

# NEST – 2010



# NATIONAL ENTRANCE SCREENING TEST (NEST – 2010)

Total Marks: 200

Time: 3 hours

## General instructions

1. This question booklet contains 5 sections, with mark distribution as follows.

Section 1	General	60 marks	Compulsory
Section 2	Biology	70 marks	} Choose any two sections
Section 3	Chemistry	70 marks	
Section 4	Mathematics	70 marks	
Section 5	Physics	70 marks	

$$\text{Total} = 60 + 2 \times 70 = 200 \text{ marks}$$

2. Section 1 is a **General** section and is **compulsory**.
3. Sections 2 to 5 are subject sections (**Biology, Chemistry, Mathematics and Physics**). **Choose any two**. That is, omit any two of the four subject sections.
4. Carefully read and follow the instructions given in each section.
5. Answers to the questions are to be marked in the Answer Sheet provided.
6. **Ensure that you have received Answer Sheet A.**
7. Rough work should be done on the sheets provided.
8. Return the Answer Sheet to the invigilator at the end of the examination.
9. Calculators, log tables, cell phones, etc. are not permitted in the examination hall.

## Instructions for writing on Answer Sheet

1. Read and follow the instructions given on the Answer Sheet.
2. Write your name, roll number and other required information with ball point pen in the appropriate boxes provided. Sign your name with ball point pen in the box provided.
3. **Your roll number (given on the admit card) must be entered correctly. If entered wrongly or not entered, the Answer Sheet may not be graded.**
4. In the remaining part of Answer Sheet use **HB pencil only as instructed. Make sure that the bubbles are filled properly** (as shown in Answer Sheet).
5. Each question has four options. Fill the appropriate bubble(s). Some questions (as specified in the question paper) have more than one correct option.
6. **Ensure that you are filling up the bubbles corresponding to correct sections.**
7. Fill in the answers only when you are sure that you do not need to change the answer. As far as possible, avoid erasing the answer. In case you have to erase the answer, do so properly so that there is no black spot inside the bubble.

## Section 1: GENERAL

Marks for Section 1: 60

*This section contains 22 questions.*

*For each question, only one of the 4 options is a correct answer. For questions 1.1 to 1.16, a correct answer will earn 3 marks. For questions 1.17 to 1.22, a correct answer will earn 2 marks. For this section, a wrong answer or an unattempted question will earn 0 mark.*

Read the following passage carefully and answer questions 1.1 to 1.3.

### Metabolism and Life

Living systems are open, non-equilibrium systems which continually exchange energy and matter with their surroundings through the process of metabolism. The metabolic network of a living organism consists of a large number of pathways. Each pathway consists of a number of biochemical reactions. The pathways are integrated with/linked to other pathways to various extents. There are no stand-alone pathways.

A metabolite is a molecule in a metabolic network. A precursor is a metabolite in a pathway from which other metabolites are synthesised either in a single step or in multiple steps. An end-product is a metabolite that is at the dead end of a pathway. Some steps of a metabolic pathway are committed steps for that pathway (i.e., they are not reversible).

Metabolic pathways are regulated. “Repression” is one type of regulatory mechanism where the synthesis of the enzyme from DNA is inhibited, i.e., no new molecules of the enzyme which is under inhibition are synthesised. “Feedback inhibition” is another type where the existing enzyme molecules are prevented from catalysing the reaction.

In the “common feedback inhibition” mechanism, the end-product of a pathway inhibits the committed step.

A “sequential feedback inhibition” mechanism is seen in branched pathways. In this mechanism, the intermediate before the branch point inhibits the committed step.

In the “cumulative feedback inhibition” mechanism, multiple end-products partially inhibit a committed step.

1.1 Suppose all the reactions in a unicellular organism have come to equilibrium. This

- (A) signals the birth of the organism.
- (B) happens when the organism is at rest.
- (C) is true at all the times.
- (D) leads to death.

1.2 P, Q, R, S, T and U are metabolites. Within this set of metabolites, P is the precursor of all others. T and U are the only end-products. Q is the precursor of R and S. R is the precursor of T. Both the metabolites T and U partially inhibit the synthesis of Q. This is an example of

- (A) common feedback inhibition.
- (B) cumulative feedback inhibition.
- (C) repression.
- (D) sequential feedback inhibition.

1.3 In its natural environment, a bacterium *E. coli* can digest lactose by synthesising  $\beta$ -galactosidase enzyme. If the medium contains minimal lactose, a specific protein binds to the nearby region of the gene and inhibits enzyme synthesis. This is an example of

- (A) common feedback inhibition.
- (B) cumulative feedback inhibition.
- (C) repression.
- (D) sequential feedback inhibition.

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Read the following passage carefully and answer questions 1.4 to 1.6.

### Blue sky

Lord Rayleigh was the first to explain the colour of the sky. He showed that the sky looks blue because of scattering of light by air molecules. When light of wavelength  $\lambda$  traverses a dilute gaseous medium, its intensity gets attenuated (reduced) according to the law

$$I(x) = I(0)e^{-\alpha x}$$

where  $I(x)$  is the intensity after it has traversed a distance  $x$ . The attenuation coefficient  $\alpha$  is given by

$$\alpha = \frac{32\pi^3}{3n\lambda^4}(\mu - 1)$$

where  $n$  and  $\mu$  are the number density and refractive index of the gaseous medium, respectively. The intensity decreases because the gas molecules scatter away light.

Let us define  $\eta$  to be the ratio of light intensity at the ground to that at the top of the atmosphere in a given direction:  $\eta = \frac{I_{\text{ground}}}{I_{\text{top}}}$ . Making suitable approximations for the variation of the density of atmosphere as a function of height from ground level, this ratio may be estimated for different colours (say red, green and blue).

1.4 The sky looks blue in directions away from the sun mainly because

- (A)  $\eta_{\text{blue}} \simeq 1$  whereas  $\eta$  for other colours is much less than 1.
- (B)  $\mu_{\text{blue}} < \mu_{\text{red}}$ .
- (C)  $\alpha_{\text{blue}} > \alpha_{\text{green}} > \alpha_{\text{red}}$ .
- (D)  $\alpha_{\text{blue}} < \alpha_{\text{green}} < \alpha_{\text{red}}$ .

1.5 The redness of the sun at sunrise/sunset arises because

- (A)  $\alpha_{\text{red}} > \alpha_{\text{green}} > \alpha_{\text{blue}}$ .
- (B)  $\eta_{\text{red}} > \eta_{\text{green}} > \eta_{\text{blue}}$  at sunrise/sunset in the direction of the sun.
- (C)  $\eta_{\text{red}} < \eta_{\text{green}} < \eta_{\text{blue}}$  at sunrise/sunset in the direction of the sun.
- (D)  $\eta_{\text{red}} = 1$  at sunrise/sunset while  $\eta$  for other colours is much less than 1.

1.6 Let  $\beta = \frac{\eta_{\text{red}}}{\eta_{\text{blue}}}$ . Considering intensity in the direction of the sun at noontime (nt) and sunrise/sunset (ss), we have

- (A)  $\beta_{\text{nt}} < \beta_{\text{ss}}$ .
  - (B)  $\beta_{\text{nt}} > \beta_{\text{ss}}$ .
  - (C)  $\beta_{\text{nt}} = \beta_{\text{ss}} = 1$ .
  - (D)  $\beta_{\text{nt}} = \beta_{\text{ss}} \neq 1$ .
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1.7 Using numbers from 1 to 6, the number 9 could be obtained as a sum of three numbers in six ways: 1+2+6, 1+3+5, 1+4+4, 2+2+5, 2+3+4, 3+3+3 and the number 10 can be obtained similarly also in six ways: 1+4+5, 1+3+6, 2+4+4, 2+2+6, 2+3+5, 3+3+4. In a throw of three dice, let  $p$  be the probability of getting the sum 9 and  $q$  be the probability of getting 10 be  $q$ . Then

- (A)  $p = q = \frac{1}{36}$
- (B)  $p = q = \frac{1}{6}$
- (C)  $p > q$
- (D)  $p < q$

1.8 The bulk energy of a spherical system of radius  $R$  is proportional to its volume and the surface energy is proportional to its area. The surface energy per unit area,  $\sigma$ , and bulk energy per unit volume,  $u$ , of the system are characteristic properties of the system that are constant at a given temperature and vary with temperature as  $\sigma \propto T^{1/2}$  and  $u \propto T$ . In estimating the total energy the approximation of neglecting the surface energy is best when

- (A)  $R$  is large and  $T$  is large.
- (B)  $R$  is large and  $T$  is small.
- (C)  $R$  is small and  $T$  is large.
- (D)  $R$  is small and  $T$  is small.

