PART 01 - MATHEMATICS

(Common to all candidates)

(Answer ALL questions)

1. The unit normal to the surface $x^2y + 2xz = 4$ at the point (2, -2, **3)** is

   1. $-i + 2j + 2\bar{k}$
   2. $\frac{1}{3}(-i + 2j + 2\bar{k})$
   3. $\frac{1}{3}(i - 2j + 2\bar{k})$
   4. $i - 2j - 2\bar{k}$

2. If r = $\sqrt{x^2 + y^2 + z^2}$, then V$\left(\frac{1}{r}\right)$ is equal to

   1. $\frac{\bar{r}}{r^3}$
   2. $\frac{\bar{r}}{r^2}$
   3. $\frac{-\bar{r}}{r^2}$
   4. $\frac{-\bar{r}}{r^3}$

3. If $\bar{A} = x^2zi - 2y^3z^2\bar{j} + xy^2z\bar{k}$, then $div\bar{A}$ at $(1, -1, 1)$ is

   1. 0
   2. -3
   3. 3
   4. 1

4. If $\bar{A} = x^2yi - 2xz\bar{j} + 2yz\bar{k}$, then $curlcurl\bar{A}$ is

   1. $(x+2)\bar{j}$
   2. $(2x+2)\bar{j}$
   3. $(2x+1)\bar{j}$
   4. $(2x+2y)\bar{j}$

5. If $\bar{V} = (x + 2y + az)i + (bx - 3y - z)\bar{j} + (4x + cy + 2z)\bar{k}$ is irrotational, then

   1. $a = 4, b = -1, c = 2$
   2. $a = 2, b = -1, c = 4$
   3. **$a = 4, b = 2, c = -1$**
   4. $a = 4, b = -2, c = 1$

6. Which of the following is a factor of the determinant?

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$$

   1. a
   2. a - b
   3. $a + b$
   4. $a + b + c$

7. If $a + b + c = 0$, one root of $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ is

   1. $x = 1$
   2. $x = 2$
   3. $x = a^2 + b^2 + c^2$
   4. $x = 0$

8. If $\boldsymbol{A}$ is a 4×4 matrix. **A** second order minor of A has its value as 0. Then the rank of $\boldsymbol{A}$ is
   1. <2
   2. $=\mathbf{2}$
   3. $>\mathbf{2}$
   4. anything

9. Given $A=\begin{pmatrix}2&0&0\\0&4&0\\0&0&8\end{pmatrix}$, then the determinant value of A^{-1} is
   1. **32**
   2. $\frac{1}{32}$
   3. $\frac{1}{64}$
   4. 64

10. If $\begin{pmatrix}3&1\\4&1\end{pmatrix}X=\begin{pmatrix}5&-1\\2&3\end{pmatrix}$, then
   1. $X=\begin{pmatrix}-3&4\\14&13\end{pmatrix}$
   2. $X=\begin{pmatrix}3&-4\\-14&13\end{pmatrix}$
   3. $X=\begin{pmatrix}-3&4\\14&-13\end{pmatrix}$
   4. $X=\begin{pmatrix}-3&-4\\-14&13\end{pmatrix}$

11. C-R equations for a function w = $P(r,\theta)+iQ(r,\theta)$ to be analytic, in polar form are
   1. $\frac{\partial P}{\partial r}=\frac{1}{r}\frac{\partial Q}{\partial\theta},\ \frac{\partial Q}{\partial r}=\frac{-1}{r}\frac{\partial P}{\partial\theta}$
   2. $\frac{\partial Q}{\partial\theta}=\frac{1}{r}\frac{\partial P}{\partial r},\ \frac{\partial P}{\partial\theta}=\frac{1}{r}\frac{\partial Q}{\partial r}$
   3. $\frac{\partial P}{\partial r}=\frac{-1}{r}\frac{\partial Q}{\partial\theta},\ \frac{\partial Q}{\partial r}=\frac{1}{r}\frac{\partial P}{\partial\theta}$
   4. $\frac{\partial P}{\partial\theta}=\frac{1}{r}\frac{\partial Q}{\partial r},\ \frac{\partial Q}{\partial\theta}=\frac{-1}{r}\frac{\partial P}{\partial r}$

12. If f (z) = u $+iv$ is an analytic function and u and v are harmonic, then u and v will satisfy
   1. one dimensional wave equation
   2. one dimensional heat equation
   3. Laplace equation
   4. Poisson equation

13. In the analytic function f $(z)=u+iv$, the curves $u(x,y)=c_1$ and $v(x,y)=c_2$ are orthogonal if the product of the slopes m_1 and m_2 are
   1. $m_1m_2=0$
   2. $m_1m_2=-\pi$
   3. $m_1m_2=\frac{-\pi}{2}$
   4. $m_1m_2=-1$

14. If the imaginary part of the analytic function $f(z)=u+iv$ is constant, then
   1. u is not a constant
   2. $f(z)$ is not a complex constant
   3, f (z) is equal to zero
   4. u is a constant

