CET – MATHEMATICS – 2013

VERSION CODE: C – 2

1. If $\sin^{-1} a$ is the acute angle between the curves $x^2 + y^2 = 4x$ and $x^2 + y^2 = 8$ at (2, 2), then a = ___________

(1) 1 (2) 0 (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{\sqrt{3}}{2}$

Ans: (3)

Slope of first curve $m_1 = 0$; slope of second curve $m_2 = -1$ therefore angle is 45^o

$A = \sin 45^o = \frac{1}{\sqrt{2}}$

2. The maximum area of a rectangle that can be inscribed in a circle of radius 2 units is _______

(1) 8π sq. units (2) 4 sq. units (3) 5 sq. units (4) 8 sq. units

Ans: (4)

r = 2; maximum rectangle is a square with each side $a = \sqrt{2}\, r = 2\sqrt{2}$

therefore area = $a^2 = 8$

3. If the length of the sub-tangent at any point to the curve $xy^n = a$ is proportional to the abscissa, then 'n' is ______

(1) any non-zero real number (2) 2

(3) -2 (4) 1

Ans: (1)

Differentiating $xy^n = a$ we get $y^I = \frac{-y}{nx}$ ST = - nx since it is proportional to x n can be any non-zero real number.

4. $\int \frac{\cos^{n-1} x}{\sin^{n+1} x} dx,\ n \neq 0$ is _________

(1) $\frac{\cot^n x}{n}$ (2) $\frac{-\cot^{n-1} x}{n-1}$ (3) $\frac{-\cot^n x}{n}$ (4) $\frac{\cot^{n-1} x}{n-1}$

Ans: (3)

Given integral can be expressed as $\int \frac{\cot^{n-1} x}{\sin^2 x} dx = \frac{-\cot^n x}{n}$

5. $\int \frac{(x-1)e^x}{(x+1)^3} dx$ = _________

(1) $\frac{e^x}{x+1}$ (2) $\frac{e^x}{(x+1)^2}$ (3) $\frac{e^x}{(x+1)^3}$ (4) $\frac{x.e^x}{(x+1)}$

Ans: (2)

$$\int \frac{(x+1-2)e^x}{(x+1)^3} dx = \int e^x\left[\frac{1}{(x+1)^2} - \frac{2}{(x+1)^3}\right] dx = \frac{e^x}{(x+1)^2}$$

6. If $I_1 = \int_0^{\pi/2} x \cdot \sin x \, dx$ and $I_2 = \int_0^{\pi/2} x \cdot \cos x \, dx$, then which one of the following is true?

(1) $I_1 = I_2$ (2) $I_1 + I_2 = 0$ (3) $I_1 = \frac{\pi}{2} . I_2$ (4) $I_1 + I_2 = \frac{\pi}{2}$

Ans: (4)

$I_1 = \int_0^{\pi/2} x \cdot \sin x \, dx = -x\cos x + \sin x|_0^1 = 1$

$I_2 = \int_0^{\pi/2} x \cos x \, dx = x \sin x - \int \sin x \int_0^{\pi/2} = \frac{\pi}{2} - 1$

$I_1 + I_2 = \frac{\pi}{2}$

7. The value of $\int_{-1}^{2} \frac{|x|}{x} dx$ is __________

(1) 0 (2) 1 (3) 2 (4) 3

Ans: (2)

$\int_{-1}^{2} \frac{|x|}{x} dx = \int_{-1}^{1} \frac{|x|}{x} dx + \int_{1}^{2} \frac{|x|}{x} dx$

$= 0 + \int_1^2 |dx| = 2 - 1 = 1$

8. $\int_0^{\pi} \frac{\cos^4 x}{\cos^4 x + \sin^4 x} dx =$

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{8}$ (4) π

Ans: (2)

$\int_0^{\pi} \frac{\cos^4 x}{\cos^4 x + \sin^4 x} dx = 2\int_0^{\frac{\pi}{2}} \frac{\cos^4 x}{\cos^4 x + \sin^4 x} dx = 2\left(\frac{\pi}{4}\right) = \frac{\pi}{2}$

9. The area bounded by the curve $y = \sin\left(\frac{x}{3}\right)$, x-axis and lines $x = 0$ and $x = 3\pi$ is _____

(1) 9 (2) 0 (3) 6 (4) 3

Ans: (3)

