v = \gamma T^3$ , where  $\gamma$  is a constant. The cavity is expanded to twice its original volume and then allowed to equilibrate at the same temperature  $T$ . The new internal energy per unit volume is

1.  $4\gamma T^4$
2.  $2\gamma T^4$
3.  $\gamma T^4$
4.  $\gamma T^4 / 4$

**PART C**

**(COMPULSORY)**

41. Consider a system of  $N$  non-interacting spins, each of which has classical magnetic moment of magnitude  $\mu$ . The Hamiltonian of this system in an external magnetic field

$$\vec{H} \text{ is } \mathcal{H} = -\sum_{i=1}^N \vec{\mu}_i \cdot \vec{H}, \text{ where } \vec{\mu}_i \text{ is the}$$

magnetic moment of the  $i^{\text{th}}$  spin. The magnetization per spin at temperature  $T$  is

1.  $\frac{\mu^2 H}{k_B T}$
2.  $\mu \left[ \coth\left(\frac{\mu H}{k_B T}\right) - \frac{k_B T}{\mu H} \right]$
3.  $\mu \sinh\left(\frac{\mu H}{k_B T}\right)$
4.  $\mu \tanh\left(\frac{\mu H}{k_B T}\right)$



42. Which of the following is an analytic function of the complex variable  $z = x + iy$  in the domain  $|z| < 2$ ?

1.  $(3 + x - iy)^7$
2.  $(1 + x + iy)^4 (7 - x - iy)^3$
3.  $(1 - 2x - iy)^4 (3 - x - iy)^3$
4.  $(x + iy - 1)^{1/2}$

43. A particle in one dimension moves under the influence of a potential  $V(x) = ax^6$ , where  $a$  is a real constant. For large  $n$  the quantized energy level  $E_n$  depends on  $n$  as:

1.  $E_n \sim n^3$
2.  $E_n \sim n^{4/3}$
3.  $E_n \sim n^{6/5}$
4.  $E_n \sim n^{3/2}$

44. The Lagrangian of a particle of charge  $e$  and mass  $m$  in applied electric and magnetic

fields is given by  $L = \frac{1}{2}m\vec{v}^2 + e\vec{A} \cdot \vec{v} - e\phi$ ,

where  $\vec{A}$  and  $\phi$  are the vector and scalar potentials corresponding to the magnetic and electric fields, respectively. Which of the following statements is correct?

1. The canonically conjugate momentum of the particle is given by  $\vec{p} = m\vec{v}$
2. The Hamiltonian of the particle is given by

$$H = \frac{\vec{p}^2}{2m} + \frac{e}{m}\vec{A} \cdot \vec{p} + e\phi$$

3.  $L$  remains unchanged under a gauge transformation of the potentials
4. Under a gauge transformation of the potentials,  $L$  changes by the total time derivative of a function of  $\vec{r}$  and  $t$

45. A static, spherically symmetric charge distribution is given by  $\rho(r) = \frac{A}{r}e^{-Kr}$  where  $A$  and  $K$  are positive constants. The electrostatic potential corresponding to this charge distribution varies with  $r$  as

- |                           |                               |
|---------------------------|-------------------------------|
| 1. $re^{-Kr}$             | 2. $\frac{1}{r}e^{-Kr}$       |
| 3. $\frac{1}{r^2}e^{-Kr}$ | 4. $\frac{1}{r}(1 - e^{-Kr})$ |

46. Consider two independently diffusing non-interacting particles in 3-dimensional space, both placed at the origin at time  $t = 0$ . These particles have different diffusion constants  $D_1$  and  $D_2$ . The quantity  $\langle [\vec{R}_1(t) - \vec{R}_2(t)]^2 \rangle$  where  $\vec{R}_1(t)$  and  $\vec{R}_2(t)$  are the positions of the particles at time  $t$ , behaves as: <sup>TM</sup>

1.  $6t(D_1 + D_2)$
2.  $6t|D_1 - D_2|$
3.  $6t\sqrt{D_1^2 + D_2^2}$
4.  $6t\sqrt{D_1 D_2}$

47. A resistance is measured by passing a current through it and measuring the resulting voltage drop. If the voltmeter and the ammeter have uncertainties of 3% and 4%, respectively, then