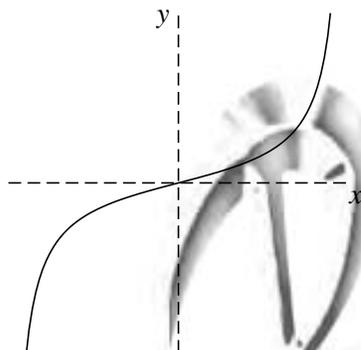
1.9 The expression  $y^{\log_y(xw)}$  is

- (A)  $\frac{1}{xw}$       (B)  $x + w$       (C)  $xw$       (D)  $(xw)^y$

1.10 Let  $M$  be the product of integers from 1 to 50, which has a factor  $2^n$ . The maximum value of  $n$  is

- (A) 19.      (B) 33.      (C) 47.      (D) 50.

1.11 The curve shown in the accompanying figure best approximates

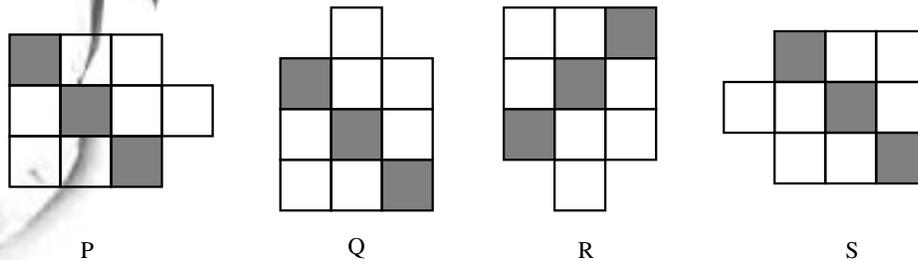


- (A)  $y = \log(1 + x)$       (B)  $y = \tan x$       (C)  $y = x^4$       (D)  $y = \tan^{-1} x$

1.12 A “straight line” between two points on a sphere is defined to be an arc of the great circle passing through these points. Given two points on the equator on the Earth (considered as a perfect sphere), one at longitude 0 degree and the other at longitude 180 degrees, the number of “straight lines” passing through these two points is

- (A) 1.      (B) 2.      (C) 4.      (D) infinite.

1.13 Three of the following four figures are related to one another by rotation in the plane of the paper.



The figure that cannot be related to the others by this operation is:

- (A) P      (B) Q      (C) R      (D) S

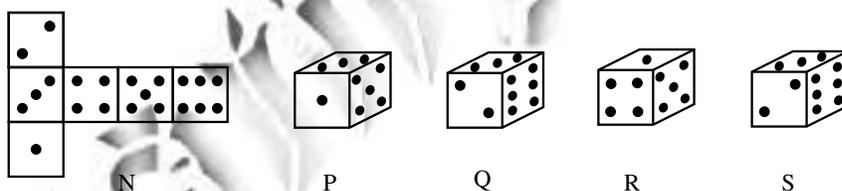
1.14 The continued fraction  $\alpha$  is defined as  $\alpha = 1 + \frac{1}{1 + \frac{1}{1 + \dots}}$ . The value of  $\alpha$  is

- (A) 2.      (B)  $\frac{\sqrt{5}}{2}$ .      (C)  $\frac{\sqrt{5} + 1}{2}$ .      (D)  $\frac{\sqrt{5} - 1}{2}$ .

1.15 The average  $\bar{x}$  and standard deviation  $\sigma$  of marks  $x_i$  obtained by students in an examination are defined as  $\bar{x} = \frac{1}{N} \sum_i f_i x_i$  and  $\sigma = \sqrt{\frac{1}{N} \sum_i f_i (x_i - \bar{x})^2}$ , where  $\sum_i f_i = N$  is the total number of students,  $f_i$  is the number of students having marks  $x_i$ , and  $\sum_i$  stands for summation. In an examination of 100 students for a 50-mark paper, the maximum score obtained was 44. Each student was awarded 6 additional marks and the resulting marks were scaled up to a total of 100 marks for the paper. Then

- (A)  $\bar{x}_{\text{new}} = 2\bar{x}_{\text{old}} + 12$  and  $\sigma_{\text{new}} = 2\sigma_{\text{old}}$   
 (B)  $\bar{x}_{\text{new}} = \bar{x}_{\text{old}} + 12$  and  $\sigma_{\text{new}} = 2\sigma_{\text{old}}$   
 (C)  $\bar{x}_{\text{new}} = \bar{x}_{\text{old}} + 6$  and  $\sigma_{\text{new}} = \sigma_{\text{old}}$   
 (D)  $\bar{x}_{\text{new}} = 2\bar{x}_{\text{old}} + 12$  and  $\sigma_{\text{new}} = \sigma_{\text{old}}$

1.16 From the net N, as shown here, one can make a cube by making folds at the appropriate edges of the squares, such that the dots appear on the outside. No cutting is allowed.



Which of the cubes shown above can be made from the net N?

- (A) P      (B) Q      (C) R      (D) S

1.17 Cerenkov radiation is

- (A) the light emitted by a charged particle travelling in a medium with a speed greater than the speed of light in the medium.  
 (B) the  $\gamma$ -radiation emitted by an excited heavy nucleus coming to its ground state.  
 (C) the  $\beta$ -radiation emitted by a radioactive element in the actinide series.  
 (D) the radiation emitted when  $\alpha$ -particles emitted by a radioactive source are stopped by a target.

**1.18** The British scientist Rosalind Franklin is known for her work on

- (A) gene cloning.
- (B) the origin of atmospheric electricity.
- (C) X-ray diffraction studies of DNA.
- (D) super-heavy elements.

**1.19** Consider the four vitamins, A, B, C, D and their properties as listed:

1. Water soluble
2. Curing scurvy
3. Preventing rickets
4. Fat soluble

Choose the correct match from the options below.

- (A) A-4, B-1, C-3, D-2
- (B) A-4, B-1, C-2, D-3
- (C) A-1, B-4, C-2, D-3
- (D) A-4, B-3, C-2, D-1

**1.20** The famous number 1729 (associated with the great Indian mathematician Ramanujan) has the interesting property that

- (A) it is the cube of a prime number.
- (B) it is the difference of squares of two prime numbers.
- (C) it is the smallest number that can be expressed as sum of cubes of two integers.
- (D) it is the smallest number that can be expressed as sum of cubes of two integers in two different ways.

**1.21** While classifying elements, Mendeleev left some blank spaces in his periodic table. He believed these elements to exist. He named one such element as ekasilicon and predicted its properties. The element was discovered later and is now known as

- (A) gallium.
- (B) germanium.
- (C) scandium.
- (D) titanium.

**1.22** The clue to possible existence of water on the moon from the Chandrayaan-1 mission came from

- (A) spectroscopic studies.
- (B) heat-sensing devices.
- (C) acoustic interferometry.
- (D) chemical analysis.

## Section 2: BIOLOGY

Marks for Section 2: 70

*This section contains 20 questions.*

*For questions 2.1 to 2.15 only one of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn (-1) mark, and an unattempted question will earn 0 mark.*

**2.1** Enzymes are biocatalysts that catalyse reactions at very high rates compared to chemical catalysts. They are specific to the substrate and reaction they catalyse. A few statements about enzymes are made below:

- (i) Not every enzyme is proteinaceous in nature.
- (ii) Some RNAs also are enzymes.
- (iii) The active site of the enzyme is complementary to the transition state.
- (iv) Enzymes alter the equilibrium constant of the reaction.
- (v) Enzymes catalyse only irreversible reactions.