Put $\frac{x}{3} = t$ given integral

$= 3\int_0^{\pi} \sin t \, dt = 3\ (2) = 6$

10. The general solution of the differential equation $\sqrt{1 - x^2 y^2}\,.dx = y\,.\,dx + x\,.\,dy$ is ______

(1) $\sin(xy) = x + c$ (2) $\sin^{-1}(xy) + x = c$

(3) $\sin(x + c) = xy$ (4) $\sin(xy) + x = c$

Ans: (3)

Put xy = z

Diff. equation is $\sqrt{1-z^2}$ dx = dz $\Rightarrow \frac{dz}{\sqrt{1-z^2}}$ = dx integral

$\sin^{-1} z = x + c$

$z = \sin (x + c)$

$xy = \sin (x + c)$

11. If 'm' and 'n' are the order and degree of the differential equation

$(y^{II})^5 + 4 . \frac{(y^{II})^3}{y^{III}} + y^{III} = \sin x$, then

(1) m = 3, n = 5 (2) m = 3, n = 1 (3) m = 3, n = 3 (4) m = 3, n = 2

Ans: (4)

Multiply by y^{III}

Order = 3 degree = 2

12. If $\frac{(x+1)^2}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$, then $\sin^{-1} A + \tan^{-1} B + \sec^{-1} C$ = __________

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{6}$ (3) 0 (4) $\frac{5\pi}{6}$

Ans: (4)

Multiply by $x^3 + x$

$(x + 1)^2 = A (x^2 + 1) + (Bx + C) x$

Compare coefficient $\therefore$ A = 1 B = 0 C = 2

$\sin^{-1} 1 + \tan^{-1} 0 + \sec^{-1} 2 = \frac{5\pi}{6}$

13. The sum of the series, $\frac{1}{2.3}.2 + \frac{2}{3.4}.2^2 + \frac{3}{4.5}.2^3 +$ .... to an terms is _______

(1) $\frac{2^{n+1}}{n+2}+1$ (2) $\frac{2^{n+1}}{n+2}-1$ (3) $\frac{2^{n+1}}{n+2}+2$ (4) $\frac{2^{n+1}}{n+2}-2$

Ans: (2)

Checking with options Putting n = 2

$S_2 = \frac{1}{3} + \frac{2}{3} = 1$ satisfies only

14. If the roots of the equation $x^3 + ax^2 + bx + c = 0$ are in A.P., then $2a^3 - 9ab$ = _______

(1) 9c (2) 18c (3) 27c (4) -27c

Ans: (4)

$x^3 + ax^2 + bx + c = 0$

Let $\alpha = -1$ $\beta = 1$ $\gamma = 3$ and

$(x + 1) (x - 1) (x - 3) = 0$

$x^3 - 3x - x + 3 = 0 \Rightarrow a = -3$ b = -1 and c = 3

Substitute in options $2a^3 - 9ab = -27c$ satisfies

15. If the value of $C_0 + 2 . C_1 + 3 . C_2 + ... + (n + 1) . C_n = 576$, then n is _______

(1) 7 (2) 5 (3) 6 (4) 9

Ans: (1)

$aC_0 + (a + d) C_1 + (a + 2d) C_2 + ... + (a + nd) C_n = (2a + nd) 2^{n-1}$

$a = 1 \quad d = 1 \Rightarrow (2 + n) 2^{n-1} = 576 \Rightarrow n = 7$

16. The inverse of the proposition $(p \wedge \sim q) \to r$ is ____

(1) $(\sim r) \to (\sim p) \vee q$ (2) $(\sim p) \vee q \to (\sim r)$ (3) $r \to p \wedge (\sim q)$ (4) $(\sim p) \vee (\sim q) \to r$

Ans: (2)

Inverse is, $\sim [p \wedge \sim q] \to \sim r \equiv (\sim p) \vee q \to \sim r$

17. The range of the function f (x) = sin [x], $-\frac{\pi}{4} < x < \frac{\pi}{4}$ where [x] denotes the greatest integer $\le x$, is ______

(1) {0} (2) {0, -1} (3) $\{0, \pm \sin 1\}$ (4) {0, -sin 1}

Ans: (4)