A. The uncertainty in the value of the resistance is

1. 7.0%
2. 3.5%
3. 5.0%
4. 12.0%

B. The uncertainty in the computed value of the power dissipated in the resistance is

1. 7%
2. 5%
3. 11%
4. 9%

48. In the absence of an applied torque a rigid body with three distinct principal moments of inertia given by  $I_1, I_2$  and  $I_3$  is rotating freely about a fixed point inside the body. The Euler equations for the components of its angular velocity  $(\omega_1, \omega_2, \omega_3)$  are

$$\dot{\omega}_1 = \frac{I_2 - I_3}{I_1} \omega_2 \omega_3, \quad \dot{\omega}_2 = \frac{I_3 - I_1}{I_2} \omega_1 \omega_3,$$

$$\dot{\omega}_3 = \frac{I_1 - I_2}{I_3} \omega_1 \omega_2$$

A. The equilibrium points in  $(\omega_1, \omega_2, \omega_3)$  space are

1.  $(1, -1, 0), (-1, 0, 1)$  and  $(0, -1, 1)$
2.  $(1, 1, 0), (1, 0, 1)$  and  $(0, 1, 1)$
3.  $(1, 0, 0), (0, 1, 0)$  and  $(0, 0, 1)$
4.  $(1, 1, 1), (-1, -1, -1)$  and  $(0, 0, 0)$

B. The constants of motion are

1.  $\omega_1^2 + \omega_2^2 + \omega_3^2$  and  $I_1\omega_1 + I_2\omega_2 + I_3\omega_3$
2.  $I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2$  and  $I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2$
3.  $I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2$  and  $\omega_1 + \omega_2 + \omega_3$
4.  $\omega_1^2 + \omega_2^2 + \omega_3^2$  and  $I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2$

49. In a system consisting of two spin  $\frac{1}{2}$

particles labeled 1 and 2, let  $\bar{S}^{(1)} = \frac{\hbar}{2} \bar{\sigma}^{(1)}$

and  $\bar{S}^{(2)} = \frac{\hbar}{2} \bar{\sigma}^{(2)}$  denote the corresponding

spin operators. Here  $\bar{\sigma} \equiv (\sigma_x, \sigma_y, \sigma_z)$  and  $\sigma_x, \sigma_y, \sigma_z$  are the three Pauli matrices.

A. In the standard basis the matrices for the operators  $S_x^{(1)}S_y^{(2)}$  and  $S_y^{(1)}S_x^{(2)}$  are respectively,

$$1. \frac{\hbar^2}{4} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \frac{\hbar^2}{4} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$2. \frac{\hbar^2}{4} \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad \frac{\hbar^2}{4} \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$$

$$3. \frac{\hbar^2}{4} \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \quad \frac{\hbar^2}{4} \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$$

$$4. \frac{\hbar^2}{4} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}, \quad \frac{\hbar^2}{4} \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

B. These two operators satisfy the relation

1.  $\{S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}\} = S_z^{(1)}S_z^{(2)}$
2.  $\{S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}\} = 0$
3.  $[S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}] = iS_z^{(1)}S_z^{(2)}$
4.  $[S_x^{(1)}S_y^{(2)}, S_y^{(1)}S_x^{(2)}] = 0$

50. Consider the matrix  $M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

A. The eigenvalues of  $M$  are

1. 0, 1, 2
2. 0, 0, 3
3. 1, 1, 1
4. -1, 1, 3

B. The exponential of  $M$  simplifies to ( $I$  is the  $3 \times 3$  identity matrix)

1.  $e^M = I + \left(\frac{e^3 - 1}{3}\right)M$
2.  $e^M = I + M + \frac{M^2}{2!}$
3.  $e^M = I + 3^3 M$
4.  $e^M = (e - 1)M$

Choose any 10 of the remaining questions

51. The radius of a  ${}^{64}_{29}\text{Cu}$  nucleus is measured to be  $4.8 \times 10^{-13}$  cm.

A. The radius of a  ${}^{27}_{12}\text{Mg}$  nucleus can be estimated to be

1.  $2.86 \times 10^{-13}$  cm
2.  $5.2 \times 10^{-13}$  cm
3.  $3.6 \times 10^{-13}$  cm
4.  $8.6 \times 10^{-13}$  cm

