MATHEMATICAL SCIENCES

Mathematical Scinces question paper will consist of multiple choice questions for 100 marks from mathematics. There will be negative marking for negative answer.

SYLLABUS

1. Algebra :

Theory of Equations :

Relations between roots and Coefficients, Newton's identities, Rolle's theorem, Reciprocal Equations, Des Cartes, Rule of Signs, Cubic and quartic equations, Complex numbers and De Moivre's Theorem.

Determinants :

Cofactors, Properties of determinants, Solution of a Linear System, Cramer's Rule.

Inequalities :

AM-GM inequality, Cauchy-Schwarz inequality.

Set Theory :

Relations, Functions, Cardinality.

Algebraic Structures :

Binary Operations, Groups, Rings: Definitions, Examples and Elementary Theorems.

Vector Spaces :

Subspaces, Linear Independence, Bases, Dimension, Linear Transformations, Matrices, Rank, Nullity, Eigenvalues and Eigenvectors.

2. Geometry :

Two-dimensional Co-ordinate Geometry :

Conics and their equations in Cartesian and Polar Coordinates, Ellipse, Parabola and Hyperbola.

Three-dimensional Co-ordinate Geometry :

Planes, Lines, Spheres and Cones.

3. Vector Algebra and Vector Calculus :

Vectors addition, Scalar multiplication, Dot Product, Cross Product, Triple Product, Equations to the Line and the Plane, Grad, Divergence and Curl, Vector Integration, Green's Gauss' and Stokes' Theorems.

4. Calculus and Analysis

Real Number System, Sequence and Series, Continuity, Differentiability, Mean Value Theorems, Indeterminate Value Theorem, L'Hospital Rule, Tangents and Normals, Maxima and Minima, Riemann Integration, Multiple Integrals, Partial differentiations, Lengths, areas and volumes by integration.

5. Differential Equations :

First Order ODE; Method of Separation of Variables; Exact equations; Euler's equation; Orthogonal Family of curves, Second Order Linear ODE : Variation of Parameters.

MODEL QUESTIONS

Four possible answers are provided for each question.
Select the correct answer by making (√) against (A), (B), (C) or (D).

1. Let ρ be a non-trivial relation on a set X. If ρ is symmetric and antisymmetric then ρ is

   (A) reflexive, (B) transitive, (C) an equivalence relation, (D) the diagonal relation.

2. The set $\{5, 15, 25, 35\}$ is a group under multiplication modulo 40. The identity element of this group is

   (A) 5, (B) 15, (C) 25, (D) 35.

3. Let $\mathbb{Z}_n$ be the additive group of integers modulo n. The number of homomorphisms from $\mathbb{Z}_n$ to itself is

   (A) 0, (B) 1, (C) n, (D) n^2.

4. Let $v = (1, 1)$ and $w = (1, -1) \in \mathbb{R}^2$. Then a vector $u = (a, b) \in \mathbb{R}^2$ is in the $\mathbb{R}$-linear span of v and w

   (A) only when $a = b$, (B) always, (C) for exactly one value of (a, b), (D) for at most finitely many values of (a, b).

5. Let A be a 3×3 real matrix. Suppose $A^4 = 0$. Then A has

   (A) exactly two distinct real eigenvalues, (B) exactly one non-zero real eigenvalue, (C) exactly 3 distinct real eigenvalues, (D) no non-zero real eigenvalue.

6. Let a, b, c, d be real numbers and let $f : \mathbb{C} \to \mathbb{C}$ be the map defined by $f(x + iy) := (ax + by) + i(cx + dy)$. Then f is linear over $\mathbb{C}$ if and only if

   (A) $(a, b) = (d, c)$, (B) $(a, b) = (d, -c)$, (C) $(a, b) = (-d, c)$, (D) $(a, b) = (-d, -c)$.

7. The function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \max\{1 - |x|, 0\}$ is differentiable

   (A) at all points, (B) at all except one point, (C) at all except three points, (D) nowhere.

8. Let $f: [0, 1] \to \mathbb{R}$ be a continuous function with $f(0) = f(1)$. If f is differentiable on $(0, 1)$ and the derivative f' is continuous on $(0, 1)$ then f' is

   (A) strictly positive in $(0, 1)$, (B) strictly negative in $(0, 1)$, (C) identically zero in $(0, 1)$, (D) zero at some point in $(0, 1)$.

9. A unit normal vector to the curve $\mathbf{C} := \{(x, x^2) : x \in \mathbb{R}\}$ in the plane $\mathbb{R}^2$ at the point $(0, 0)$ is given by

   (A) $(0, -1)$, (B) $(-1, 0)$, (C) $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$, (D) $(1, 0)$.

10. The differential equation $\frac{d^2y}{d^2x} + 4\frac{dy}{dx} + 4y = 0$ has general solution of the form:

    (A) $A\cos 2x + B\sin 2x$, (B) $Ae^{-2x} + Bxe^{-2x}$, (C) $Ae^{2x} + Bxe^{2x}$, (D) $Ae^{2x} + Be^{-2x}$.