15. If f (z) = P (r,θ) + iQ $(r,8)$ is analytic, then f $'(z)$ is equal to
   1. $e^{i\theta}\left(\frac{\partial P}{\partial r}+i\frac{\partial Q}{\partial\theta}\right)$
   2. $e^{-i\theta}\left(\frac{\partial P}{\partial r}+i\frac{\partial Q}{\partial\theta}\right)$
   3. $e^{-i\theta}\left(\frac{\partial P}{\partial r}+i\frac{\partial Q}{\partial r}\right)$
   4. $e^{+i\theta}\left(\frac{\partial P}{\partial r}+i\frac{\partial Q}{\partial r}\right)$

16. The formula for the radius of curvature in cartesian coordinate is

1. $\dfrac{\left(1+(y')^2\right)^{1/2}}{y''(x)}$
2. $\dfrac{\left(1+(y')^2\right)^{3/2}}{y''(x)}$
3. $\dfrac{\left(1+(y')^2\right)^{3/2}}{(y'')^2}$
4. $\dfrac{\left(1+(y')^2\right)^{1/2}}{(y''(x))^2}$

17. The condition for the function $z = f(x, y)$ to have a extremum at (a, a) is $\dfrac{\partial z}{\partial x} = 0$, $\dfrac{\partial z}{\partial y} = 0$. and $A = \dfrac{\partial^2 z}{\partial x^2}, B = \dfrac{\partial^2 z}{\partial x \partial y}, C = \dfrac{\partial^2 z}{\partial y^2}$, $\Delta = AC - B^2$. Then the function z has a maximum value at (a, a) if

1. $\Delta > 0, A < 0$
2. $\Delta > 0, A = 0$
3. $\Delta < 0, A < 0$
4. $\Delta > 0, A > 0$

18. The stationary point of $f(x, y) = x^2 - xy + y^2 - 2x + y$ is

1. $(0, 1)$
2. $(1, 0)$
3. $(-1, 0)$
4. $(1, -1)$

19. $\displaystyle\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\, dx$ is

1. $\dfrac{\pi}{2}$
2. π
3. $\dfrac{\pi}{4}$
4. 2π

20. $\int x \cos x \, dx$ is

1. $x \sin x + \cos x$
2. $x \sin x - x \cos x$
3. $x \sin x + x \cos x$
4.

21. For the following data :

x:	0	2	4	6
y:	-1	3	7	11

the straight line $y = mx + c$ by the method of least square is

1. $y = -2x - 1$ $y = x - 1$
2. $y = 1 - 2x$
3. $y = 2x - 1$
4.

22. The velocity v (km/min) of a train which starts from rest, is given at fixed intervals of time t (min) as follows :

t :	2	4	6	8	10	12	14	16	18	20
v :	10	18	25	29	32	20	11	5	2	0

The approximate distance covered by Simpson's 1/3 rule is

1. 306.3
2. 309.3
3. 310.3
4. 307.3

23. Find the cubic polynomial by Newton's forward difference which takes the following

x : 0 1 2 3

$f(x)$: 1 2 1 10

Then f (4) is

1. 40
2. 41
3. 39
4. 42

24. The first derivative $\frac{dy}{dx}$ at $x = 0$ for the given data

x : 0 1 2 3

$f(x)$: 2 1 2 5

is

1. 2
2. –2
3. –1
4. 1

25. Error in Simpson's $\frac{1}{3}$ rule is of the order

1. $-h^2$
2. h^3
3. h^4
4. $\frac{2h^3}{3}$

26. A lot consists of ten good articles, four with minor defects and two with major defects. Two articles are chosen from the lot a t random (without replacement). Then the probability that neither of them good is

1. $\frac{5}{8}$
2. $\frac{7}{8}$
3. $\frac{3}{8}$
4. $\frac{1}{8}$

27. If A , B, C are any three events such that

$P(A) = P(B) = P(C) = \frac{1}{4}$;

$P(A \cap B) = P(B \cap C) = 0$, $P(C \cap A) = \frac{1}{8}$.

Then the probability that atleast one of the events A , B, C occurs, is

1. $\frac{1}{32}$
2. $\frac{3}{32}$
3. $\frac{7}{8}$
4. $\frac{5}{8}$

28. To establish the mutual independence of n events, the equations needed are

1. $2^n + n + 1$
2. $n^2 + n + 1$
3. $2^n - (n+1)$
4. $2^n + 2(n+1)$

29. If atleast one child in a family with two children is a boy, then the probability that both children are boys is

1. 3/4
2. 1/3
3. 1/4
4. 1/2

30. A discrete random variable X takes the values $a, ar, ar^2, \cdots, ar^{n-1}$ with equal probability. Then Arithmetic Mean (A.M) is

1. $a(1-r^n)$
2. $\frac{1}{n}a(1-r^n)$
3. $\frac{a}{n}\frac{(1-r^n)}{1-r}$
4. $\frac{a}{n}\frac{(r^n-1)}{1-r}$