Which of the above statements are true?

- (A) (i), (ii), and (iii).
- (B) (ii), (iii), and (iv).
- (C) (iii), (iv), and (v).
- (D) (i), (ii), and (v).

**2.2** An mRNA ready for translation would have

- (A) introns, coding exons and non-coding exons.
- (B) coding exons and non-coding exons.
- (C) only coding exons.
- (D) only coding exons and introns.

**2.3** During oogenesis in mammals, the second meiotic division occurs

- (A) during the formation of primary oocyte.
- (B) before ovulation.
- (C) after fertilisation.
- (D) after implantation.

**2.4** The Kyoto Protocol specifies regulations on the emission of greenhouse gases. It defines a term known as “Carbon-Credits”. The following statements pertain to Carbon-Credits:

- (i) The mandatory limit of Carbon-Credit for each country is directly proportional to its size and population.
- (ii) One Carbon-Credit defines the emission of one ton of carbon dioxide or equivalent gases responsible for greenhouse effect.
- (iii) Carbon-credits are exchangeable among countries/industries.
- (iv) An industry emitting higher than prescribed limit can do so by purchasing Carbon-Credits.

Which of the above statements are true?

- (A) (i) and (ii) only.
- (B) (ii) and (iv) only.
- (C) (ii), (iii), and (iv) only.
- (D) (i), (ii), and (iv) only.

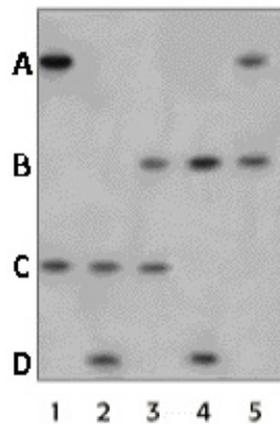
**2.5** Which of the following techniques were used by Messelson and Stahl to show that DNA replication is semi-conservative?

- (i) Growing *E. coli* in defined synthetic medium.
  - (ii) Use of [<sup>3</sup>H]-thymidine and analysis of the products by autoradiography.
  - (iii) Use of heavy nitrogen and analysis of the products by equilibrium density gradient centrifugation.
- (A) (i) and (ii) only.
  - (B) (i) and (iii) only.
  - (C) (ii) and (iii) only.
  - (D) (i), (ii), and (iii).

**2.6** Hormones are regulatory molecules secreted by the endocrine systems in vertebrates. Which of the following statements is **NOT TRUE** for hormones?

- (A) Hormones are secreted into the blood circulation system and they act on target organs.
- (B) Amino acids are precursors for some hormones.
- (C) Hormones can act on the organ that secretes them.
- (D) Hormones are always short-lived molecules.

2.7 Single locus probe DNA fingerprinting was done for a father and his four children. The resultant gel electrophoretic pattern is shown below.



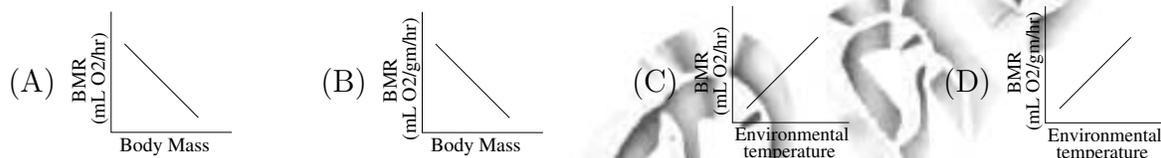
Which lanes contain the DNA of the children?

- (A) 1, 2, 3, and 4.
  - (B) 2, 3, 4, and 5.
  - (C) 1, 2, 4, and 5.
  - (D) 1, 2, 3, and 5.
- 2.8 When the lipid content of leaves from two plants was analysed, it was found that the thylakoid membranes of plant P1 had much more percentage of unsaturated fatty acids compared to plant P2. What can be deduced from this observation?
- (A) P1 is less likely to show low-temperature photoinhibition compared to P2.
  - (B) P1 will perform photosynthesis at much higher rates compared to P2.
  - (C) P2 will perform photosynthesis even at very low light intensities compared to P1.
  - (D) P2 will perform photosynthesis at very low partial pressures of  $\text{CO}_2$  compared to P1.
- 2.9 In an experiment, mature leaves on the plant were enclosed for a fixed amount of time in a transparent bag that had radioactive  $\text{CO}_2$ . In which part of the plant will maximum radioactivity be found after some time?
- (A) Actively growing leaves.
  - (B) Guard cells of all the leaves.
  - (C) In mature leaves.
  - (D) Senescing leaves and roots.

2.10 The major difference between the mosses and ferns is:

- (A) Ferns lack alternation of generation while mosses show the same.
- (B) Mosses are facultative aerobes while ferns are obligate aerobes.
- (C) Vascular bundles of ferns show xylem vessels while those of mosses lack it.
- (D) Sporophytes of ferns live much longer as compared to the sporophytes of mosses.

2.11 The basal metabolic rate (BMR) of a mammal is greatly influenced by its surface area to volume ratio and environmental temperature. Which graph correctly depicts this relationship?



2.12 Microbes “GE” and “OG” are facultative aerobes. Their doubling time is comparable to each other. When grown together, they do not affect the growth of each other. GE can utilise either glucose or ethanol as carbon source. OG can use only glucose as carbon source.

An inoculum containing equal number of GE and OG were grown together with glucose as the sole source of carbon. After a certain duration, only GE was present and not OG. The reason for this is that culture was grown under

- (A) aerobic condition throughout.
- (B) anaerobic condition throughout.
- (C) anaerobic condition initially and then under aerobic condition.
- (D) aerobic condition initially and then under anaerobic condition.

2.13 Lipid molecules have a polar headgroup and one or two hydrophobic tails. A variety of lipid assemblies are known: they can be lamellar (layer-like) or spherical and they can be monolayered or bilayered. A lipid molecule has one hydrophobic tail of length  $l$  nm. Suppose that  $l \ll a$ , where  $a$  nm<sup>2</sup> is the surface area of the headgroup. In an aqueous medium, molecules of such a lipid assemble to form

- (A) monolayered lamellar structures.
- (B) monolayered spherical structures.
- (C) bilayered lamellar structures.
- (D) bilayered spherical structures.

**2.14** Hemoglobin (Hb) transports oxygen from lungs to tissues. The partial pressure of oxygen in lungs is different from that in tissues. Each Hb can bind to up to four oxygen molecules. Suppose we have an equal number of Hb and oxygen molecules and all the oxygen molecules are in bound form. Then, which of the following is TRUE?

- (A) Almost all the Hb molecules have one bound oxygen molecule.
- (B) Nearly half of all the Hb molecules are each bound to two oxygen molecules.
- (C) Nearly one-fourth of all the Hb molecules are bound to four oxygen molecules each.
- (D) Most of the Hb molecules have one bound oxygen molecule each; the rest either have no bound oxygen or have two or more bound oxygen molecules.

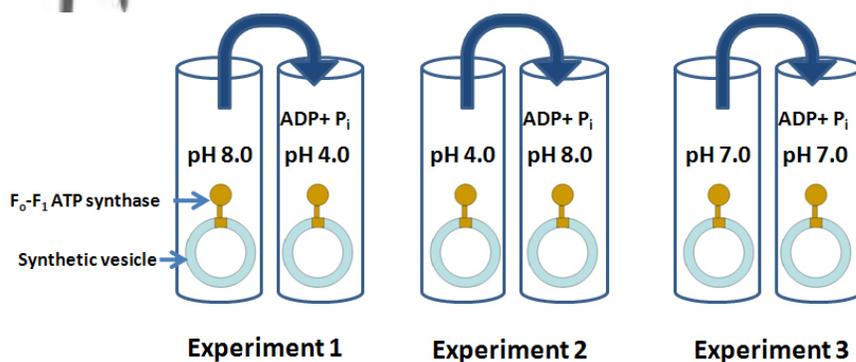
**2.15** Dr. Venkataraman Ramakrishnan is one of the three recipients of the 2009 Nobel Prize for Chemistry. He worked towards the elucidation of the three-dimensional structure of ribosomes. Ribosomes are involved in the biosynthesis of proteins and

- (A) one of their protein components is the catalyst.
- (B) one of their RNA components is the catalyst.
- (C) they bind to DNA for the purpose of protein synthesis.
- (D) they bind either to tRNA or to mRNA at any given time.