B. The root-mean-square (rms) energy of a nucleon in a nucleus of atomic number  $A$  in its ground state varies as:

1.  $A^{4/3}$
2.  $A^{1/3}$
3.  $A^{-1/3}$
4.  $A^{-2/3}$

52. The character table of  $C_{3v}$ , the group of symmetries of an equilateral triangle is given below

	$\chi^{(0)}$	$\chi^{(1)}$	$\chi^{(2)}$
$1C_1$	1	1	$b$
$3C_2$	1	$a$	$c$
$2C_3$	1	1	$d$

In the above  $C_1, C_2, C_3$  denote the three classes of  $C_{3v}$  containing 1, 3 and 2 elements respectively, and  $\chi^{(0)}, \chi^{(1)}$  and  $\chi^{(2)}$  are the characters of the three irreducible representations  $\Gamma^{(0)}, \Gamma^{(1)}$  and  $\Gamma^{(2)}$  of  $C_{3v}$ .

A. The entries  $a, b, c$  and  $d$  in this table are, respectively,

1. 2, 1, -1, 0
2. -1, 2, 0, -1
3. -1, 1, 0, -1
4. -1, 1, 1, -1

B. The reducible representation  $\Gamma$  of  $C_{3v}$ ,

with character  $\chi = (4, 0, 1)$  decomposes into its irreducible representations  $\Gamma^{(0)}, \Gamma^{(1)}, \Gamma^{(2)}$  as

1.  $2\Gamma^{(0)} + 2\Gamma^{(1)}$
2.  $\Gamma^{(0)} + \Gamma^{(1)} + \Gamma^{(2)}$
3.  $\Gamma^{(0)} + 3\Gamma^{(1)}$
4.  $2\Gamma^{(2)}$

53. Light of wavelength 660 nm and power of 1 mW is incident on a semiconductor photodiode with an absorbing layer of thickness of  $(\ln 4)\mu\text{m}$ .

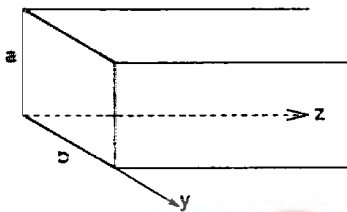
A. If the absorption coefficient at this wavelength is  $10^4 \text{ cm}^{-1}$  and if 1% power is lost on reflection at the surface, the power absorbed will be

- |                     |                     |
|---------------------|---------------------|
| 1. $750\mu\text{W}$ | 2. $675\mu\text{W}$ |
| 3. $250\mu\text{W}$ | 4. $225\mu\text{W}$ |

53. The generated photo-current for a quantum efficiency of unity will be

1.  $400\mu\text{A}$
2.  $360\mu\text{A}$
3.  $133\mu\text{A}$
4.  $120\mu\text{A}$

54. The magnetic field of the  $TE_{11}$  mode of a rectangular waveguide of dimensions  $a \times b$  as shown in the figure is given by  $H_z = H_1 \cos(0.3\pi x) \cos(0.4\pi y)$ , where  $x$  and  $y$  are in cm.



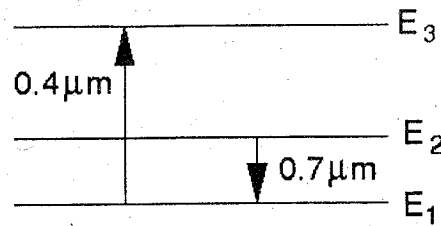
A. The dimensions of the waveguide are

1.  $a = 3.33\text{cm}, b = 2.50\text{cm}$
2.  $a = 0.40\text{cm}, b = 0.30\text{cm}$
3.  $a = 0.80\text{cm}, b = 0.60\text{cm}$
4.  $a = 1.66\text{cm}, b = 1.25\text{cm}$

B. The entire range of frequencies  $f$  for which the  $TE_{11}$  mode will propagate is

1.  $6.0\text{GHz} < f < 7.5\text{GHz}$
2.  $7.5\text{GHz} < f < 9.0\text{GHz}$
3.  $7.5\text{GHz} < f < 12.0\text{GHz}$
4.  $7.5\text{GHz} < f$

55. Consider the energy level diagram (as shown in the figure below) of a typical three level ruby laser system with  $1.6 \times 10^{19}$  Chromium ions per cubic centimetre. All the atoms excited by the  $0.4\mu\text{m}$  radiation decay rapidly to level  $E_2$  which has a lifetime  $\tau = 3\text{ms}$ .