Clearly sin 0 = 0

$\left[\frac{\pi}{4}\right] = \left[\frac{3.1}{4}\right] = 0$

$\therefore \forall x \in \left[0, \frac{\pi}{4}\right]$, sin [x] = sin 0 = 0

$\forall x \in \left[\frac{-\pi}{4}, 0\right]$, [x] = -1

$\therefore$ sin [x] = sin (-1) = -sin 1

18. If the line $6x - 7y + 8 + \lambda (3x - y + 5) = 0$ is parallel to y – axis, then λ = ______

(1) -7 (2) -2 (3) 7 (4) 2

Ans: (1)

$m = -\frac{[6 + 3\lambda]}{-7 - \lambda} = \infty \Rightarrow \lambda = -7$

19. The angle between the lines $\sin^2\alpha . y^2 - 2xy . \cos^2 \alpha + (\cos^2 \alpha - 1) x^2 = 0$ is _____

(1) 90^o (2) α (3) $\frac{\alpha}{2}$ (4) 2α

Ans: (1)

Clearly $a + b = \sin^2 \alpha + (\cos^2 \alpha - 1) = 0 \Rightarrow \theta = 90^o$

20. The minimum area of the triangle formed by the variable line $3 \cos \theta . x + 4 \sin \theta . y = 12$ and the co-ordinate axes is ______

(1) 144 (2) $\frac{25}{2}$ (3) $\frac{49}{4}$ (4) 12

Ans: (4)

GE $\Rightarrow \frac{x}{\frac{12}{3\cos\theta}} + \frac{y}{\frac{12}{4\sin\theta}} = 1$

Area $= \frac{1}{2} . \frac{4}{\cos\theta} . \frac{3}{\sin\theta} = \frac{12}{\sin 2\theta}$

When Area is minimum, $\sin 2\theta$ is maximum = 1 $\quad \therefore A_{min} = \frac{12}{1} = 12$

21. log (sin 1^o) . log (sin 2^o) . log (sin 3^o) ……… log (sin 179^o)

(1) is positive (2) is negative

(3) lies between 1 and 180 (4) is zero

Ans: (4)

log sin 1. log sin2…………log sin 90…….log sin179

= log sin1.log sin2……….log1…………..log sin 179 = 0

22. If sin x – sin y = $\frac{1}{2}$ and cos x – cos y = 1, then tan (x + y) = _____

(1) $\frac{3}{8}$ (2) -$\frac{3}{8}$ (3) $\frac{4}{3}$ (4) -$\frac{4}{3}$

Ans: (3)

$$\frac{2\cos\frac{x+y}{2}\sin\frac{x-y}{2}}{-2\sin\frac{x+y}{2}\sin\frac{x-y}{2}} = \frac{\frac{1}{2}}{1} \Rightarrow \tan\frac{x+y}{2} = -2$$

$$\Rightarrow \tan(x+y) = \frac{2\tan\frac{x+y}{2}}{1-\tan^2\frac{x+y}{2}} = \frac{2(-2)}{1-(-2)^2} = \frac{-4}{1-4} = \frac{4}{3}$$

23. In a triangle ABC, if $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ and a = 2, then its area is ………..

(1) $2\sqrt{3}$ (2) $\sqrt{3}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\frac{\sqrt{3}}{4}$

Ans: (2)

We know $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ ……….. (1)

Given $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ ……….. (2)

$\frac{(1)}{(2)}$; tan A = tan B = tan C

$\Rightarrow \Delta ABC$ is equilateral

$\therefore$ Area = $\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}2^2 = \sqrt{3}$

24. $\lim_{x\to 0} \frac{\log_e(1+x)}{3^x - 1}$ = ...

(1) $\log_e 3$ (2) 0 (3) $\log_3 e$ (4) 1

Ans: (3)

$$\lim_{x\to 0} \frac{\frac{1}{1+n}}{3^n \log 3} = \frac{\frac{1}{1+0}}{3^o \log 3} = \log_3 e$$

25. Let f (x) = $\begin{cases} x, & \text{if } x \text{ is irrational} \\ 0, & \text{if } x \text{ is rational} \end{cases}$ then f is ....