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*For questions 2.16 to 2.20 one or more than one of the 4 options may be correct. Your answer is regarded correct only if you choose all the correct option(s) and no incorrect option(s). A correct answer will earn 5 marks, a wrong answer or an unattempted question will earn 0 mark.*

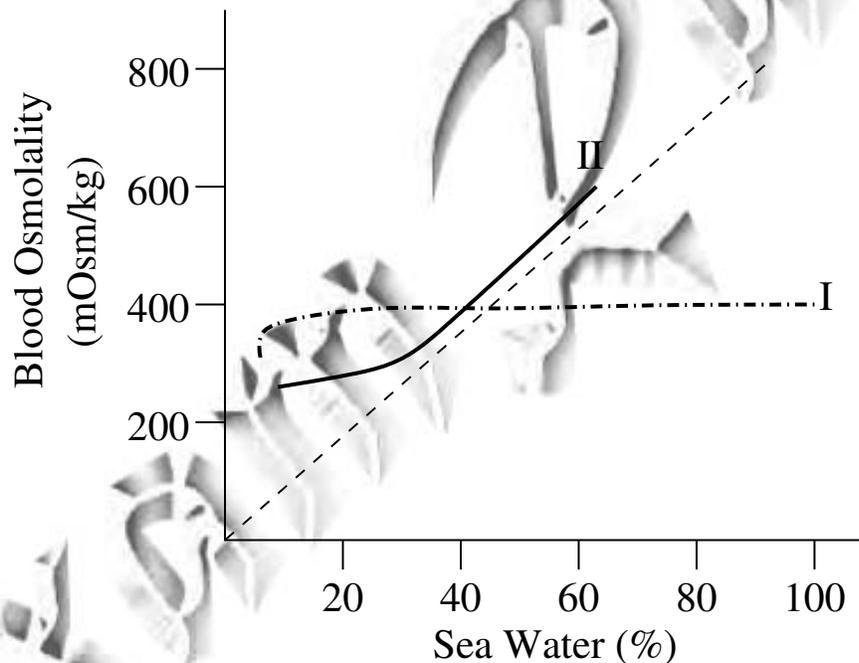
**2.16** Experiments 1, 2, and 3 were conducted wherein synthetic vesicles containing  $F_o$ - $F_1$  ATP synthase were prepared and incubated overnight in a tube. Subsequently, the vesicles were transferred to another tube which also contained ADP and  $P_i$  (inorganic phosphate).



Which of the following statements is/are true for the above experiments?

- (A) A proton gradient across the vesicular membrane will be present in both experiments 1 and 2 at the time of transfer.
- (B) As a consequence of the proton gradient, ATP will be synthesised in both experiments 1 and 2.
- (C) ATP will be synthesised in experiment 3 because  $F_0-F_1$  ATP synthase has the inherent property to catalyse the synthesis of ATP from ADP and  $P_i$ .
- (D) ATP will be synthesised in experiment 2 because the proton has to flow out of the vesicles through the  $F_0-F_1$  ATP synthase for ATP synthesis.

2.17 Blood osmolalities and the adaptable ranges of environmental salinities of two types of animals are shown in the graph below.



Mark the correct descriptions of these animals.

- (A) Type I animals maintain more or less stable internal osmolality by physiological means.
- (B) Type II animals maintain blood osmolality within narrow physiological ranges under different salinity conditions.
- (C) Type I animals are not likely to be found in estuaries or river mouths where fresh and salt water meet and the salinity fluctuates greatly.
- (D) Type II animals gain and lose water at equal rates and have no need to expend energy expelling water or salt from the body.

**2.18** Troglolites are animals that spend their entire life in a very stable, unchanging cave environment. Which of the following adaptations will be seen in these animals?

- (A) detrivory or carnivory.
- (B) loss of pigmentation.
- (C) reduced antennae.
- (D) reduced photoreceptors.

**2.19** The DNA content of individual cells and the number of cells in each phase of a “cell cycle” can be determined using flow cytometry. Which of the following combinations of “phase of a cell cycle and its corresponding DNA content” can be considered normal?

- (A) Diploid cells found in the  $G_0$  or  $G_1$  phase.
- (B) Cells with twice the normal DNA content in the early M phase.
- (C) Cells with intermediate amounts of DNA in the S phase.
- (D) Cells with twice the normal DNA content in the  $G_2$  phase.

**2.20** The regions of the *lac*-operon system of *E. coli* which contain the repressor gene (*I*), operator (*O*), and the structural gene (*Z*) for  $\beta$ -galactosidase are denoted by the symbols *a*, *b*, and *c*, but it is not known which symbol represents which region.

The following genotypes of *E. coli* for the *lac*-operon were generated. The activity of the enzyme  $\beta$ -galactosidase was measured separately in the absence and in the presence of the inducer. The activity of the enzyme  $\beta$ -galactosidase is shown by the symbol (++) and the absence of the activity is shown by the symbol (--).

Note that the order in which the symbols are written in the table is not necessarily the actual sequence in which they occur in the *lac*-operon. The symbols (+) and (-) written as superscript on *a*, *b* and *c* represent the wild type and mutant phenotypes, respectively.

Genotype	Inducer absent	Inducer present
$a^+b^+c^+$	--	++
$a^-b^-c^-$	--	--
$a^-b^+c^+$	++	++
$a^+b^+c^-$	++	++
$a^+b^-c^-$	--	--

Which of the following statements is/are correct for the above experiment?

- (A) The genotypes  $a^-b^+c^-$  and  $a^+b^-c^-$  are likely to produce similar results.
- (B) Mutations in the region *a* or *c* make the *lac*-operon constitutively expressed.
- (C) The symbol *b* represents the region containing the *lac Z* gene.
- (D) Whether region *a* contains the *lac I* or *lac O* gene cannot be inferred from these data.

## Section 3: CHEMISTRY

Marks for Section 3: 70

*This section contains 20 questions.*

*For questions 3.1 to 3.15 only one of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn (-1) mark, and an unattempted question will earn 0 mark.*

**3.1** The molecular orbital diagram of carbide ion ( $C_2^{2-}$ ) would show the following molecular orbital occupancy.

- (A)  $\sigma 1s^2$   $\sigma^* 1s^2$   $\sigma 2s^2$   $\sigma^* 2s^2$   $\pi 2p^4$
- (B)  $\sigma 1s^2$   $\sigma^* 1s^2$   $\sigma 2s^2$   $\sigma^* 2s^2$   $\pi 2p^4$   $\sigma 2p^2$
- (C)  $\sigma 1s^2$   $\sigma^* 1s^2$   $\sigma 2s^2$   $\sigma^* 2s^2$   $\pi 2p^2$   $\sigma 2p^2$   $\pi^* 2p^2$
- (D)  $\sigma 1s^2$   $\sigma^* 1s^2$   $\sigma 2s^2$   $\sigma^* 2s^2$   $\pi 2p^3$   $\sigma 2p^3$

**3.2** The activation energies of two reactions are  $E_a$  and  $E'_a$  with  $E_a > E'_a$ . If the temperature of the reaction systems is increased from  $T_1$  to  $T_2$ , predict which of the following alternatives is correct for the corresponding rate constants ( $k$ ) ( $k_1, k'_1$  at  $T_1$  and  $k_2, k'_2$  at  $T_2$ ).