A. Assuming that there is no radiation of wavelength  $0.7\mu\text{m}$  present in the pumping cycle and that the pumping rate is  $R$  atoms per  $\text{cm}^3$ , the population density in the level  $N_2$  builds up as :

1.  $N_2(t) = R\tau(e^{t/\tau} - 1)$
2.  $N_2(t) = R\tau(1 - e^{-t/\tau})$
3.  $N_2(t) = \frac{Rt^2}{\tau}(1 - e^{-t/\tau})$
4.  $N_2(t) = Rt$

B. The minimum pump power required (per cubic centimetre) to bring the system to transparency, i.e., zero gain, is

1.  $1.52\text{ kW}$
2.  $2.64\text{ kW}$
3.  $0.76\text{ kW}$
4.  $1.32\text{ kW}$

56. A flux quantum (fluxoid) is approximately equal to  $2 \times 10^{-7}$  gauss-cm<sup>2</sup>. A type II superconductor is placed in a small magnetic field, which is then slowly increased till the field starts penetrating the superconductor. The strength of the field at this point is  $\frac{2}{\pi} \times 10^5$  gauss.

A. The penetration depth of this superconductor is

1.  $100\text{ \AA}$
2.  $10\text{ \AA}$
3.  $1000\text{ \AA}$
4.  $314\text{ \AA}$

B. The applied field is further increased till superconductivity is completely destroyed. The strength of the field is now  $\frac{8}{\pi} \times 10^5$  gauss. The correlation length of the superconductor is

1. 20 Å
2. 200 Å
3. 628 Å
4. 2000 Å

57. A beam of pions ( $\pi^+$ ) is incident on a proton target, giving rise to the process  $\pi^+ p \rightarrow n + \pi^+ + \pi^+$

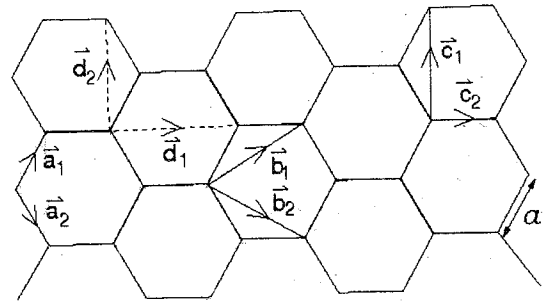
A. Assuming that the decay proceeds through strong interactions, the total isospin  $I$  and its third component  $I_3$  for the decay products, are

1.  $I = \frac{3}{2}, I_3 = \frac{3}{2}$
2.  $I = \frac{5}{2}, I_3 = \frac{5}{2}$
3.  $I = \frac{5}{2}, I_3 = \frac{3}{2}$
4.  $I = \frac{1}{2}, I_3 = -\frac{1}{2}$

B. Using isospin symmetry, the cross-section for the above process can be related to that of the process

1.  $\pi^- n \rightarrow p \pi^- \pi^-$
2.  $\pi^- \bar{p} \rightarrow \bar{n} \pi^- \pi^-$
3.  $\pi^+ n \rightarrow p \pi^+ \pi^-$
4.  $\pi^+ \bar{p} \rightarrow n \pi^+ \pi^-$

58. The two dimensional lattice of graphene is an arrangement of Carbon atoms forming a honeycomb lattice of lattice spacing  $a$ , as shown below. The Carbon atoms occupy the vertices.



A. The Wigner-Seitz cell has an area of

1.  $2a^2$
2.  $\frac{\sqrt{3}}{2}a^2$
3.  $6\sqrt{3}a^2$
4.  $\frac{3\sqrt{3}}{2}a^2$

B. The Bravais lattice for this array is a

1. rectangular lattice with basis vectors  $\bar{a}_1$  and  $\bar{a}_2$
2. rectangular lattice with basis vectors  $\bar{c}_1$  and  $\bar{c}_2$
3. hexagonal lattice with basis vectors  $\bar{a}_1$  and  $\bar{a}_2$
4. hexagonal lattice with basis vectors  $\bar{b}_1$  and  $\bar{b}_2$

59. Consider the decay process  $\tau^- \rightarrow \pi^- + \nu_\tau$  in the rest frame of the  $\tau^-$ . The masses of the  $\tau^-$ ,  $\pi^-$  and  $\nu_\tau$  are  $M_\tau$ ,  $M_\pi$  and zero respectively.