(1) continuous everywhere (2) discontinuous everywhere

(3) continuous only at x = 0 (4) continuous at all rational numbers

Ans: (3)

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x = 0$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x = 0$

$f(0) = 0$

$\therefore$ f (x) is continuous at x = 0

Note: that between every two rationals there exists one irrational number and viceversa.

26. In a regular graph of 15 vertices the sum of the degree of the vertices is 60. Then the degree of each vertex is ....

(1) 5 (2) 3 (3) 4 (4) 2

Ans: (3)

Let degree of each vertex be k

Thus $15k = 60 \Rightarrow k = 4$

27. The remainder when, $10^{10} . (10^{10} + 1) (10^{10} + 2)$ is divided by 6 is ....

(1) 2 (2) 4 (3) 0 (4) 6

Ans: (3)

$10^{10}.(10^{10} + 1) (10^{10} + 2)$ is a product of 3 consecutive integers and hence is divisible by $3! = 6$.

$\therefore$ Remainder = 0

28. A value of x satisfying $150x \equiv 35 \pmod{31}$ is ....

(1) 14 (2) 22 (3) 24 (4) 12

Ans: (3)

$150x \equiv 35 \pmod{31} \Rightarrow 30x \equiv 7 \pmod{31} \Rightarrow 31 \mid 30x - 7$

clearly 24 satisfies this relation

29. The smallest positive divisor greater than 1 of a composite number 'a' is ....

(1) $< \sqrt{a}$ (2) $= \sqrt{a}$ (3) $> \sqrt{a}$ (4) $\leq \sqrt{a}$

Ans: (4)

Standard result (Property)

30. If A and B are square matrices of order 'n' such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be true?

(1) Either of A or B is zero matrix (2) A = B

(3) AB = BA (4) Either of A or B is an identity matrix

Ans: (3)

$A^2 - B^2 = A^2 - BA + AB - B^2$

$\Rightarrow 0 = -BA + AB \Rightarrow AB = BA$

Note: Even though (1), (2) and (4) satisfy the given equation none of those is a necessary condition for $A^2 - B^2 = (A - B)(A + B)$

31. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$, then α = ...

(1) ± 1 (2) ± 2 (3) ± 3 (4) ± 5

Ans: (3)

$|A^3| = |A|^3 = 125 = 5^3 \therefore |A| = 5 \Rightarrow \alpha^2 - 4 = 5$

$\alpha^2 = 9 \Rightarrow \alpha = \pm 3$

32. If $A = \begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$ and $B = \begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix}$, then $\frac{dA}{dx}$ = ...

(1) 3B + 1 (2) 3B (3) -3B (4) 1 – 3B

Ans: (2)

$A = x(x^2 - 1) - 1(x - 1) + 1(1 - x) = x^3 - x - x + 1 + 1 - x$

$A = x^3 - 3x + 2$

$\frac{dA}{dx} = 3x^2 - 3 \quad (B = x^2 - 1) = 3B$

33. If the determinant of the adjoint of a (real) matrix of order 3 is 25, then the determinant of the inverse of the matrix is

(1) 0.2 (2) ± 5 (3) $\frac{1}{\sqrt[5]{625}}$ (4) ± 0.2

Ans: (4)

|adj A| = 25

x = 3

we have $|\text{adj } A| = |A|^{n-1}$

$25 = |A|^2 \Rightarrow |A| = \pm 5$

$\therefore |A^{-1}| = \frac{1}{|A|} = \pm\frac{1}{5} = \pm 0.2$

34. If the matrix $\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$ = A + B, where A is symmetric and B is skew symmetric, then B = ...

(1) $\begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}$ (2) $\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$ (3) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ (4) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Ans: (4)

$B = \frac{1}{2}(A - A^{l}) = \frac{1}{2}\left[\begin{pmatrix} 2 & 3 \\ 5 & -1 \end{pmatrix} - \begin{pmatrix} 2 & 5 \\ 3 & -1 \end{pmatrix}\right] = \frac{1}{2}\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

35. In a group (G, ∗), for some element 'a' of G, if $a^2 = e$, where e is the identity element, then

(1) $a = a^{-1}$ (2) $a = \sqrt{e}$ (3) $a = \frac{1}{a^2}$ (4) a = e

Ans: (1)