- (A)  $k'_1/k_1 = k'_2/k_2$
- (B)  $k'_1/k_1 = 2k'_2/k_2$
- (C)  $k'_1/k_1 < k'_2/k_2$
- (D)  $k'_1/k_1 > k'_2/k_2$

**3.3** The alcohol that is chiral and can react with acidified dichromate under controlled conditions to give an aldehyde is

- (A) 2-ethyl-1-butanol
- (B) 2-pentanol
- (C) 2-methyl-1-butanol
- (D) 2,2-dimethyl-1-butanol

**3.4** Equal volumes of 1.0 M KCl (aq) and 1.0 M  $AgNO_3$  (aq) solutions are mixed. The depression of freezing point of the resulting solution (with respect to water) will be (assume  $K_f$  for water =  $1.86 \text{ K kg mol}^{-1}$  and Molarity  $\approx$  Molality)

- (A) 1.86 K
- (B) 3.72 K
- (C) 0.93 K
- (D) 7.44 K

3.5 Which of the following compounds is the strongest Bronsted base?

- (A) NaOH      (B) NaF      (C) NaCH<sub>3</sub>      (D) NaNH<sub>2</sub>

3.6 The standard electrode potentials for the following redox couples are given as:

$$E_0(\text{Fe}^{2+}/\text{Fe}) = x \text{ V}, E_0(\text{Fe}^{3+}/\text{Fe}^{2+}) = y \text{ V}.$$

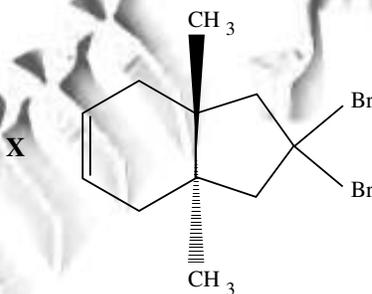
The potential  $E_0(\text{Fe}^{3+}/\text{Fe})$  (in V) will be

- (A)  $(2x + y)$       (B)  $(x + y)$       (C)  $(x + 2y)/3$       (D)  $(2x + y)/3$

3.7 The reaction of 2,4-hexadiene with one equivalent of bromine at 0° C gives a mixture of two compounds X and Y. X is 4,5-dibromo-2-hexene. Y is

- (A) 3,4-dibromohexane  
(B) 2,5-dibromo-3-hexene  
(C) 2,2-dibromo-4-hexene  
(D) 1,5-dibromo-3-hexene

3.8 For the compound X shown below, the number of asymmetric centres, and optical property are



- (A) 2, optically active.  
(B) 2, optically inactive.  
(C) 3, optically active.  
(D) 3, optically inactive.

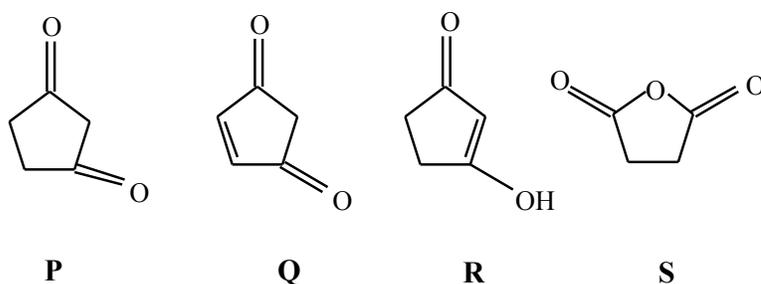
3.9 The correct order of decreasing atomic radii is

- (A) Rb > Cs > Na > F > Cl  
(B) Cs > Rb > Na > I > Cl  
(C) Cs > I > Rb > Cl > Na  
(D) I > Cl > F > Cs > Rb

3.10 The electron in H atom-like species makes a transition from an excited state to the ground state. In this process, its

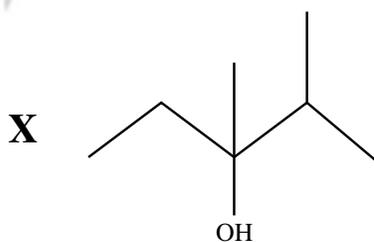
- (A) kinetic and total energies decrease but potential energy increases.
- (B) kinetic energy decreases whereas the potential and total energies remain the same.
- (C) kinetic energy increases but potential and total energies decrease.
- (D) kinetic, potential and total energies decrease.

3.11 For the four compounds P, Q, R, and S, shown below, the appropriate relationship is

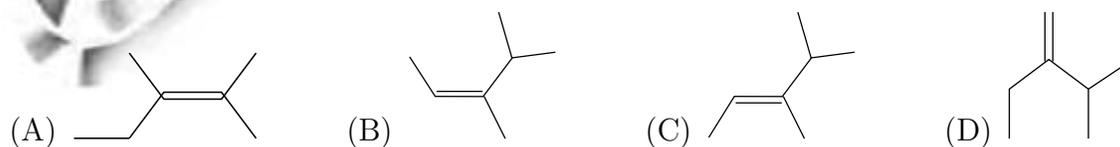


- (A) P and Q are tautomers.
- (B) P and R are tautomers.
- (C) P and S are isomers.
- (D) Q and R are resonance structures.

3.12 A certain compound X is shown below:



On heating with conc.  $\text{H}_2\text{SO}_4$ , it gives mainly



- 3.13 The species having tetrahedral shape is  
(A)  $[\text{PdCl}_4]^{2-}$  (B)  $[\text{Ni}(\text{CN})_4]^{2-}$  (C)  $[\text{Pd}(\text{CN})_4]^{2-}$  (D)  $[\text{NiCl}_4]^{2-}$

- 3.14  $\text{MgSO}_4$ , on reaction with  $\text{NH}_4\text{OH}$  and  $\text{Na}_2\text{HPO}_4$ , forms a white crystalline precipitate. The precipitate is  
(A)  $\text{Mg}(\text{NH}_4)\text{PO}_4$  (B)  $\text{Mg}_3(\text{PO}_4)_2$  (C)  $\text{Mg}(\text{OH})_2$  (D)  $\text{MgHPO}_4$

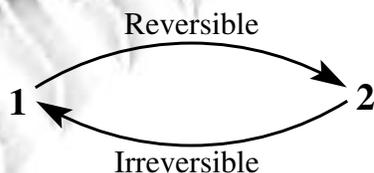
- 3.15 The ratio of molar heat capacities at constant pressure ( $C_p$ ) and at constant volume ( $C_v$ ) of nitrogen gas at ordinary temperatures is close to  
(A) 11/9 (B) 9/7 (C) 7/5 (D) 5/3

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For questions 3.16 to 3.20 one or more than one of the 4 options may be correct. Your answer is regarded correct only if you choose all the correct option(s) and no incorrect option(s). A correct answer will earn 5 marks, a wrong answer or an unattempted question will earn 0 mark.

- 3.16 The compound(s) which will be oxidised by  $\text{O}_3$  is/are  
(A) KI (B)  $\text{FeSO}_4$  (C)  $\text{KMnO}_4$  (D)  $\text{K}_2\text{MnO}_4$

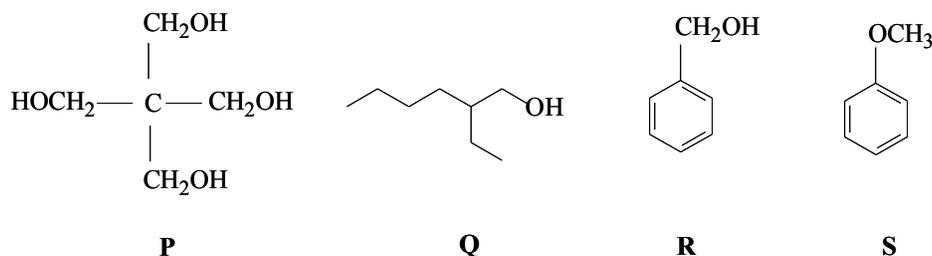
- 3.17 For the cyclic process considered below, which of the following relations is/are correct?



