PHYSICAL SCIENCES

PART A

1. The calculation of the probability of excitation of an atom originally in the ground state to an excited state, involves the contour integral

$$\int_{-\infty}^{\infty} \frac{\tau e^{i\omega t}}{(t^2+\tau^2)^2}\,dt$$

Evaluate the above integral.

2. Evaluate $\frac{d^2}{dx^2}|x|$ in terms of the Dirac δ-function.

3. Give the circuit diagram and explain how the photo diode operates in the (a) voltage mode and (b) current mode. What are the practical applications of photo diodes ?

4. An electric field that is constant with time is given in cylindrical polar coordinates by

$$\vec{E} = \begin{cases} k\rho\hat{\phi} & \text{for } 0 \le \rho \le R \\ k\dfrac{R^2}{\rho}\hat{\phi} & \text{for } \rho > R \end{cases}$$

Here ($\rho = \sqrt{x^2+y^2}$, ϕ, z) are the cylindrical polar coordinates.

(a) Evaluate $\oint_C \vec{E}.d\vec{l}$ along $x^2 + y^2 = a^2$ for $a < R$ and for $a > R$. The contour C is taken in the counter clockwise direction.

(b) Can $\vec{E}$ be an electrostatic field?

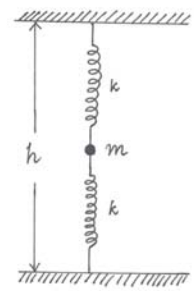
(c) Show that there must be a magnetic field in this space of the form

$$\vec{B}(\vec{r},t) = \begin{cases} \vec{B}_0(\vec{r}) + \alpha t\hat{k} & \text{for } \rho \le R \\ \vec{B}_1(\vec{r}) & \text{for } \rho > \text{R} \end{cases}$$

5. A linear homogeneous isotropic dielectric sphere of radius a has a small spherical concentric cavity of radius r. A point charge $+q$ is placed at the center. Find the electric displacement $\vec{D}$ in the dielectric. Find all bound charges and show that they add up to zero.

6. A particle of mass m moves in one dimension in a medium in which the frictional drag force on it is proportional to the square of its instantaneous speed: $F(t) = -\beta v^2(t)$, $\beta > 0$. Write the equation of motion and express the position and velocity as a function of time, for the initial conditions $x(0) = 0$, $v(0) = u$.

7. A particle of mass m is connected to the roof and floor of a room in a vertical configuration by two springs, the equilibrium length of each being $h/2$, (h is the height of the room and k is the spring constant. (See the figure). This system is in a uniform gravitational field on Earth's surface, but we neglect all other forces. The particle is displaced slightly in the vertical direction and allowed to oscillate with a small amplitude. Write the Lagrangian describing the dynamics of the particle. Find the equation of motion, the equilibrium position and the frequency of oscillation.



8 The energy eigenvalue and the corresponding eigenfunction for a particle of mass m in a one dimensional potential, $V(x)$ are

$$E = 0,\ \psi(x) = \frac{A}{x^2 + a^2}.$$

Deduce the potential $V(x)$.

9. Estimate the bound state energy of a particle in an attractive one-dimensional δ–function potential, $V(x) = -a\,\delta(x)$, (where $a > 0$) using the trial function $\psi(x) = Ae^{-Bx^2}$, using B as the variational parameter.

10. Consider a system of two non-interacting identical particles, each having only three single-particle states of energies $-\varepsilon$, 0, $+\varepsilon$. The system is in thermal equilibrium with a heat reservoir at temperature T. Write down the partition function of the system, given that the particles obey

(a) Maxwell-Boltzmann statistics (assuming that the particles are distinguishable)
(b) Bose-Einstein statistics

PART B

11. The 4×4 matrix A satisfies the equations $A^2 = I$ and Trace A = 0. Find all its eigenvalues and the value of det A.

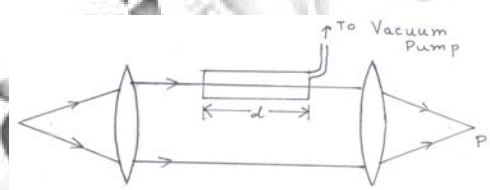
12. The graph of the inverse square of a junction capacitance varies linearly with reverse voltage.

    a. What is the nature of the doping profile?
    b. How will the slope of the above graph vary if the electron and hole concentrations are both doubled?
    c. Assuming that the concentration of electrons and holes is the same in Si and Ge, compare the intercepts of the above graph on the voltage axis for the two materials.

13. Draw the circuit diagram of a *JK* flipflop based up-counter. Explain with timing diagrams how the counter works as a frequency divider.

14. Sketch and explain the I – V characteristic of a solar cell in dark and under illumination. Identify on the curve, the point of maximum power delivery. Explain how this point is chosen.

15. Two thin parallel beams of monochromatic light from a common source are made to interfere at a point P. A glass tube is introduced in the path of one of the



beams (see figure). The length of the tube is d. The interference pattern is then observed. Next, we slowly evacuate the tube of all the air in it. In this process, we notice that 5 fringes go past the point of observation P. If the wave length of light used is λ, obtain an expression for the refractive index of air.

16. A system of non-interacting spin-half particles with gyromagnetic ratio γ is in thermal equilibrium at temperature T in a magnetic field of strength B_0. The system has two energy levels . In the high temperature approximation in which $k_B T >>$ the energy difference ΔE between the two levels, find the fractional excess population in the lower level.