Direct since group is abetian

$a = a^{-1}$

36. In the group (Z, ∗), if a ∗ b = a + b – n ∀ a, b ∈ Z, where n is a fixed integer, then the inverse of (-n) is ....

(1) n (2) –n (3) -3n (4) 3n

Ans: (4)

a * e = a ⇒ a + e – n = a ⇒ e = n (identity)

To find inverse : α * (-n) = n

α - n – n = n ⇒ α = 3n

37. If $\vec{a}$ = (1, 2, 3), $\vec{b}$ = (2, -1, 1), $\vec{c}$ = (3, 2, 1) and $\vec{a} \times (\vec{b} \times \vec{c}) = \alpha \vec{a} + \beta \vec{b} \times \gamma \vec{c}$, then

(1) $\alpha = 1, \beta = 10, \gamma = 3$ (2) $\alpha = 0, \beta = 10, \gamma = -3$

(3) $\alpha + \beta + \gamma = 8$ (4) $\alpha = \beta = \gamma = 0$

Ans: (Question is wrong)

Question would have been $\vec{a} \times (\vec{b} \times \vec{c}) = \alpha \vec{a} + \beta \vec{b} + \gamma \vec{c}$

$(\vec{a} . \vec{c})\vec{b} - (\vec{a} . \vec{b})\vec{c} = \alpha \vec{a} + \beta \vec{b} + \gamma \vec{c}$

$\alpha = 0$	$\beta = \vec{a} . \vec{c}$	$\gamma = -(\vec{a} . \vec{b})$
	$= 3 + 4 + 3$	$= -(2 - 2 + 3)$
	$= 10$	$= -3$

38. If $\vec{a} \perp \vec{b}$ and $(\vec{a} + \vec{b}) \perp (\vec{a} + m\vec{b})$, then m = ______

(1) -1 (2) 1 (3) $\frac{-|\vec{a}|^2}{|\vec{b}|^2}$ (4) 0

Ans: (3)

$\vec{a} . \vec{b} = 0$

$(\vec{a} + \vec{b}).(\vec{a} + m\vec{b}) = \vec{a} . \vec{a} + m(\vec{a} . \vec{b}) + (\vec{b} . \vec{a}) + m(\vec{b} . \vec{b})$

$0 = |\vec{a}|^2 + 0 + 0 + m|\vec{b}|^2 \Rightarrow m = -\frac{|\vec{a}|^2}{|\vec{b}|^2}$

39. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then $\vec{a} . \vec{b} + \vec{b} . \vec{c} + \vec{c} . \vec{a} = ...$

(1) $\frac{3}{2}$ (2) $-\frac{3}{2}$ (3) $\frac{2}{3}$ (4) $\frac{1}{2}$

Ans: (2)

$|\vec{a} + \vec{b} + \vec{c}| = 0$

$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2GE = 0$

$GE = -\frac{3}{2}$

40. If $\vec{a}$ is vector perpendicular to both $\vec{b}$ and $\vec{c}$, then

(1) $\vec{a} . (\vec{b} \times \vec{c}) = 0$ (2) $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{0}$

(3) $\vec{a} \times (\vec{b} + \vec{c}) = \vec{0}$ (4) $\vec{a} + (\vec{b} + \vec{c}) = \vec{0}$

Ans: (2)

Take $\vec{a} = i$, $\vec{b} = j$, $\vec{c} = k$

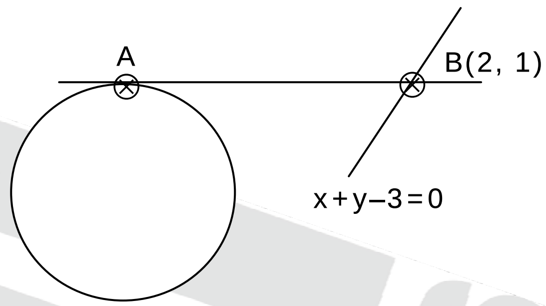
$i \times (j \times k) = i \times i = 0$

Other options will be not correct

41. A tangent is drawn to the circle $2x^2 + 2y^2 - 3x + 4y = 0$ at point 'A' and it meets the line $x + y = 3$ at B (2, 1), then AB = ...

1) $\sqrt{10}$ 2) 2 3) $2\sqrt{2}$ 4) 0

Ans: (2)



AB = length of Tangent to the circle from B.