A. The energy of  $\pi^-$  is

1.  $\frac{(M_\tau^2 - M_\pi^2)c^2}{2M_\tau}$
2.  $\frac{(M_\tau^2 + M_\pi^2)c^2}{2M_\tau}$
3.  $(M_\tau - M_\pi)c^2$
4.  $\sqrt{M_\tau M_\pi} c^2$

B. The velocity of  $\pi^-$  is

1.  $\frac{(M_r^2 - M_\pi^2)c}{M_r^2 + M_\pi^2}$
2.  $\frac{(M_r^2 + M_\pi^2)c}{M_r^2 - M_\pi^2}$
3.  $\frac{M_\pi c}{M_r}$
4.  $\frac{M_r c}{M_\pi}$

60. A narrow beam of X-rays with wavelength  $1.5 \text{ \AA}$  is reflected from an ionic crystal with an fcc lattice structure with a density of  $3.32 \text{ g cm}^{-3}$ . The molecular weight is 108 AMU ( $1 \text{ AMU} = 1.66 \times 10^{-24} \text{ g}$ ).

A. The lattice constant is

1.  $6.00 \text{ \AA}$
2.  $4.56 \text{ \AA}$
3.  $4.00 \text{ \AA}$
4.  $2.56 \text{ \AA}$

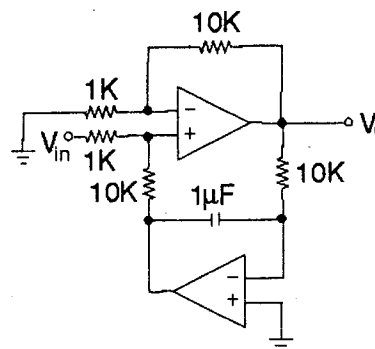
B. The sine of the angle corresponding to (111) reflection is

1.  $\frac{\sqrt{3}}{4}$
2.  $\frac{\sqrt{3}}{8}$
3.  $\frac{1}{4}$
4.  $\frac{1}{8}$

61. If an electron is in the ground state of the hydrogen atom, the probability that its distance from the proton is more than one Bohr radius is approximately

1. 0.68
2. 0.48
3. 0.28
4. 0.91

62. A time varying signal  $V_{in}$  is fed to an op-amp circuit with output signal  $V_o$  as shown in the figure below.



The circuit implements a

1. high pass filter with cutoff frequency 16Hz
2. high pass filter with cutoff frequency 100 Hz
3. low pass filter with cutoff frequency 16 Hz
4. low pass filter with cutoff frequency 100 Hz

63. The Hamiltonian of a particle of unit mass moving in the  $xy$ -plane is given to be:

$$H = xp_x - yp_y - \frac{1}{2}x^2 + \frac{1}{2}y^2 \text{ in suitable units.}$$

The initial values are given to be

$$(x(0), y(0)) = (1, 1) \text{ and}$$

$$(p_x(0), p_y(0)) = \left(\frac{1}{2}, -\frac{1}{2}\right). \text{ During the}$$

motion, the curves traced out by the particles in the  $xy$ -plane and the  $p_x p_y$ -plane are

1. both straight lines
2. a straight line and a hyperbola respectively
3. a hyperbola and an ellipse, respectively
4. both hyperbolas

64. Consider an ideal Bose gas in three dimensions with the energy-momentum relation  $\varepsilon \propto p^s$  with  $s > 0$ . The range of  $s$  for which this system may undergo a Bose-Einstein condensation at a non-zero temperature is

1.  $1 < s < 3$
2.  $0 < s < 2$
3.  $0 < s < 3$
4.  $0 < s < \infty$

65. Two gravitating bodies  $A$  and  $B$  with masses  $m_A$  and  $m_B$ , respectively, are moving in circular orbit. Assume that  $m_B \gg m_A$  and let the radius of the orbit of body  $A$  be  $R_A$ . If the body  $A$  is losing mass adiabatically, its orbital radius  $R_A$  is proportional to

1.  $1/m_A$
2.  $1/m_A^2$
3.  $m_A$
4.  $m_A^2$



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