17. A system, described by the Hamiltonian $H = \alpha L^2$ where L^2 is the total angular momentum and α is a constant, exhibits a line spectrum where the line A represents transitions from the second excited state to the first excited state. The system is now placed in an external magnetic field and the Hamiltonian changes to $H = \alpha L^2 + \beta L_z$, where L_z is the z component of the angular momentum. How many distinct lines will the original line A split into ?

18. A laser resonator cavity is made up of two mirrors M_1 and M_2 with reflectivities $R_1 = 0.98$ and $R_2 = 0.90$. The mirrors are separated by 10 cm. Determine the contribution to the loss coefficient due to the finite reflectivities of the mirrors. What happens when $R_1 = R_2 = 1$?

19. (a) Using the relation between carrier energy ε, band gap E_g, and wavevector of magnitude k, show that the optical absorption in a direct band-gap semiconductor is proportional to $\sqrt{\varepsilon - E_g}$.

(b) Why does this argument not hold for an indirect band-gap semiconductor?

20. The interaction between two neighbouring atoms in solid Argon is modelled by the potential

$V(r) = V_0\left[\left(\frac{a}{r}\right)^{12} - 2\left(\frac{a}{r}\right)^{6}\right]$ where V_0 and a are positive constants.

(a) Sketch $V(r)$ versus r schematically.

(b) Find the equilibrium distance between a nearest-neighbour pair of atoms.

(c) Find the angular frequency of small oscillations of an atom about its equilibrium position.

21. Consider a semiconductor with p and n type carriers, with charges +e and –e, respectively, present in *equal* concentration. Show that, in the usual Hall effect experiment (current imposed in the x direction, magnetic field B in the z direction) in the steady state, the electric field $\vec{E}$ satisfies $E_y = \frac{eB}{c}(\mu_p - \mu_n) E_x$ where μ_p and μ_n are the mobilities of p-type and n-type carriers respectively, when terms of order B^2 and higher are neglected.

22. A two-dimensional harmonic oscillator has the Hamiltonian (in suitable units) $H = \frac{1}{2}\,(q_1^2 + p_1^2 + q_2^2 + p_2^2)$. The canonical Poisson bracket relations are

$\{q_i, q_j\} = 0$; $\{p_i, p_j\} = 0$; $\{q_j, p_j\} = \delta_{ij}$. Show that $J = (q_1 p_2 - q_2 p_1)$ and $K = (q_1 q_2 + p_1 p_2)$ are constants of the motion.

23. The single particle potential in the nuclear shell model is given by

$$V = \frac{1}{2} m\omega^2 r^2 - \lambda \vec{L} \cdot \vec{S}\,.$$

where $\hbar\omega = 7\,\text{MeV}$ and $\lambda\hbar^2 = 1\,\text{MeV}$. Use the above potential to give the energy-level scheme for ^{13}C. Draw the required energy levels and indicate *all* the quantum numbers clearly. Give the spin and isospin of ^{13}C.

24. Consider a radioactive series decay $A \rightarrow B \rightarrow C$, where, the decay constants for A and B are, respectively,

$\lambda_A = 0.05\,\text{sec}^{-1}$ and $\lambda_B = 0.10\,\text{sec}^{-1}$.

The number of B-type nuclei at time t is given by

$$N_B(t) = \frac{\lambda_A N_A(t=0)}{\lambda_B - \lambda_A}\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right)$$

(i) Calculate the time t at which $N_B(t)$ becomes maximum

(ii) Calculate the ratio $\frac{N_B(t)}{N_A(t)}$ at that instant of time.

25. Suppose that in a particle process the $\bar{u}$ quark in π^- is replaced by an $\bar{s}$ quark. What is the new particle? How are the quantities *Q, B, S* and I_3 affected? What are their new values? Check that the Gell-Mann Nishijima formula holds for the new particle, as well as for π^-.

26. Obtain the hyperfine splitting in the ground state of the hydrogen atom to first order in perturbation theory, for the perturbation

$$H' = A\,\vec{S}_p \cdot \vec{S}_e\,\delta^{(3)}(\vec{r})$$

where A is a constant, $\vec{S}_p$ and $\vec{S}_e$ denote the spins of the proton and electron, respectively.

27. Draw schematically, with labelled axes, the power spectrum of the Fourier transform of the following signals:

(a) $S(t) = \sin(44t) + \cos(88t)$
(b) a square wave of frequency 3 Hz.

The signals are sampled at 20Hz. Mark the x-axis at intervals of 1Hz (take $\pi = 22/7$).

28. Consider a thermocouple, a platinum resistor and a thermistor as temperature measuring devices. Arrange them in the increasing order of (i) range, (ii) accuracy, and (iii) resolution.

29. The temperatures of a bath determined with a thermometer of least count 0.1°C are 19.2 °C, 19.5°C, 19.3°C, 19.7°C , 19.5°C.
Determine and report the mean temperature. How many data points have to be collected in order to achieve a standard deviation of 0.1°C?

30. The free energy of a magnetic system as a function of the reduced temperature $t = (T - T_c)/T_c$ and the magnetization m is given by

$$F(t, m) = F_0 + \frac{a}{2} tm^2 + \frac{b}{4} m^4 ,$$

where F_0 , a and b are positive constants.

(a) Find the extrema of this potential with respect to m for $t > 0$ and for $t < 0$.
(b) What type of phase transition is represented by this potential?