$AB = \sqrt{x^2 + y^2 - \frac{3}{2}x + 2y} = \sqrt{4+1-3+2} = 2$ units.

42. The area of the circle having its centre at (3, 4) and touching the line $5x + 12y - 11 = 0$ is ....

1) 16π sq. units 2) 4π sq. units 3) 12π sq. units 4) 25π sq. units

Ans: (1)

$C \equiv (3, 4)$

$r = \left|\frac{5(3)+12(4)-11}{\sqrt{25+144}}\right| = \left|\frac{15+48-11}{\sqrt{169}}\right| = \left|\frac{52}{13}\right| = 4 \Rightarrow A = \pi r^2 = 16\pi \text{ units}^2$

43. The number of real circles cutting orthogonally the circle $x^2 + y^2 + 2x - 2y + 7 = 0$ is ......

1) 0 2) 1 3) 2 4) infinitely many

Ans: (1)

$x^2 + y^2 + 2x - 2y + 7 = 0$

$r = \sqrt{1+1-7} = \sqrt{-5}$, imaginary $\therefore$ Given circle is an imaginary circle.

$\therefore$ Number of real circles cutting orthogonally given imaginary circle is zero.

44. The length of the chord of the circle $x^2 + y^2 + 3x + 2y - 8 = 0$ intercepted by the y-axis is

1) 3 2) 8 3) 9 4) 6

Ans: (4)

$x^2 + y^2 + 3x + 2y - 8 = 0$

Intercept made by y-axis = $2\sqrt{f^2 - C} = 2\sqrt{(1)+8} = 6$

45. $A \equiv (\cos\theta, \sin\theta)$, $B \equiv (\sin\theta, -\cos\theta)$ are two points. The locus of the centroid of ΔOAB, where 'O" is the origin is ....

1) $x^2 + y^2 = 3$ 2) $9x^2 + 9y^2 = 2$ 3) $2x^2 + 2y^2 = 9$ 4) $3x^2 + 3y^2 = 2$

Ans: (2)

Take $\theta = \frac{\pi}{4}$

$A = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $B = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$, O = (0, 0) $\therefore$ Centroid $\left(\frac{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + 0}{3}, \frac{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + 0}{3}\right) = \left(\frac{\sqrt{2}}{3}, 0\right)$

only equation $9x^2 + 9y^2 = 2$ holds.

46. The sum of the squares of the eccentricities of the conics $\frac{x^2}{4} + \frac{y^3}{3} = 1$ and $\frac{x^2}{4} - \frac{y^2}{3} = 1$ is ....

1) 2   2) $\sqrt{\frac{7}{3}}$ 3) $\sqrt{7}$ 4) $\sqrt{3}$

Ans: (1)

$\frac{x^2}{4}+\frac{y^2}{3}=1$ $e_1 = \sqrt{\frac{4-3}{4}} = \frac{1}{2}$

$\frac{x^2}{4}-\frac{y^2}{3}=1$ $e_1{}^2 = \sqrt{\frac{4+3}{4}} = \frac{\sqrt{7}}{2}$ $\therefore e_1{}^2 + 2_2{}^2 = \frac{1}{4}+\frac{7}{4} = \frac{8}{4} = 2$

47. The equation of the tangent to the parabola $y^2 = 4x$ inclined at an angle of $\frac{\pi}{4}$ to the +ve direction of x-axis is ....

1) $x + y - 4 = 0$ 2) $x - y + 4 = 0$ 3) $x - y - 1 = 0$ 4) $x - y + 1 = 0$

Ans: (4)

$y = mx + c$ be a tangent $\theta = \frac{\pi}{4} \Rightarrow m = 1$

$\therefore y = 1.x + c$ $a = 1$

Condition is $c = \frac{a}{m} = \frac{1}{1}$

$\therefore y = x + 1 \Rightarrow x - y + 1 = 0$

48. If the distance between the foci and the distance between the directrices of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are in the ratio 3 : 2, then a : b is ....