- (A)  $\Delta S = S_2 - S_1 = \int_1^2 \left( \frac{dq_{\text{rev}}}{T} \right)$   
(B)  $\Delta S = S_1 - S_2 = \int_2^1 \left( \frac{dq_{\text{irr}}}{T} \right)$   
(C)  $\Delta S_{\text{cycle}} = 0 = \int_1^2 \left( \frac{dq_{\text{rev}}}{T} \right) + \int_2^1 \left( \frac{dq_{\text{irr}}}{T} \right)$   
(D)  $\Delta S_{\text{cycle}} = 0 > \int_1^2 \left( \frac{dq_{\text{rev}}}{T} \right) + \int_2^1 \left( \frac{dq_{\text{irr}}}{T} \right)$

- 3.18 Amongst the following, the species having one unpaired electron is/are  
(A) BN (B)  $\text{O}_2^-$  (C)  $\text{F}_2^-$  (D) XeF

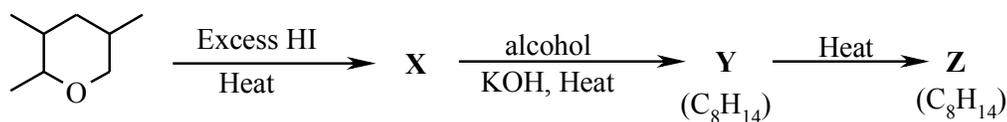
3.19 The compounds P, Q, R, and S, shown below, are commercially very important.



Which of the following statements is/are true regarding the synthesis of these compounds?

- (A) P can be prepared by the reaction of acetaldehyde and formaldehyde in the presence of an alkali.
- (B) Q can be prepared from acetaldehyde alone through a series of reactions.
- (C) R is formed when a mixture of benzaldehyde and formaldehyde is treated with an alkali.
- (D) S is formed when phenol is reacted with formaldehyde in the presence of  $\text{HBF}_4$ .

3.20 Consider the sequence of reactions shown below.



Which of the following statements is/are true?

- (A) X is 5-iodo-2,4-dimethylhexan-1-ol.
- (B) Y on reaction with chlorine gives a tetrachloro derivative.
- (C) X is a diiodo compound.
- (D) Z is 2,4-dimethyl-1,3-hexadiene.

## Section 4: MATHEMATICS

Marks for Section 4: 70

This section contains 20 questions.

For questions 4.1 to 4.15 **only one** of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn (-1) mark, and an unattempted question will earn 0 mark.

4.1 If  $a, b, c, d$  are positive real numbers such that  $a+b+c+d = 2$ , then  $\lambda = (1+a+b)(1+c+d)$  satisfies

- (A)  $2 < \lambda \leq 3$       (B)  $3 < \lambda \leq 4$       (C)  $4 < \lambda \leq 5$       (D)  $5 < \lambda \leq 6$

4.2 In an ellipse, the ratio of the area of the rectangle formed by the end points of its each latus rectum to that of the ellipse is  $1/\pi$ . The eccentricity of the ellipse is

- (A)  $\frac{1}{\sqrt{2}}$       (B)  $\frac{1}{2}$       (C)  $\frac{2 \pm \sqrt{3}}{4}$       (D)  $\frac{\sqrt{3} \pm 1}{2\sqrt{2}}$

4.3 When all possible four-member committees were formed from a set of people, it was found that exactly  $1/20$ -th of the committees contained the same two members. The number of persons in the set was

- (A) 20      (B) 17      (C) 16      (D) 15

4.4 The value of the integral

$$\int_0^{\infty} \frac{5}{(x^2 + 4)(x^2 + 9)} dx$$

is

- (A)  $\pi$       (B)  $\frac{\pi}{12}$       (C)  $\frac{\pi}{6}$       (D)  $\frac{\pi}{4}$

4.5 If  $\sin \alpha \sin 2\alpha \sin 3\alpha \sin 4\alpha$  is written as a polynomial in  $\sin^2 \alpha$ , the sum of the coefficients of the polynomial is

- (A) 0      (B) 1      (C) -1      (D) 4

4.6 If  $X$  and  $Y$  are two singular matrices such that  $XY = Y$  and  $YX = X$ , then  $X^2 + Y^2$  equals

- (A)  $X + Y$       (B)  $XY$       (C)  $YX$       (D)  $2(X + Y)$

4.7 Let  $ABCD$  be a rectangle and a semicircle be drawn with  $AB$  as the diameter, externally to the rectangle. If the perimeter of the figure so obtained is a constant  $p$  and the figure has the maximum possible area, then the ratio of the area of the rectangular portion to the area of the rest of the figure is

- (A)  $2 : \pi$       (B)  $\pi : 2$       (C)  $\pi : 4$       (D)  $4 : \pi$

4.8 Let  $a, b$  be positive real numbers such that  $10 < a < b$ . Then the number of positive integers  $b$  such that (i)  $10, a, b$  are in geometric progression, and (ii)  $10, a, b$  form the sides of a triangle is

- (A) 15      (B) 16      (C) 17      (D) 26

4.9 The set  $\{z \in \mathbb{C} : |z + 1| = |z - 1|\}$  is

- (A) a circle.  
 (B) an ellipse.  
 (C) the imaginary axis.  
 (D) the empty set.

4.10 Let  $A, B$  be points of intersection of the ellipse  $\frac{x^2}{36} + \frac{y^2}{25} = 1$  and the parabola  $y^2 = 20x$ . The area bounded by the parabola and the line  $AB$  lies in the interval

- (A) (6, 7)      (B) (7, 8)      (C) (8, 9)      (D) (9,  $\infty$ )

4.11 Let  $f(x), g(x), h(x)$  be real non-constant polynomials such that

$$\begin{aligned} f(g(x)) &= g(x)h(x), \\ g(h(x)) &= h(x)f(x), \\ h(f(x)) &= f(x)g(x). \end{aligned}$$

The sum of the degrees of the polynomials  $f(x), g(x), h(x)$  is

- (A) 6      (B) 9      (C) 12      (D) not determinable.

4.12  $\lim_{x \rightarrow 0} \frac{1}{x^2} \int_0^x \frac{t dt}{t^4 + 1}$  is equal to

- (A) 0  
 (B) 1  
 (C)  $\frac{1}{2}$   
 (D) does not exist.

4.13 If  $\omega$  is an imaginary cube root of unity then the system of linear equations

$$\begin{aligned}2x + 2\omega y - \omega^2 z &= 0 \\x + y + z &= 0 \\x - y &= 0\end{aligned}$$

- (A) has only the trivial solution:  $(x, y, z) = (0, 0, 0)$ .
- (B) has only one non-trivial solution.
- (C) has infinitely many non-trivial solutions.
- (D) has no solution at all.

4.14 The coefficient of  $x^{100}$  in  $\frac{(1+x)^{100}(1-x^{101})}{(1-x)}$  is

- (A) 1
- (B) 200
- (C)  ${}^{100}C_2$
- (D)  $2^{100}$

4.15 The image of the function  $\tan^{-1}(2x^2)$  is

- (A)  $[0, \frac{\pi}{2})$
- (B)  $(-\frac{\pi}{2}, \frac{\pi}{2})$
- (C)  $[0, \frac{\pi}{2}]$
- (D)  $(0, \frac{\pi}{2})$

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*For questions 4.16 to 4.20 one or more than one of the 4 options may be correct. Your answer is regarded correct only if you choose all the correct option(s) and no incorrect option(s). A correct answer will earn 5 marks, a wrong answer or an unattempted question will earn 0 mark.*

4.16 Let  $f(x) = \min\{x, x^2\}$ , where  $x$  belongs to  $\mathbb{R}$ . The function  $f(x)$  is

- (A) continuous everywhere.
- (B) continuous except 0 and 1.
- (C) differentiable everywhere.
- (D) differentiable everywhere except 0 and 1.

**4.17** Let  $f(x, y) = x^2 - axy + y^2$  be a quadratic polynomial in real variables  $x, y$ ,  $a$  being a real constant. Then

- (A)  $f(x, y)$  is nonnegative for all real values of  $x$  and  $y$ , if  $-1 \leq a \leq 1$ .
- (B) there are real values of  $a$  outside the interval  $[-1, 1]$  for which  $f(x, y)$  is again nonnegative for all real values of  $x$  and  $y$ .
- (C) the graph of  $f(x, y) = 0$  is a single point in the  $xy$ -plane, when  $a = 1$ .
- (D) the graph of  $f(x, y) = c$  is a circle, ellipse or a hyperbola but never a pair of lines for any given real numbers  $a$  and  $c$ , where  $c$  is positive.