1) $\sqrt{2} + 1$ 2) 1 : 2   3) $\sqrt{3} : \sqrt{2}$ 4) 2 : 1

Ans: (1)

Given $\frac{2ae}{2a/e}=\frac{3}{2}$ $\Rightarrow e^2 = \frac{3}{2}$

$\Rightarrow \frac{a^2+b^2}{a^2}=\frac{3}{2} \Rightarrow \left(\frac{b}{a}\right)^2=\frac{1}{2}$ $\therefore$ $\frac{b}{a}=\frac{1}{\sqrt{2}}$

49. If the area of the auxillary circle of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ is twice the area of the ellipse, then the eccentricity of the ellipse is ....

1) $\frac{1}{\sqrt{3}}$ 2) $\frac{1}{2}$ 3) $\frac{1}{\sqrt{2}}$ 4) $\frac{\sqrt{3}}{2}$

Ans: (4)

Area of auxiliary circle $x^2 + y^2 = a^2$ is πa^2 area of ellipse $= \pi ab$

Given, $\pi a^2 = 2\pi ab$

$a = 2b$

$\therefore e = \sqrt{1-\left(\frac{b}{a}\right)^2} = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$

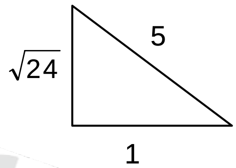
50. $\cos\left[2\cos^{-1}\frac{1}{5}+\sin^{-1}\frac{1}{5}\right]$ = ...

1) $\frac{1}{5}$ 2) $\frac{-2\sqrt{6}}{5}$ 3) - $\frac{1}{5}$ 4) $\frac{\sqrt{6}}{5}$

Ans: (2)

$$\cos\left[\cos^{-1}\left(\frac{1}{5}\right)+\cos^{-1}\left(\frac{1}{5}\right)+\sin^{-1}\left(\frac{1}{5}\right)\right] = \cos\left[\cos^{-1}\left(\frac{1}{5}\right)+\frac{\pi}{2}\right]$$

$$= \cos\left(\frac{\pi}{2}+\cos^{-1}\frac{1}{5}\right) = -\sin\left(\cos^{-1}\frac{1}{5}\right) = -\sin\left(\sin^{-1}\frac{\sqrt{24}}{5}\right) = -\frac{2\sqrt{6}}{5}$$



51. The value of $\tan^{-1}\left(\frac{x}{y}\right)$ - $\tan^{-1}\left(\frac{x-y}{x+y}\right)$, x, y > 0 is

1) $\frac{\pi}{4}$ 2) - $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) - $\frac{\pi}{2}$

Ans: (1)

Take x = 1, y = 1

LHS = $\tan^{-1}\left(\frac{1}{1}\right)$ - $\tan^{-1}(0)$ = $\frac{\pi}{4}$

52. The general solution of sin x – cos x = $\sqrt{2}$, for any integer 'n' is ....

1) $2n\pi + \frac{3\pi}{4}$ 2) $n\pi$ 3) $(2n + 1)\pi$ 4) $2n\pi$

Ans: (1)

sin x – cos x = $\sqrt{2}$

Method of Inspection

For n = 0 (1) x = $\frac{3\pi}{4}$ holds

(2) x = 0, doesn't hold

(3) x = π doesn't hold

(4) x = 0 doesn't hold

53. The modulus and amplitude of $\frac{1+2i}{1-(1-i)^2}$ are ....

1) $\sqrt{2}$ and $\frac{\pi}{6}$ 2) 1 and $\frac{\pi}{4}$ 3) 1 and 0 4) 1 and $\frac{\pi}{3}$

Ans: (3)

$$\frac{1+2i}{1-(1-i)^2} = \frac{1+2i}{1-1-i^2+2i} = \frac{1+2i}{1+2i} = 1 + i.0$$

$\therefore$ Modulus = 1 amplitude = $\tan^{-1}\left|\frac{0}{1}\right|$ = 0

54. If $2x = -1 + \sqrt{3}\, I$, then the value of $(1 + x^2 + x)^6 - (1 - x + x^2)^6 =$ ....

1) 32 2) 64 3) – 64 4) 0

Ans: (4)

$2x = -1 + \sqrt{3}\, i$

$x = \dfrac{-1 + \sqrt{3}\, i}{2} = \omega$

$\text{LHS} = (1 - \omega^2 + \omega)^6 - (1 - \omega + \omega^2)^6$

$= (-2\omega^2)^6 - (-2\omega)^6 = 64 - 64 = 0$

55. If $x + y = \tan^{-1} y$ and $\dfrac{d^2y}{dx^2} = f(y)\dfrac{dy}{dx}$, then $f(y) =$ ....

1) $\dfrac{-2}{y^3}$ 2) $\dfrac{2}{y^3}$ 3) $\dfrac{1}{y}$ 4) $\dfrac{-1}{y}$

Ans: (2)

$x + y = \tan^{-1} y \Rightarrow x + y - \tan^{-1} y = 0$

$\dfrac{dy}{dx} = \dfrac{1}{1 - \dfrac{1}{1+y^2}} \Rightarrow \dfrac{dy}{dx} = -\dfrac{1}{y^2} - 1$

$\dfrac{d^2y}{dx^2} = -\dfrac{(-2)}{y^3}\dfrac{dy}{dx} = \dfrac{2}{y^3}\dfrac{dy}{dx}$

56. $f(x) = \begin{cases} 2a - x \text{ when } -a < x < a \\ 3x - 2a \text{ when } a \le x \end{cases}$

Then which of the following is true?

1) f (x) is not differentiable at x = a. 2) f (x) is discontinuous at x = a.

3) f (x) is continuous for all x < a. 4) f (x) is differentiable for all $x \ge a$.

Ans: (1)

$f^{\prime}(a^-) = -1$ and $f^{\prime}(a^+) = 3$

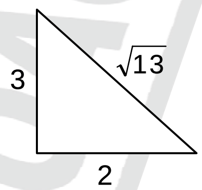
$\therefore f^{\prime}(a^-) \ne f^{\prime}(a^+)$

57. Let $f(x) = \cos^{-1}\left[\dfrac{1}{\sqrt{13}}(2\cos x - 3\sin x)\right]$. Then $f'(0.5) =$ ....

1) 0.5 2) 1 3) 0 4) – 1

Ans: (2)

$f(x) = \cos^{-1}\left[\dfrac{2}{\sqrt{13}}\cos x - \dfrac{3}{\sqrt{13}}\sin x\right]$



$= \cos^{-1}[\cos\alpha \cdot \cos x - \sin\alpha \cdot \sin x]$

$= \cos^{-1}[\cos(x + \alpha)] = x + \alpha$

$\Rightarrow f^{\prime}(x) = 1 \therefore f^{\prime}(0.5) = 1$

58. If f (x) is a function such that f” (x) + f (x) = 0 and g (x) = $[f(x)]^2 + [f'(x)]^2$ and g (3) = 8, then g (8) = ……..

1) 0 2) 3 3) 5 4) 8

Ans: (4)

$g(x) = [f(x)]^2 + [f'(x)]^2$

$\Rightarrow g'(x) = 2 f(x) f'(x) + 2f'(x) f''(x)$

$= 2 f'(x) [f(x) + f''(x)]$

$= 2f'(x) [0] = 0$

$\Rightarrow$ g (x) = constant

Given that g (3) = 8

$\therefore$ g (x) = 8

59. If f (x) = f’ (x) = f” (x) + f”’ (x) + ……… and f (0) = 1, then f (x) = …………

a) $e^{\frac{x}{2}}$ 2) e^x 3) e^{2x} 4) e^{4x}

Ans: (1)

If f (x) = $e^{\frac{x}{2}}$ then

$$f(x) = e^{\frac{x}{2}} . \frac{1}{2} + e^{\frac{x}{2}} . \frac{1}{2^2} + e^{\frac{x}{2}} . \frac{1}{2^3} + ...$$

$$f(x) = e^{\frac{x}{2}} \left[\frac{\frac{1}{2}}{1 - \frac{1}{2}}\right] = e^{\frac{x}{2}}$$

and f (0) = e^o = 1

60. The function f (x) = $\frac{x}{3} + \frac{3}{x}$ decreases in the interval

1) (-3, 3) 2) $(-\infty, 3)$ 3) $(3, \infty)$ 4) (-9, 9)

Ans: (1)

$$f(x) = \frac{x}{3} + \frac{3}{x}$$

$$f'(x) = \frac{1}{3} - \frac{3}{x^2} < 0$$

$$\Rightarrow \frac{1}{3} < \frac{3}{x^2} \Rightarrow x^2 < 9$$

$\Rightarrow x \in (-3, 3)$