**4.18** Let  $ABC$  be an acute-angled triangle;  $L, M, N$  be the feet of perpendiculars respectively from  $A, B, C$  to the opposite sides;  $D, E, F$  be the midpoints of the sides  $BC, CA, AB$  respectively; and  $I_1, I_2, I_3$  be the ex-centres of triangle  $ABC$ . Then

- (A) the ortho-centre of triangle  $ABC$  is the in-centre of triangle  $LMN$ .
- (B) The circum-centre of triangle  $ABC$  is the ortho-centre of triangle  $DEF$ .
- (C) The in-centre of triangle  $ABC$  is the ortho-centre of triangle  $I_1I_2I_3$ .
- (D) the centroid of triangle  $ABC$  is the centroid of triangle  $DEF$ .

**4.19** Consider the second order ordinary differential equation:  $y'' - y = 0$ . Then

- (A) there is a non-zero solution of the equation which is bounded on  $\mathbb{R}$ .
- (B) any solution of this equation may be represented by  $y = ae^x + be^{-x}$ , where  $a$  and  $b$  are two suitable constants.
- (C) any solution of this equation may also be represented by  $y = a \cosh x + be^{-x}$ , where  $a$  and  $b$  are two suitable constants.
- (D) there are infinitely many solution curves  $y = ae^x + be^{-x}$ , passing through  $(0, 0)$  in the  $xy$ -plane.

**4.20** If  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$  are unit vectors given by

$$\mathbf{v}_1 = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$$

$$\mathbf{v}_2 = b\mathbf{i} + c\mathbf{j} + a\mathbf{k}$$

$$\mathbf{v}_3 = c\mathbf{i} + a\mathbf{j} + b\mathbf{k},$$

where  $a, b, c$  are nonnegative real numbers, and  $\mathbf{v}_\alpha \cdot \mathbf{v}_\beta = 0$ , for  $\alpha \neq \beta$ , then

- (A)  $|\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3| = 1$
- (B)  $a + b + c = 1$
- (C)  $\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3 = \mathbf{0}$
- (D)  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$  are coplanar.

## Section 5: PHYSICS

Marks for Section 5: 70

*This section contains 20 questions.*

*For questions 5.1 to 5.15 only one of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn (-1) mark, and an unattempted question will earn 0 mark.*

**5.1** A car of mass  $m$ , moving to the right, collides elastically with a truck of mass  $3m$  moving to the left with equal speed. The collision is head-on. After the collision

- (A) the car comes to a stop and the truck moves to the right.
- (B) both the car and the truck move to the left.
- (C) both the car and the truck come to a stop.
- (D) the truck comes to a stop and the car moves to the left.

**5.2** A non-conducting sphere of radius  $R$  has a positive charge  $Q$  uniformly distributed over its entire volume. A smaller, concentric, spherical volume of radius  $r$  ( $r < R$ ) is scooped out of the sphere. The magnitude of the electric field  $E$  at a point inside the body at a distance  $x$  from the centre ( $r < x < R$ ) is ( $K = 1/(4\pi\epsilon_0) = \text{constant}$ ):

- (A)  $\frac{KQ}{x^2}$
- (B)  $\frac{KQ r^3}{x^2 R^3}$
- (C)  $\frac{KQ}{x^2} \left( \frac{x^3 - r^3}{R^3} \right)$
- (D) 0

**5.3** Unpolarised light of intensity  $I$  falls on a polaroid. The emerging light is passed through another polaroid whose axis makes an angle of  $30^\circ$  with respect to the first one. The final emergent light has an intensity of

- (A)  $3I/4$       (B)  $3I/8$       (C)  $I/8$       (D)  $I/2$

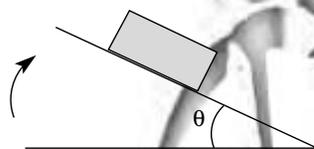
**5.4** A parallel-plate capacitor is formed with plates of area  $A$ , and separation between the plates  $d$ . A second parallel-plate capacitor has plates of area  $A$  and separation  $d/2$ . The two capacitors are connected in series. The resultant capacitance is:

- (A)  $\frac{\epsilon_0 A}{d}$       (B)  $2 \frac{\epsilon_0 A}{d}$       (C)  $\frac{3 \epsilon_0 A}{2 d}$       (D)  $\frac{2 \epsilon_0 A}{3 d}$

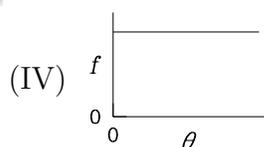
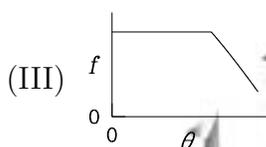
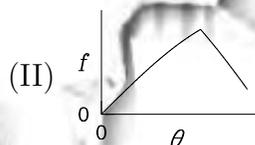
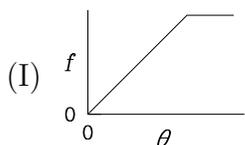
**5.5** We want to determine the value of Planck's constant,  $h$ , by doing an experiment using the photoelectric effect. This experiment is performed by varying both the frequency of the incident light as well as its intensity, and measuring both the voltage and the current. In order to calculate  $h$ , it would be best to plot

- (A) current as a function of frequency.
- (B) current as a function of intensity.
- (C) stopping potential as a function of intensity.
- (D) stopping potential as a function of frequency.

**5.6** A block is placed on a rough inclined plane. The angle of the incline,  $\theta$ , is slowly increased, starting from the horizontal position. At a certain angle, the block starts to slide along the plane. The angle of the incline is increased further.



Consider the following graphs:



Which of the above graphs correctly depicts the variation of the frictional force,  $f$ , exerted by the plane on the block, as a function of  $\theta$ ? (Assume that the block does not topple.)

- (A) (I)
- (B) (II)
- (C) (III)
- (D) (IV)

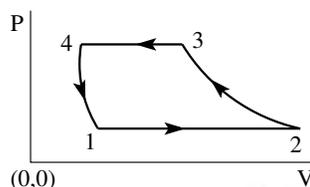
**5.7** In a repeat of one of Michael Faraday's classic experiments, a solenoid of length 10 cm with 200 turns and radius 1 cm is surrounded by another solenoid of the same length, but with 400 turns and radius 2 cm. The two solenoids are not in contact. A current is switched on through the inner solenoid. Suppose that the current increases at the rate of  $100 \text{ A s}^{-1}$  during the switch-on. What is the emf induced in the outer solenoid? (Assume that a current carrying solenoid has no magnetic field outside it.)

- (A) Zero
- (B) 31.6 mV
- (C) 126.4 mV
- (D) 316 mV

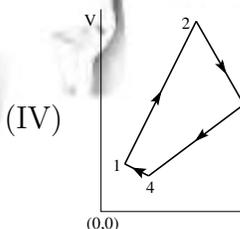
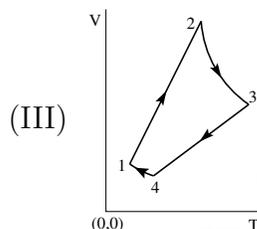
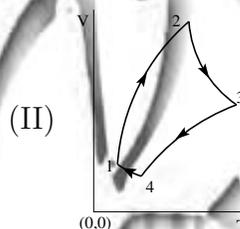
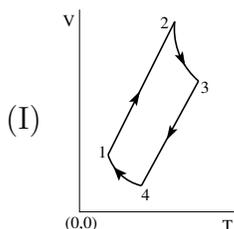
5.8 A shaft is rotating at a speed of 4000 revolutions per minute. If the power expended in driving the shaft is 12 kW, the magnitude of the driving torque is:

- (A)  $90/\pi$  N m      (B) 90 N m      (C) 180 N m      (D)  $36/\pi$  N m

5.9 An ideal gas undergoes a cyclic process  $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$  depicted on the  $P$ - $V$  diagram, where the processes  $1 \rightarrow 2$  and  $3 \rightarrow 4$  are isobaric, and  $2 \rightarrow 3$  and  $4 \rightarrow 1$  are adiabatic.



Consider the following diagrams on the  $V$ - $T$  plane.



The correct representation of the cycle is given by the graph

- (A) (I)      (B) (II)      (C) (III)      (D) (IV)

5.10 An astronomical telescope has an objective of focal length 100 cm and a diameter  $D$ . The eyepiece has a focal length of 2.5 cm and the eye is held close to the eyepiece for viewing. Let  $M_1$  be the magnifying power when the telescope is in normal adjustment (i.e., final image is at infinity), and  $M_2$  be the magnifying power when the final image is at the least distance of distinct vision (25 cm). Then

- (A)  $M_1 = M_2$ .  
 (B)  $M_1$  is slightly greater than  $M_2$ .  
 (C)  $M_1$  is slightly less than  $M_2$ .  
 (D) The ratio  $M_1/M_2$  depends on  $D$ .

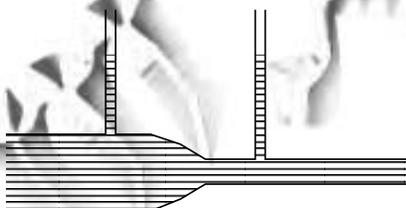
**5.11** The wall of a concert hall consists of a uniform brick wall of thickness 25 cm on the outside. On the inside, it is completely covered with a uniform wooden layer of thickness 5 cm. The thermal conductivities of brick and wood are, respectively,  $0.60 \text{ Wm}^{-1}\text{K}^{-1}$  and  $0.06 \text{ Wm}^{-1}\text{K}^{-1}$ . The thermal conductivity of the composite wall of the hall is

- (A)  $0.51 \text{ Wm}^{-1}\text{K}^{-1}$
- (B)  $0.24 \text{ Wm}^{-1}\text{K}^{-1}$
- (C)  $0.33 \text{ Wm}^{-1}\text{K}^{-1}$
- (D)  $0.15 \text{ Wm}^{-1}\text{K}^{-1}$

**5.12** Assume that the battery in your mobile phone can be treated as a simple capacitor that stores charge, and discharges when you use the phone. Suppose you start to charge a completely discharged phone and it takes 100 minutes to charge it to half its full capacity. How long more will you have to charge it to reach  $3/4$ th its full capacity?

- (A) 25 minutes
- (B) 50 minutes
- (C) 100 minutes
- (D) 200 minutes

**5.13** In the tube shown below, the wider part has cross-sectional area  $A$ , and the narrower part has cross-sectional area  $A/3$ . When water inside the tube is at rest, the vertical manometers show the same height of water, as shown.



When water is made to flow steadily through the tube, the difference in heights of water in the two manometers is 40 cm. Neglecting viscosity effects, the speed of water flow in the narrower part of the tube is (taking  $g = 10 \text{ m s}^{-2}$ )

- (A)  $1 \text{ m s}^{-1}$
- (B)  $0.5 \text{ m s}^{-1}$
- (C)  $2 \text{ m s}^{-1}$
- (D)  $3 \text{ m s}^{-1}$

**5.14** Electrons accelerated in an X-ray tube strike a nickel target ( $Z_{\text{Ni}} = 28$ ) and come to rest. The resulting continuous X-ray spectrum has a minimum wavelength given by  $\lambda_{\text{min}} = 3.0 \times 10^{-11} \text{ m}$ . If the nickel target is replaced by a molybdenum target ( $Z_{\text{Mo}} = 42$ ), the minimum wavelength will be

- (A)  $3.0 \times 10^{-11} \text{ m}$ .
- (B)  $2.0 \times 10^{-11} \text{ m}$ .
- (C)  $4.5 \times 10^{-11} \text{ m}$ .
- (D)  $6.8 \times 10^{-11} \text{ m}$ .

**5.15** An excited state of doubly ionised Lithium ( $\text{Li}^{2+}$ ) has an orbital radius that is about 1.33 times that of the ground state of hydrogen (H) (in Bohr's theory). The ratio of energy of the two states,  $E(\text{Li}^{2+})/E(\text{H})$ , is

- (A) 2.25      (B) 4.5      (C) 1      (D) 9

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*For questions 5.16 to 5.20 one or more than one of the 4 options may be correct. Your answer is regarded correct only if you choose all the correct option(s) and no incorrect option(s). A correct answer will earn 5 marks, a wrong answer or an unattempted question will earn 0 mark.*

**5.16** A machine gun mounted on a stationary vehicle fires bullets at a fixed target at the speed of  $100 \text{ m s}^{-1}$ . The sound waves produced by the firing travel at the speed of  $330 \text{ m s}^{-1}$ . The vehicle now moves toward the target with a speed of  $20 \text{ m s}^{-1}$ . Which of the following statements is/are true?

- (A) The fractional change in frequency ( $\delta\nu/\nu$ ) of the sound of firing coming from the moving vehicle, as measured by a stationary ground observer, is approximately  $2/33$ .
- (B) For a stationary ground observer, the speed of sound from the moving vehicle is  $350 \text{ m s}^{-1}$  and that of the bullets is  $120 \text{ m s}^{-1}$ .
- (C) An echo is heard when the sound waves are reflected by the target. For a ground observer stationed close to the target, the intensity of echo is more than the intensity of sound received directly from the machine gun.
- (D) Suppose the vehicle now stops and the observer moves towards the vehicle at the speed of  $20 \text{ m s}^{-1}$ . In this case the moving observer will find the wavelength of the sound of firing to be the same as when both he and the vehicle were at rest.

**5.17** A solid sphere and a solid cylinder, both of mass  $M$  and radius  $R$ , are released from rest at the same height on an inclined plane. Both roll down the plane without slipping. Which of the following statements is/are true?

- (A) The moment of inertia of the rolling sphere is greater than the moment of inertia of the rolling cylinder.
- (B) The frictional force on the cylinder is greater than that on the sphere.
- (C) The sphere will reach the bottom before the cylinder.
- (D) When the two objects reach the bottom, the speed of the cylinder will be greater than that of the sphere.

**5.18** An electron moving in the positive  $x$ -direction, enters a region that has a constant electric field pointing in the positive  $y$ -direction, and a constant magnetic field pointing in the positive  $z$ -direction. Then, which of the following statements is/are true?

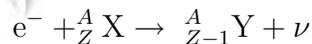
- (A) The electron will be undeflected and travel in a straight line.
- (B) The net work done on the electron is zero.
- (C) The velocity component of the electron in the negative  $y$ -direction will increase continuously.
- (D) The electron will be confined in the  $xy$ -plane.

**5.19** Consider positron ( $e^+$ ) emission from a nucleus at rest:



Which of the following statements is/are correct?

- (A) The emitted  $e^+$  has a continuous energy spectrum since the energy released in the decay is shared by  $e^+$  and  $\nu$ .
- (B) If there were no additional particle  $\nu$  emitted in the decay, the energy of the emitted  $e^+$  would be always equal to  $(M_X - M_Y)c^2$ , if the recoil energy of Y is neglected. ( $M_X$  and  $M_Y$  are rest masses of the nuclei X and Y, respectively, and  $c$  is the speed of light in vacuum.)
- (C) The emitted neutrino ( $\nu$ ) always has non-relativistic speed (i.e., speed much less than  $c$ ) due to its very small rest mass.
- (D) Consider the competing process to the positron emission, in which an electron ( $e^-$ ) from an inner orbit of the atom (K shell) is captured by the nucleus and a neutrino is emitted.



This process is energetically allowed if  $e^+$  emission process is energetically allowed. (“Energetically allowed” means “allowed by the principle of conservation of energy and linear momentum”.)

**5.20** One mole of Helium gas (assumed ideal) is subjected to a process such that  $PT^{-2}$  remains constant in the process, where  $P$  and  $T$  represent its pressure and temperature, respectively. Which of the following statements is/are true?

- (A) The heat capacity of the gas in this process is  $R/2$ .
- (B) If the temperature of the gas is doubled in the process, the entropy of the gas changes by  $0.5R \ln 2$ .
- (C) In this process, work of an amount  $R\Delta T$  is done on the gas, where  $\Delta T = T_{\text{final}} - T_{\text{initial}}$ .
- (D) This process is represented by a straight line in the  $V$ - $T$  diagram.